



HAL
open science

High order time-stepping methods for cardiac electrophysiology models

Charlie Douanla Lontsi, Yves Coudière, Charles Pierre

► **To cite this version:**

Charlie Douanla Lontsi, Yves Coudière, Charles Pierre. High order time-stepping methods for cardiac electrophysiology models. IHU Liryc-Workshop 2016, Sep 2016, Bordeaux, France. hal-01406536

HAL Id: hal-01406536

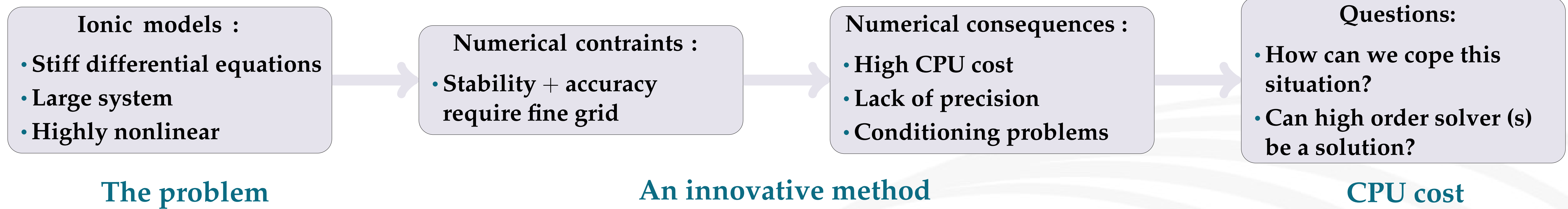
<https://inria.hal.science/hal-01406536>

Submitted on 1 Dec 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Problematic of the time numerical integration in cardiac electrophysiology



EXAMPLE IONIC MODEL (TNNP)

Behavior of the physiological parameters :

- Different time and space scales.
- Fast and slow variables.
- Stiff wave fronts.

Numerical consequences :

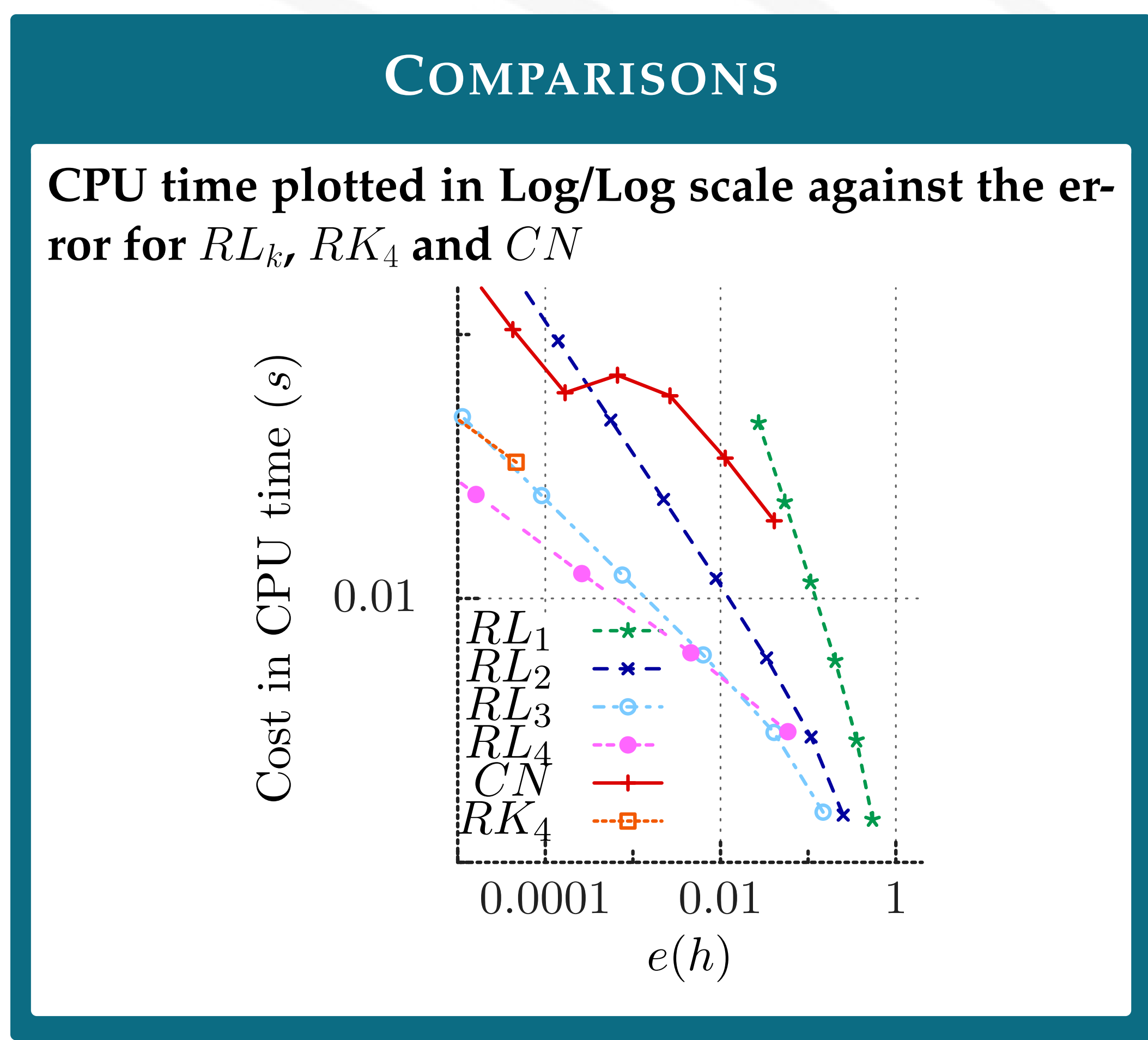
- Numerical instabilities.
- High computational cost.
- Significant loss of accuracy

HIGH-ORDER RUSH LARSEN METHODS

Consider the problem,
 $y' = a(t, y)y + b(t, y), y(0) = y_0,$
 transformed in each time discretization interval $(t_n, t_{n+1}]$ into,
 $y' = \alpha_n y + c_n(t, y),$
 with $c_n(t, y) = (a(t, y) - \alpha_n)y + b(t, y)$ and α_n a stabilizer. For $t \in (t_n, t_{n+1}]$, the exact solution of this problem satisfies the *variation of constant* formula
 $y(t) = e^{A(t)}(y_n + \int_{t_n}^t e^{-A(\tau)} c_n(\tau, y) d\tau),$
 with $A(t) = \alpha_n(t - t_n)$. If we set $t = t_{n+1}$ and approximate c_n by a constant β_n , we obtain the following definition of RL_k
 $y_{n+1} = y_n + h\varphi_1(\alpha_n h)(\alpha_n y_n + \beta_n).$
 $\varphi_1(z) = \frac{e^z - 1}{z}, \alpha_n$ and β_n are set so that the convergence order k is ensured.

RUSH LARSEN ORDER 3

$$\alpha_n = \frac{1}{12}(23a_n - 16a_{n-1} + 5a_{n-2}),$$

$$\beta_n = \frac{1}{12}(23b_n - 16b_{n-1} + 5b_{n-2}) + \frac{h}{12}(a_n b_{n-1} - a_{n-1} b_n).$$


Scheme properties

ADVANTAGES AND CONVERGENCE

- Explicit k -multi-step method:
 $y_n, y_{n-1}, \dots, y_{n-k+1} \rightarrow y_{n+1}.$
- Stability: The same critical time-step as the implicit schemes.
- Easy to implement.
- Cost: one evaluation of the model at every time step (minimal).

THEOREM. The RL_k scheme is stable under perturbation and converges with an error of order k , under the assumptions that $a(t, y)$ and $b(t, y)$ in the previous problem are C^k functions and its solution $y(t)$ is defined in $[0, T)$.

NUMERICAL ILLUSTRATION (Relative error for the BR model in Log/Log scale)

State of the art

STABILIZATION PRINCIPLE AND SOME STABILIZED SOLVERS

Equation on gating variables :
 $\frac{dW}{dt} = \frac{W_\infty(V) - W}{\tau(V)}$

Forward Backward scheme :
 $\frac{W^{n+1} - W^n}{\Delta t} = \frac{W_\infty(V^n) - W^{n+1}}{\tau(V^n)}$

This scheme allows large Δt , linear part = $\frac{-1}{\tau(V)}$.
Linear part known at time $t_n \Rightarrow$ Stabilization.

ORDER ONE SOLVERS :

- Forward backward Euler
- RL1: Rush Larsen (1978).

ORDER TWO SOLVER :

- RL2: Perego and Veneziani (2009).

HIGH ORDER METHODS :

- Exponential integrator of Adams type: Norsett (1969).
- Exponential Runge kutta and exponential multi-step (the last ten years).

Discussion and conclusion

DISCUSSION AND CONCLUSION

DISCUSSION:

- Ionic models are very stiff \Rightarrow classical explicit numerical solver must use small time-steps for stability \Rightarrow **Large CPU time.**
- **High order RL_k remain stable for large time-steps. Their critical time-steps don't depend on the stiffness but on the stabilizer's choice.**

CONCLUSION:

- We found that high order RL_k is a good alternative to improve the accuracy with negligible additional cost w/r to RL1 scheme.
- High order RL_k allow the use of large time steps unlike classical explicit schemes. They are suitable for solving stiff ODE.
- High order schemes in cardiac electrophysiology may allow more reliable simulations of long lasting events.

References

- [1] Coudiere, Y. and Douanla Lontsi, C. and Pierre, C. High order Rush Larsen solver for stiff ODEs HAL Preprint (2015).
- [2] Rush and Larsen. A practical algorithm for solving dynamic membrane equations. IEEE Trans Biomed Eng 1978.
- [3] Hochbruck. and Ostermann. Exponential multistep methods of Adams-type BIT Numer Math (2011).
- [4] Beeler, G.W. and Reuter, H. Reconstruction of the Action Potential of Ventricular Myocardial Fibers J. Physiol. (1977).
- [5] Douanla Lontsi, C. and Coudière, Y. and Pierre, C. Efficient High Order Schemes for Stiff ODEs in Cardiac Electrophysiology CARI (2016).