

Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing

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Application Perspectives

Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing

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- Main goal: To obtain a better characterization of the Earth's subsurface
- How: Recording borehole resistivity measurements

- Procedure:
 - Well
 - Transmitters
 - Receivers



Numerical Results

Second Approach

Approach Application

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MOTIVATION

- Practical Difficulties:
 - It is not easy to drill a borehole
 - It may collapse
- Practical Solutions:
 - Use a metallic casing
 - Surround with a cement layer
- Problem solved, but...



MOTIVATION

- Practical Difficulties:
 - It is not easy to drill a borehole
 - It may collapse
- Practical Solutions:
 - Use a metallic casing
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Second Approach

• Problem solved, but... Numerical problems due to the high conductivity and thinness of the casing

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CONFIGURATION OF INTEREST



We reduce the 3D problem to a 2D problem due to the axysimmetric configuration.



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REALISTIC SCENARIO



• Conductivity and casing width:

$$\left(\begin{array}{c} \varepsilon &= 1.27 \mathrm{e} - 2m \\ \sigma_c &= 4.34 \mathrm{e6} \ \Omega^{-1} m^{-1} \end{array} \right)$$

$$\Rightarrow \sigma_c \approx \varepsilon^{-3}$$

• Classical approach:

$$\sigma_{c} = \alpha \qquad \alpha \in \mathbb{R}$$

• High conductive case:

$$\sigma_{c} = \alpha \varepsilon^{-3} \qquad \alpha \in \mathbb{R}$$

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MODEL PROBLEM - 3D

STATIC ELECTRIC POTENTIAL

 $\operatorname{div}\left(\left(\sigma-i\epsilon\omega\right)\nabla u\right)=f \text{ in } \Omega$

Where f is a known data, σ is piecewise constant



$$\sigma = \begin{cases} \sigma_i & \text{in } \Omega_i^{\varepsilon} \\ \sigma_c = \alpha \varepsilon^{-3} & \text{in } \Omega_c^{\varepsilon} \\ \sigma_e & \text{in } \Omega_e^{\varepsilon} \end{cases}$$

and the solution is expressed as

$$u = \begin{cases} u_i & \text{ in } \Omega_i^{\varepsilon} \\ u_c & \text{ in } \Omega_c^{\varepsilon} \\ u_e & \text{ in } \Omega_e^{\varepsilon} \end{cases}$$

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MODEL PROBLEM - 2D

Reference problem (P) set in the domain $\Omega = \Omega_i^{\varepsilon} \cup \Omega_c^{\varepsilon} \cup \Omega_e^{\varepsilon} \cup \Gamma_i^{\varepsilon} \cup \Gamma_e^{\varepsilon}$.



$$\sigma_i \Delta u_i = f_i$$
 in Ω_i^{ε}

$$\sigma_c \Delta u_c = 0$$
 in Ω_c^{ε}

$$\sigma_e \Delta u_e = f_e \qquad \text{ in } \quad \Omega_e^{\varepsilon}$$

$$u_i = u_c$$
 on Γ_i^{ε}

$$u_c = u_e$$
 on Γ_e^{ε}

$$\sigma_i \partial_n u_i = \sigma_c \partial_n u_c$$
 on Γ_i^{ε}

$$\sigma_{c}\partial_{n}u_{c} = \sigma_{e}\partial_{n}u_{e} \quad \text{on} \quad \Gamma_{e}^{\varepsilon}$$

u = 0 on $\partial \Omega$

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EXISTENCE AND UNIQUENESS

Theorem 1

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^{\varepsilon})$ and $f_e \in L^2(\Omega_e^{\varepsilon})$, then $\exists ! u \in H_0^1(\Omega)$ solution of (P) and

$$\|u\|_{1,\Omega} \leq C\left(\|f_i\|_{0,\Omega_i^{\varepsilon}} + \|f_e\|_{0,\Omega_e^{\varepsilon}}\right).$$

Proof:

- Derive variational formulation, for all $v \in H_0^1(\Omega)$, a(u, v) = b(v).
- Prove that a is coercive in $H_0^1(\Omega)$.
- Apply the Lax-Milgram Lemma.

DEFINITIONS

Definition 1

We define the jump and mean value of the solution u across the casing as

$$[u] = u|_{\mathsf{\Gamma}_e^\varepsilon} - u|_{\mathsf{\Gamma}_i^\varepsilon}$$

$$\{u\} = \frac{1}{2} \left(u|_{\Gamma_e^{\varepsilon}} + u|_{\Gamma_i^{\varepsilon}} \right)$$

Definition2

Let u be the reference solution. We say an asymptotic model is of Order k+1, if its solution $u^{[k]}$ satisfies

$$||u - u^{[k]}||_{H^1} \le C \varepsilon^{k+1}$$
 when $\varepsilon \to 0$

• Step1: Derive an Asymptotic Expansion for u when $\varepsilon \longrightarrow 0$

• In the casing:
$$u_c(x,y) = \sum_{n \in \mathbb{N}} \varepsilon^n U^n \left(\frac{x - x_0}{\varepsilon}, y \right)$$

• Outside the casing: $u(x,y) = \sum_{n \in \mathbb{N}} \varepsilon^n u^n(x,y)$

 Step2: Obtain Equivalent Conditions of order k + 1 by identifying a simpler problem satisfied by the truncated expansion and neglecting the higher order terms in ε

•
$$u_{k,\varepsilon} := u^0 + \varepsilon u^1 + \varepsilon^2 u^2 + \ldots + \varepsilon^k u^k$$

• Step3: Prove Convergence rates for asymptotic models

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MULTISCALE EXPANSION

$$\begin{aligned} & \int \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\ & \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_e^\varepsilon \\ & \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\ & u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\ & U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\ & \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_e^\varepsilon \\ & \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\ & U^k = 0 & \text{on } \partial\Omega \end{aligned}$$



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MULTISCALE EXPANSION

$$\begin{aligned} & \int \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\ & \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\ & \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\ & u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\ & U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\ & \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_e^\varepsilon \\ & \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\ & U^k = 0 & \text{on } \partial\Omega \end{aligned}$$



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MULTISCALE EXPANSION

$$\begin{cases} \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^{\varepsilon} \\ \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^{\varepsilon} \\ \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^{\varepsilon} \\ u_i^k = U^k & \text{on } \Gamma_i^{\varepsilon} \\ U^k = u_e^k & \text{on } \Gamma_e^{\varepsilon} \\ \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_e^{\varepsilon} \\ \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^{\varepsilon} \\ U^k = 0 & \text{on } \partial\Omega \end{cases}$$



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MULTISCALE EXPANSION



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MULTISCALE EXPANSION

$$\begin{cases} \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^{\varepsilon} \\ \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^{\varepsilon} \\ \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^{\varepsilon} \\ u_i^k = U^k & \text{on } \Gamma_i^{\varepsilon} \\ U^k = u_e^k & \text{on } \Gamma_e^{\varepsilon} \\ \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_e^{\varepsilon} \\ \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^{\varepsilon} \\ U^k = 0 & \text{on } \partial\Omega \end{cases}$$



Approach Application

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EQUIVALENT MODELS

FIRST APPROACH

(Transmission conditions across the casing)

• Order 2:

$$\begin{cases}
\sigma_i \Delta u_i = f_i & \text{in } \Omega_i^{\varepsilon} \\
u_i = 0 & \text{on } \partial \Omega_i^{\varepsilon} \\
\int \sigma_e \Delta u_e = f_e & \text{in } \Omega_e^{\varepsilon} \\
u_e = 0 & \text{on } \partial \Omega_e^{\varepsilon}
\end{cases} \quad (P_2)$$
• Order 4:

$$\begin{cases}
\sigma_i \Delta u_i = f_i & \text{in } \Omega_e^{\varepsilon} \\
\sigma_e \Delta u_e = f_e & \text{in } \Omega_e^{\varepsilon} \\
[u] = 0 & (P_4) \\
[\sigma \partial_n u] = -\frac{\alpha}{\varepsilon^2} \Delta_{\Gamma^{\varepsilon}} \{u\} \\
u = 0 & \text{on } \partial \Omega \cup \partial (\Omega_e^{\varepsilon} \cup \Omega_i^{\varepsilon})
\end{cases}$$

Application Perspectives

CONVERGENCE

Theorem 2

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^{\varepsilon})$ and $f_e \in L^2(\Omega_e^{\varepsilon})$, then $\exists ! u^{[k]} \in V_{k+1}$ solution of $(P_{k+1}), k = 1, 3$, and $\|u^{[k]}\|_{1,\Omega_i^{\varepsilon}\cup\Omega_e^{\varepsilon}} \leq C\left(\|f_i\|_{0,\Omega_i^{\varepsilon}} + \|f_e\|_{0,\Omega_e^{\varepsilon}}\right)$ $\|u-u^{[k]}\|_{1,\Omega^{\varepsilon}\cup\Omega^{\varepsilon}} \leq C\varepsilon^{k+1}$

Where

$$V_{2} = \left\{ v : v_{i} \in H_{0}^{1}(\Omega_{i}^{\varepsilon}), v_{e} \in H_{0}^{1}(\Omega_{e}^{\varepsilon}) \right\}$$
$$V_{4} = \left\{ v : v_{i} \in H^{1}(\Omega_{i}^{\varepsilon}), v_{e} \in H^{1}(\Omega_{e}^{\varepsilon}), \nabla_{\Gamma^{\varepsilon}} \left\{ v \right\} \in L^{2}(\Gamma^{\varepsilon}), v|_{\Gamma_{i}^{\varepsilon}} = v|_{\Gamma_{e}^{\varepsilon}}, v|_{\Gamma_{0}} = 0 \right\}$$

Application Perspectives

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- Equivalent Conditions
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- Classic Finite Element Method code
- Straight triangular elements (h refinement)
- Lagrange shape functions of any degree (p refinement)



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CONVERGENCE RATES



Casing Thickness (ε)	0.0117	0.0234	0.0469	0.0938	Expected ($\varepsilon ightarrow 0$)
Order 2 Slopes	1.9969	1.9899	1.9638	1.8624	2
Order 4 Slopes	4.0046	4.0039	3.9879	3.8992	4

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EQUIVALENT MODELS

SECOND APPROACH

(Transmission conditions across the interface Γ)

• Order 1:

$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i \\ u_i = 0 & \text{on } \partial \Omega_i \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e \\ u_e = 0 & \text{on } \partial \Omega_e \end{cases}$$
(P1)

$$\begin{aligned} \sigma_i \Delta u_i &= f_i & \text{in } \Omega_i \\ \sigma_e \Delta u_e &= f_e & \text{in } \Omega_e \\ [u] &= -\varepsilon \left\{ \partial_n u \right\} & \text{on } \Gamma \\ [\partial_n u] &= -\frac{4}{\varepsilon} \left\{ u \right\} & \text{on } \Gamma \\ u &= 0 & \text{on } \partial\Omega \end{aligned}$$

• Order 2:

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STABILITY

Problem

Stability problems for the order 2 model of the second approach due to a non coercive term.





Solution

Use of Artificial Boundaries to move the transmission conditions and recover stability. For $\delta>0.5$

$$\Gamma_i^{\delta} = \{(x, y) : x = x_0 - \delta\varepsilon, y \in (0, y_0)\}$$

$$\Gamma_e^{\delta} = \{(x, y) : x = x_0 + \delta\varepsilon, y \in (0, y_0)\}$$

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EQUIVALENT MODELS

SECOND APPROACH

(Transmission conditions across the artificial interfaces)

Order 2:
$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e \\ [u] = -\varepsilon (1 - 2\delta) \{\partial_n u\} & (P_2) \\ [\partial_n u] = -\frac{4(1 - 2\delta)}{\varepsilon} \{u\} \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Stable if $\delta > 0.5$

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CONVERGENCE

Theorem 3

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^{\varepsilon})$ and $f_e \in L^2(\Omega_e^{\varepsilon})$, then $\exists ! u^{[k]} \in V_{k+1}$ solution of (P_{k+1}) , k = 1, 2, and $\|u^{[k]}\|_{1,\Omega_i^{\varepsilon}\cup\Omega_e^{\varepsilon}} \leq C\left(\|f_i\|_{0,\Omega_i^{\varepsilon}} + \|f_e\|_{0,\Omega_e^{\varepsilon}}\right)$ $\|u - u^{[k]}\|_{1,\Omega_i^{\varepsilon}\cup\Omega_e^{\varepsilon}} \leq C\varepsilon^{k+1}$

Where

$$V_{1} = \left\{ v : v_{i} \in H_{0}^{1}\left(\Omega_{i}\right), v_{e} \in H_{0}^{1}\left(\Omega_{e}\right) \right\}$$

$$V_{2} = \left\{ v : v_{i} \in H^{1}\left(\Omega_{i}\right), v_{e} \in H^{1}\left(\Omega_{e}\right), v_{i}|_{\Gamma} = v_{e}|_{\Gamma}, v|_{\Gamma_{0}} = 0 \right\}$$

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QUALITATIVE COMPARISON



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CONVERGENCE RATES L²



Casing Thickness (ε)	0.008	0.011	0.015	0.020	0.027	Expected ($\varepsilon ightarrow 0$)
Order 1 Slopes	0.972	0.962	0.948	0.928	0.899	1
Order 2 Slopes	2.032	2.011	2.062	2.090	1.9636	2
Order 2 displaced	2.005	2.006	2.007	2.008	2.007	2

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CONVERGENCE RATES H¹



Casing Thickness (ε)	0.008	0.011	0.015	0.020	0.027	Expected ($\varepsilon ightarrow 0$)
Order 1 Slopes	0.982	0.975	0.966	0.953	0.935	1
Order 2 Slopes	1.915	1.302	2.463	2.569	-0.125	2
Order 2 displaced	2.004	2.005	2.006	2.008	2.007	2

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COMPARISON



Casing Thickness (m)

	Order Stability		arepsilon-independent Domain		
Model 1 No Gap	1	1	1		
Model 2 No Gap	2	×	√		
Displaced Model	2	1	×		
Model 1 Gap	2	1	×		
Model 2 Gap	4	1	×		

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APPLICATION

•
$$z = z_3$$

• $z = z_2$ Receivers
• $z = z_1$
• $z = z_0$ Transmitter

- Equation: div $(\sigma \nabla u) = f$
- Right hand side:

 $f = \begin{cases} 1 & \text{In the transmitter} \\ 0 & \text{Outside the transmitter} \end{cases}$

• **Objective:** Measure the second difference of potential on the Receivers

 $U_2 = u(z_1) - 2u(z_2) + u(z_3)$

• **Expected Result:** (Model of Kaufman) Relation between second difference of potential and rock resistivity of the form

$$U_2 = k \cdot
ho_{\mathsf{rock}}^{-rac{1}{2}} \quad k \in \mathbb{R}$$

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VARYING ROCK CONDUCTIVITY



Resistivity	[1,10]	$[10, 10^2]$	$[10^2, 10^3]$	$[10^3, 10^4]$	Expected
Reference Model Slopes	-0.4914	-0.4924	-0.4978	-0.4991	-0.5
Order 4 Slopes	-0.4914	-0.4924	-0.4977	-0.4993	-0.5

Application Perspectives

TWO ROCK LAYERS



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- Obtain semianalytical solutions to reduce the computational cost.
- Consider physically more realistic scenarios.
- Develop 3D electromagnetic models.
- Study highly deviated boreholes.



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THANK YOU FOR YOUR ATTENTION