# Stability results for the parameter identification inverse problem in cardiac electrophysiology

Lassoued Jamila [(1)], Mahjoub Moncef [(2)] and Zemzemi Nejib [(3)]

[1] *LAMSIN-ENIT, B. P. 37,1002 Tunis-Belvedere, Tunis,Tunisia, lassoued.jamila@enit.rnu.tn*
[2] *LAMSIN-ENIT, B. P. 37,1002 Tunis-Belvedere, Tunis,Tunisia, moncef.mahjoub@lamsin.rnu.tn*
[3] *INRIA Bordeaux Sud-Ouest, 200 avenue de la vieille tour, 33405 Talence, France, nejib.zemzemi@inria.fr*

*The aim of this work is to establish stability estimates for the parameter identification problem in cardiac electrophysiology modeling. The propagation of the electrical wave in the heart is described by the monodomain equation. The model consists of a reaction-diffusion non linear equation coupled to an ODE system representing the electrical activity of the cell membrane.*

$$\begin{cases} \chi_m \dfrac{\partial V_m}{\partial t} - div(\sigma \nabla V_m) = I_{app} - I_{ion}(V_m, w) & in \ \ \Omega \times (0,T) \\ \dfrac{\partial w}{\partial t} + G(V_m, w) = 0 & in \ \ \Omega \times (0,T) \\ \sigma \nabla V_m . n = 0 & on \ \ \Sigma. \end{cases} \tag{1}$$

*where $\Omega$ and $\Sigma$ denote respectively the domain and the boundary of the heart. The time domain is given by $[0,T]$ and $\chi_m$ the membrane capacitance per area unit. The variable $V_m$ denote the action potential, and $\sigma$ is the bulk conductivity. The term $I_{app}$ is a given external current stimulus, $w$ represents the concentrations of different chemical species, and variables representing the openings or closures of some gates of the ionic channels. And the ionic current $I_{ion}$ and the function $G(V_m, w)$ depends on the considered ionic model. In this study, the dynamics of $w$ and $I_{ion}$ are described by the Mitchell and Schaeffer model :*

$$I_{ion}(v,w) = \frac{w}{\tau_{in}} v^2(v-1) - \frac{v}{\tau_{out}} \quad and \quad G(v,w) = \begin{cases} \dfrac{w-1}{\tau_{open}} & si \ \ v \le v_{gate} \\ \\ \dfrac{w}{\tau_{close}} & si \ \ v > v_{gate} \end{cases}$$

*The time constants $\tau_{in}, \tau_{out}$ are respectively related to the length of the depolarization and repolarization. The parameters $\tau_{open}$ and $\tau_{close}$ are the characteristic times of gate opening and closing respectively and $v_{gate}$ corresponds to the change-over voltage.*
*In this work we are interested in the identification of the parameter $\tau_{in}$ to which the solution is known to be very sensitive. Let $\tau_{in}$ and $\widetilde{\tau}_{in}$ be two different strictly positive parameters. We define $f(V_m, w) = I_{app} - I_{ion}(V_m, w)$, $a = \dfrac{1}{\tau_{in}}$, $\widetilde{a} = \dfrac{1}{\widetilde{\tau}_{in}}$ $(V_m, w) = (V_m(a), w(a))$ and $(\widetilde{V}_m, \widetilde{w}) = (V_m(\widetilde{a}), w(\widetilde{a}))$. Let's also denote*

$$V = V_m - \widetilde{V}_m \qquad and \qquad W = w - \widetilde{w}. \tag{2}$$

*Let $\omega_0$ be an arbitrary non-empty subdomain of $\Omega$, we prove the following stability result*

**Theorem 1** *Let $(V, W)$ defined as in (2) with the initial condition $V_0 \in H^2(\Omega)$.*
*Let us assume that*

$$\frac{\partial f}{\partial a}(\tilde{V}_m, \tilde{w}) \ge r_0 > 0 \quad for \ some \quad T' > 0 \quad and \quad \forall x \in \Omega \, .$$

*Then there exists $C > 0$ such that*

$$|a - \widetilde{a}| \le C N_{T', \omega_0}(V, W),$$

*where*

$$N_{T',\omega_0}(V,W) = \|V\|_{H^1(0,T;L^2(\omega_0))} + \|V\|^2_{L^4(0,T;L^4(\omega_0))} + \|V(T')\|_{H^2(\Omega)} + \|V(T')\|^3_{L^6(\Omega)} + \|W(T')\|_{L^2(\Omega)}.$$