Taking into account correlated observation errors by progressive assimilation of multiscale information

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**Abstract**

- The description of correlated observation error statistics is a challenge in data assimilation.
- Currently, the observation errors are assumed uncorrelated (the covariance matrix is diagonal), which is a severe approximation that leads to suboptimal results.
- It is possible to use multi-scale transformations to retain the diagonal matrix approximation while accounting for some correlation. However, this approach can lead to some convergence problems due to scale interactions.
- We propose an online scale selection algorithm that improves the convergence properties in such case.

**Motivation: account for correlated satellite observation errors**

- Numerical weather prediction requires the determination of the initial state of the system. Indeed, the true state, at a given moment and in all points of space, is not accessible. In order to retrieve an optimal initial condition one uses the so-called data assimilation methods that combine information from observations, model equations, and their respective error statistics.
- Since the late 70s, satellites are a dominant source of information. Errors associated to such data are highly correlated in space, which can be detrimental if this is not properly accounted for. However, their density in space allows for the efficient use of multi-scale transformation, which in turn permit a cheap but good approximation of said error statistics representation.

- For homogeneously spatially correlated Gaussian observation errors this approach is very efficient.
- For more complex errors, however, it can severely damage the convergence properties of the assimilation methods.
- Here we present, through a laboratory experiment, an illustration of the above-mentioned problem and a possible solution using scale selection during the assimilation process.

**Data assimilation: Principle**

- **What is data assimilation?**
  - Combine at best different sources of information to estimate the state of a system:
    - model equations
    - observations, data
    - background, a priori information
    - errors statistics
  - **What is data assimilation for?**
    - historically: initial state estimation, for weather forecasting,
    - today: many other application domains...
    - ... and many other application domains.

**Best Linear Unbiased Estimation (BLUE)**

- Least squares analysis: solve the inverse problem $y^* = (X'X)^{-1}X'y$ given a background estimate $x^*$ of the true input parameters $x$, where:
  - $y^*$ complete observations, errors $e$ unbiased, covariance matrix $R$ $x^* = x + e$, background error unbiased, covariance $P^B$; observation operator $H$ maps linearly the input parameters to the observations

**Variational Data Assimilation**

- Let $M$ be a dynamical model describing the evolution of the state variable $X$ in space and time: $\partial_t X = M(X_{t-1},X_{t-1})$, $M(X_{t-1},x) = 0$ $X_t = X_t(X_{t-1}) = X_{t-1}$
- Let $Y(t)$ be (partial) observations of this state variable.
- The aim of data assimilation is to estimate an optimal initial condition $X^0$ (often called analysis state) so that it is not far from the first guess $X_0^0$ (in general coming from a previous forecast), and that the model trajectory $X_t(X_0^0, t)$ is close to the observations $Y(t)$.
- This is done by defining $X^0$ as the minimum of the cost function: $J(X^0) = J_0(X^0) + J_d(X^0)$
- In order to mimic the usual approach, only diagonal approximations of $R$ are used
- In general, to be $X^0 = Y'(t)$, where $Y$ is the observation operator.

**Error covariance matrix representation**

- **Cruical choice for $R$ and $D_r$**: drive the way information is spread and how redundancy of error is dealt with.
- **So far** a strong research effort for $B$ modeling
- $B$ matrix, mostly assumed diagonal.
- $\beta = \Sigma B_{k0} B_{0k}$
- **Main reason for this**: management greatly simplified.
- **Number of observations in general non constant over time**: $\to$ diagonal $R$ matrix.
- **Consequence**: at each assimilation cycle, a new $R$ matrix should be formed and inverted.

**Results: Inhomogeneous observation error**

- The observation error is still Gaussian and spatially correlated, but these correlations are now inhomogeneous in space:
  - $y_i^* = A_i^* + \epsilon_i$ with $\epsilon_i = \epsilon_i^T A_i^T D_i B_i$ $y_i^* \sim N(0, \Sigma)$ so that $D_i$ is the exact representation of $R$ in the wavelet space.

**Results 2: Inhomogeneous observation error**

- $r = (x_i^* - X_i^0)/(\Sigma X_i^0)$ along minimisation iterations for both pixel and wavelet-based distances in presence of an inhomogeneous observation error

**Comments:** Even though the observation error is exactly represented in the Wavelet case, its minimisation is struggling to converge toward a minimum similar or better than for the pixel case. Main reason for this: these scales differ strongly in the small scales, as illustrated below.

**Results 3: Improvement by online scale selection**

- **Best improvement**: first assimilate the large scales, in order to reduce the difference between $X(X)$ and $Y(t)$, and to be in the monostatic region of the green curve, and then progressively include the smaller scales.

**References**


**Papers online:** [https://team.inria.fr/saissre/maille/](https://team.inria.fr/saissre/maille/)