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Comparison of solvers performance when solving the 3D Helmholtz elastic wave equations over the Hybridizable Discontinuous Galerkin method

M. Bonnasse-Gahot^{1,2}, H. Calandra³, J. Diaz¹ and S. Lanteri²

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³ TOTAL Exploration-Production

Motivations

Imaging methods

- ▶ Full Wave Inversion (FWI) : **inversion process** requiring to solve **many forward problems**

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Seismic imaging : time-domain or harmonic-domain ?

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Memory usage



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Resolution of the forward problem of the inversion process

- ▶ Elastic wave propagation in the frequency domain : **Helmholtz equation**

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First order formulation of Helmholtz wave equations

$$\mathbf{x} = (x, y, z) \in \Omega \subset \mathbb{R}^3,$$

$$\begin{cases} i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = \nabla \cdot \underline{\underline{\sigma}}(\mathbf{x}) + \mathbf{f}_s(\mathbf{x}) \\ i\omega\underline{\underline{\sigma}}(\mathbf{x}) = \underline{\underline{C}}(\mathbf{x}) \underline{\underline{\varepsilon}}(\mathbf{v}(\mathbf{x})) \end{cases}$$

- ▶ \mathbf{v} : velocity vector
- ▶ $\underline{\underline{\sigma}}$: stress tensor
- ▶ $\underline{\underline{\varepsilon}}$: strain tensor

Approximation methods

Discontinuous Galerkin Methods

- ✓ unstructured tetrahedral meshes
- ✓ combination between FEM and finite volume method (FVM)
- ✓ *hp*-adaptivity
- ✓ easily parallelizable method

Approximation methods

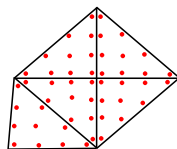
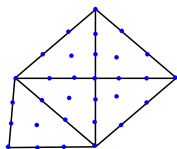
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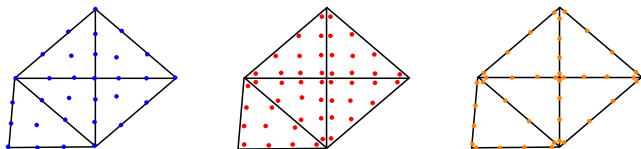
Hybridizable Discontinuous Galerkin Methods

- ✓ same advantages as DG methods : unstructured tetrahedral meshes, *hp*-adaptivity, easily parallelizable method, discontinuous basis functions
- ✓ introduction of a new variable defined only on the interfaces
- ✓ lower number of coupled DOF than classical DG methods

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Principles of the HDG method

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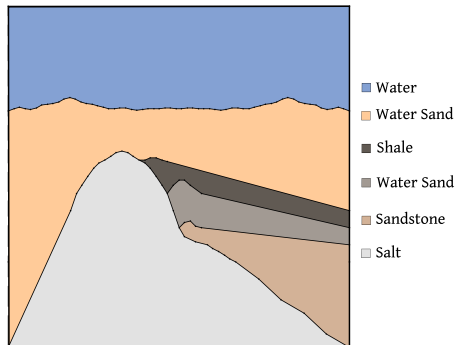
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5. Computation of the solutions of the initial problem, element by element

Contents

2D Numerical results : performances comparison of the HDG method

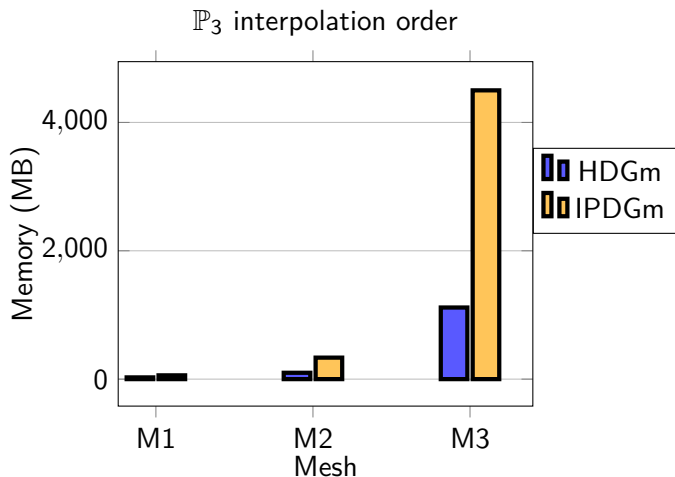
3D numerical results : focus on the linear solver

Anisotropic test case



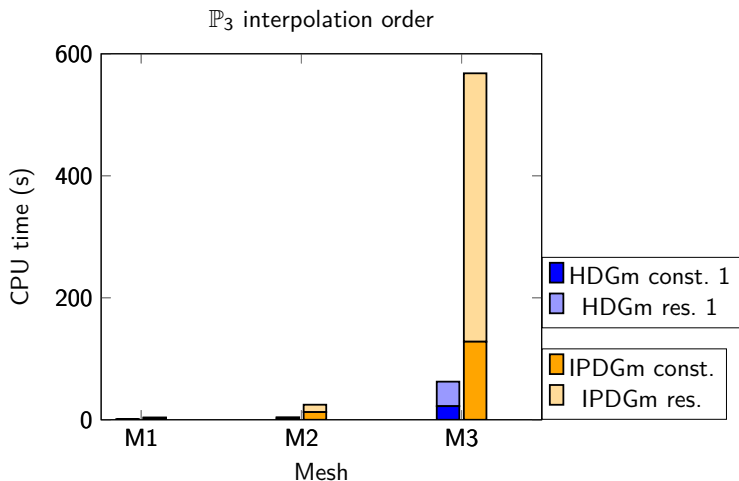
- ▶ Three meshes :
 - ▶ 600 elements
 - ▶ 3000 elements
 - ▶ 28000 elements

Anisotropic case : Memory consumption



$$\text{IPDG}_{\text{memory}} \simeq 4 \times \text{HDG}_{\text{memory}}$$

Anisotropic case : CPU time (s)



$$\text{IPDG}_{\text{CPUtime}} \simeq 9 \times \text{HDG}_{\text{CPUtime}}$$

Contents

2D Numerical results : performances comparison of the HDG method

3D numerical results : focus on the linear solver

3D plane wave in an homogeneous medium

3D geophysic test-case : Epati test-case

Resolution of the linear system $M\lambda = S$

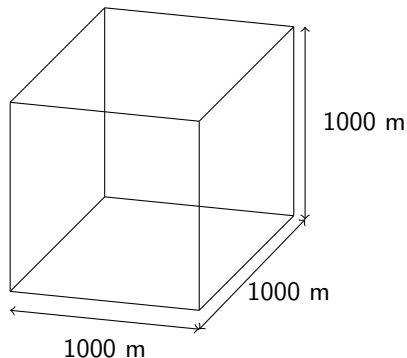
- ▶ **direct solver : MUMPS** (MULTifrontal Massively Parallel sparse direct Solver) :
 - ▶ Direct factorization $A = LU$ or $A = LDL^T$
 - ▶ Multiple RHS
- ▶ **hybrid solver : MaPhys** (Massively Parallel Hybrid Solver) :
 - ▶ combination of direct and iterative methods
 - ▶ non-overlapping algebraic domain decomposition method (Schur complement method)

Cluster configuration

Features of the nodes :

- ▶ 2 Dodeca-core Haswell Intel Xeon E5-2680
- ▶ Frequency : 2,5 GHz
- ▶ RAM : 128 Go
- ▶ Storage : 500 Go
- ▶ Infiniband QDR TrueScale : 40Gb/s
- ▶ Ethernet : 1Gb/s

3D plane wave in an homogeneous medium



Configuration of the computational domain Ω .

► Physical parameters :

- $\rho = 1 \text{ kg.m}^{-3}$
- $\lambda = 16 \text{ GPa}$
- $\mu = 8 \text{ GPa}$

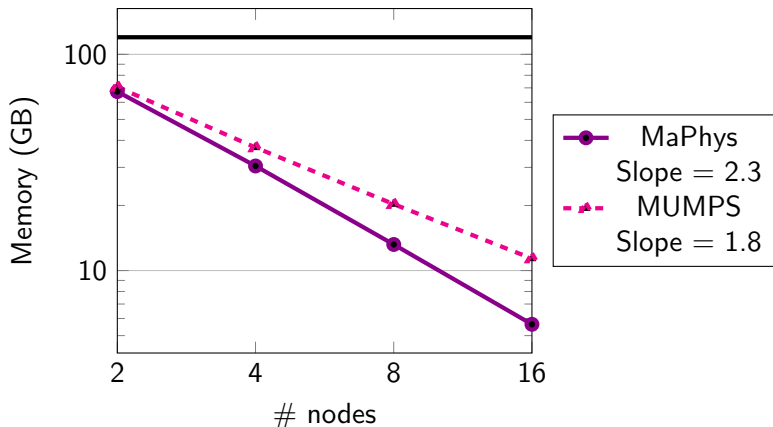
► Plane wave :

$$u = \nabla e^{i(k_x x + k_y y + k_z z)}$$

- $\omega = 2\pi f, f = 8 \text{ Hz}$
- Mesh : 21 000 elements

3D Plane wave : Memory consumption

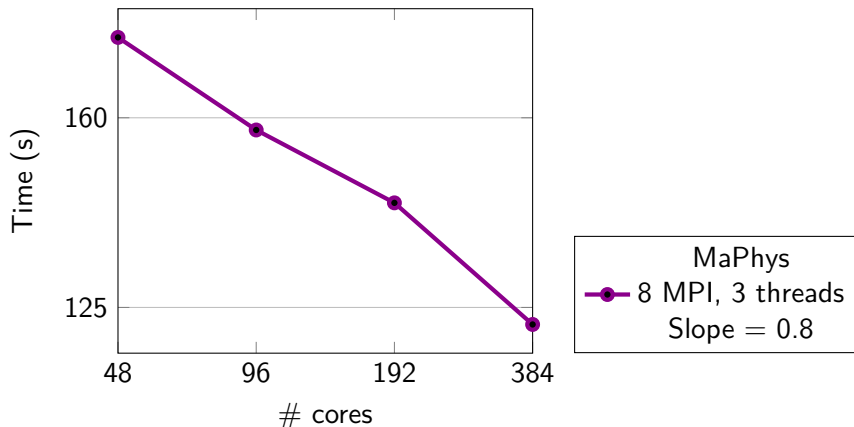
Average memory for one node (8 MPI by node and 3 threads by MPI)



(matrix order = 1 300 000, # nz=300 000 000)

3D Plane wave : Execution time

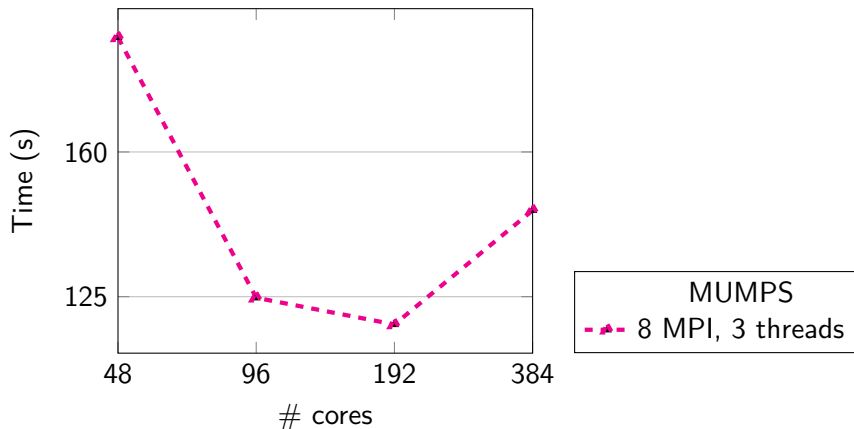
Execution time for the resolution of the HDG- \mathbb{P}_3 system



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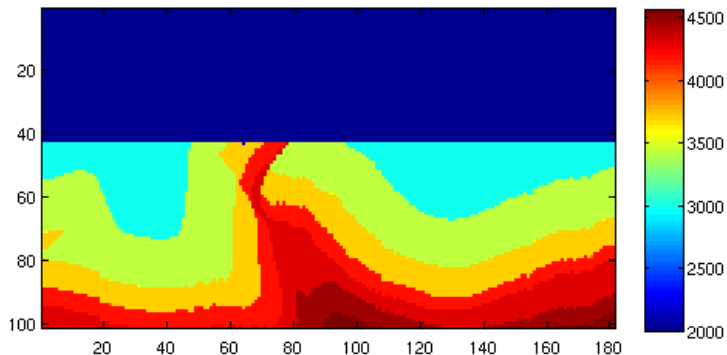
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Epati test-case

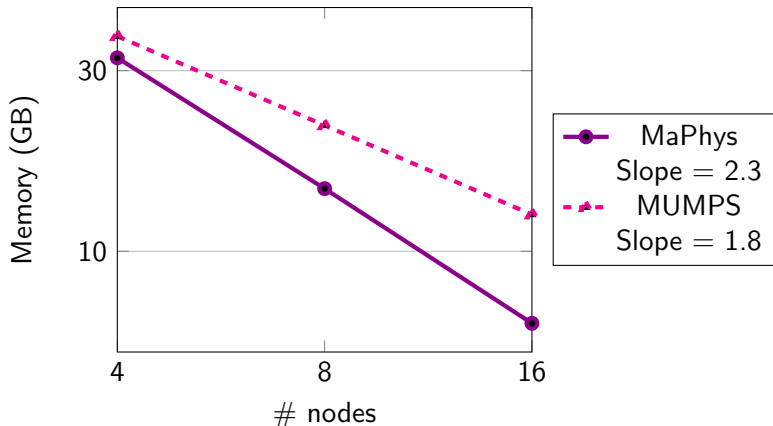


V_p -velocity model ($\text{m}\cdot\text{s}^{-1}$), vertical section at $y = 700$ m

Mesh composed of 25 000 tetrahedrons

Epati test-case : Memory consumption

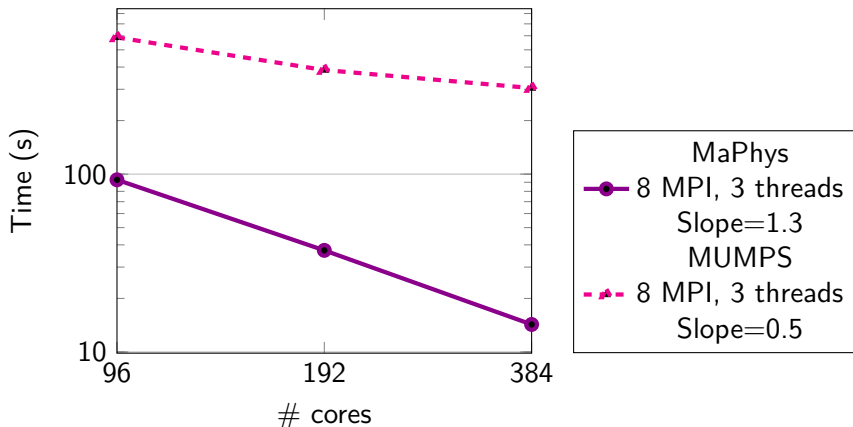
Average memory for one node (8 MPI by node and 3 threads by MPI)



(matrix order = 1 300 000, # nz=365 000 000)

Epati test-case : Execution time

Execution time for the resolution of the HDG- \mathbb{P}_3 system



(matrix order = 1 300 000, # nz=365 000 000)

Conclusion-Perspectives

- ▶ more detailed analysis of the comparison between solvers
 - ▶ larger meshes
 - ▶ more powerful clusters
- ▶ memory crash test
- ▶ extension to elasto-acoustic case

Thank you !

The logo for Inria, featuring the word "inria" in a stylized, cursive font with a color gradient from red to orange. Above the "ria" part, the words "informatiques" and "mathématiques" are written in a smaller, black, sans-serif font, separated by a small red asterisk.

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informatiques mathématiques