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B. Xu ^{1,3}

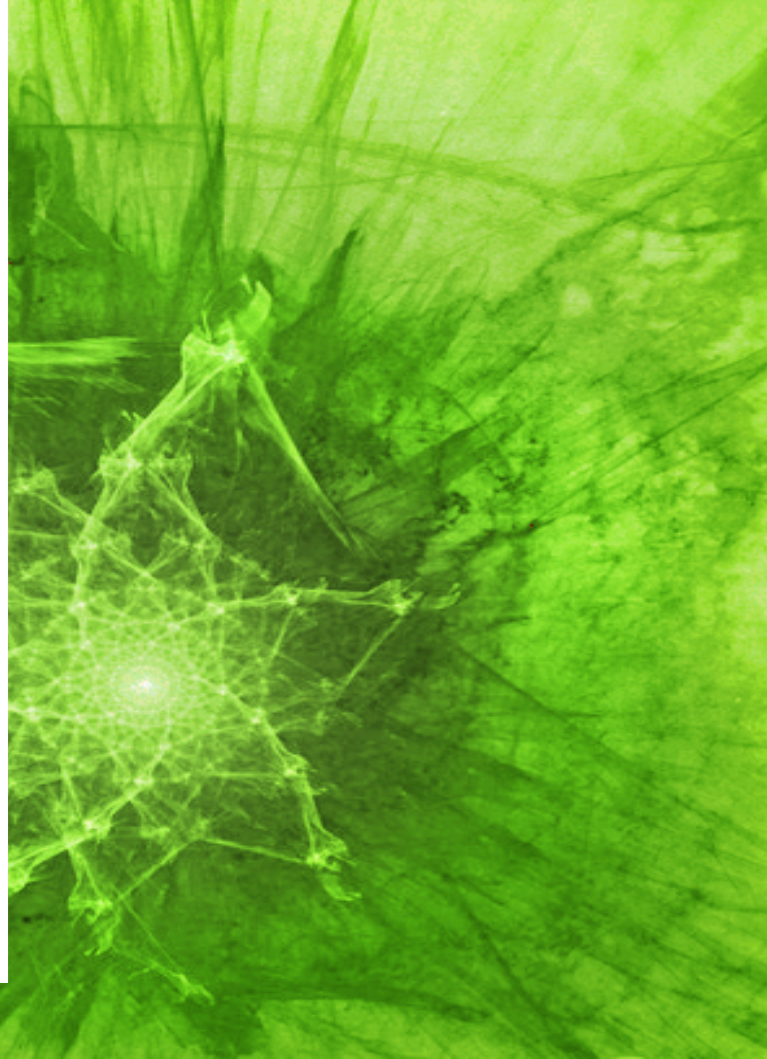
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Complexity in Electrophysiological Dynamics

Emergence and measures of organization

2nd BCAM Workshop on Nonlinear Dynamics in Biological Systems



Cosmos and chaos

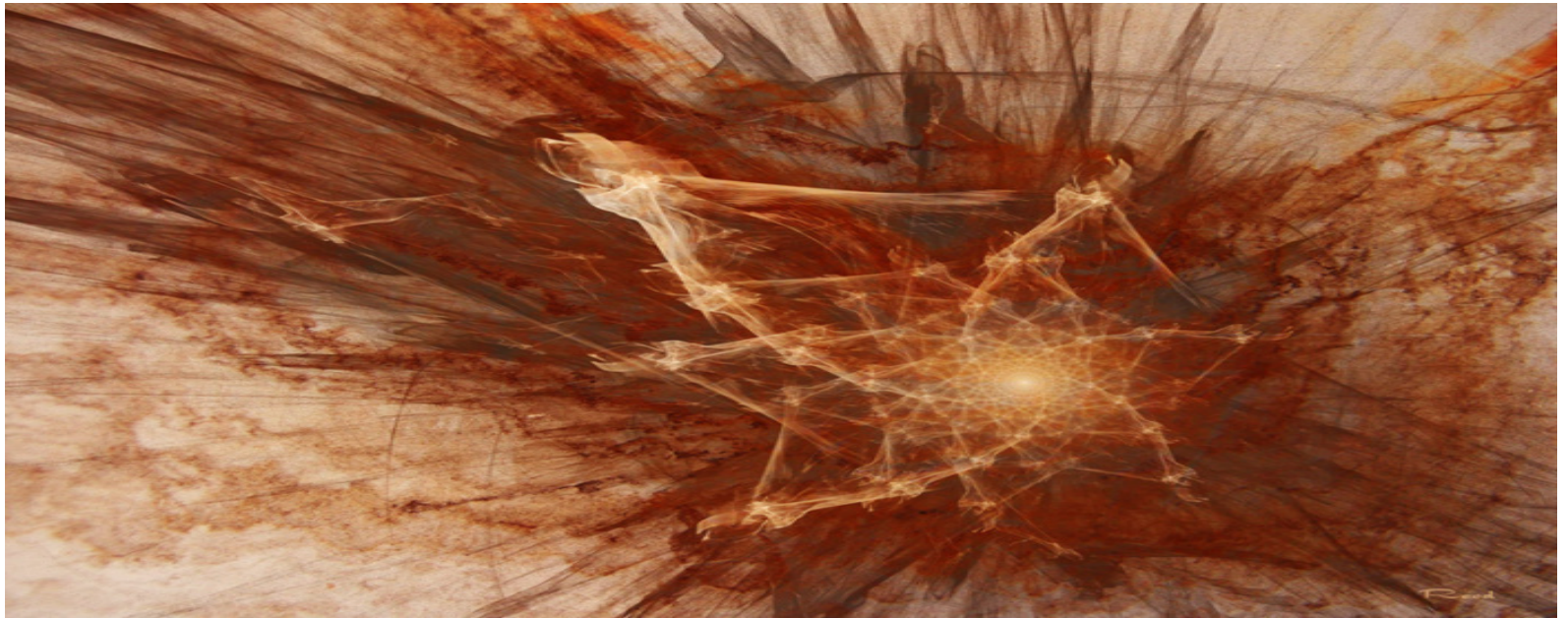
Classic view:

- Hesiod's cosmogony: gods created from chaos
- Anaximander: *Apeiron* is the principle of everything
- Ovid: in the origin, mix of the four elements
- Genesis 1:2: The earth was formless and void; darkness covered the abyss and the Spirit of God moved upon the waters



Cosmos and chaos

Modern view: entropy (from order to chaos)

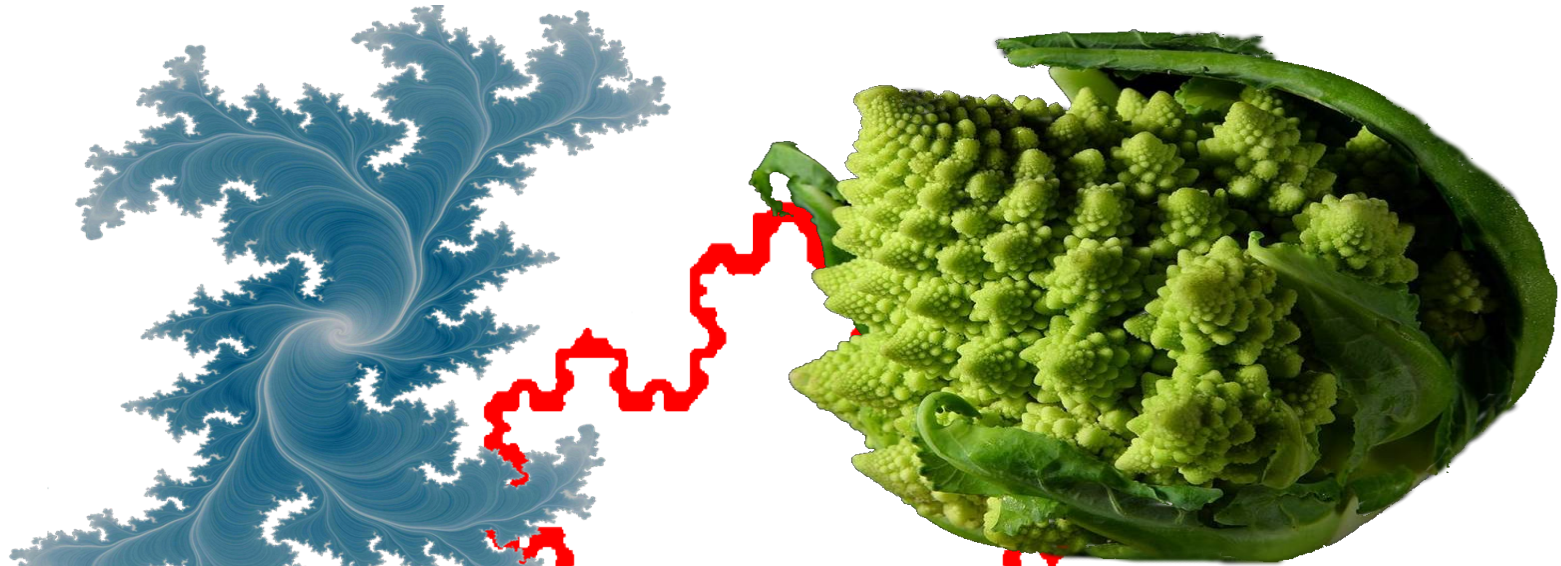


Cosmos and chaos

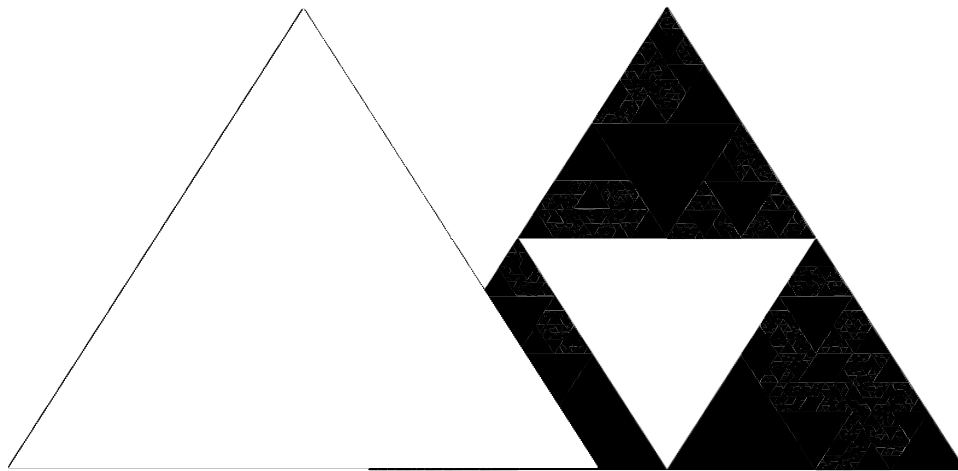
Order from chaos?



Scale invariance

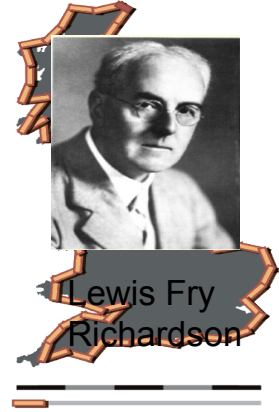
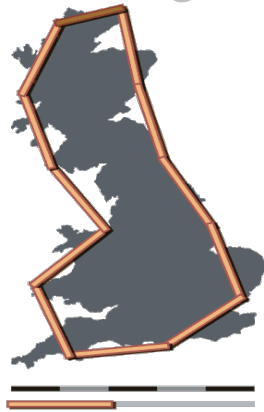


Scale invariance



Measure for scale invariance

- How long is the coastline of Britain?



$$l_r \sim r^{1-D}$$

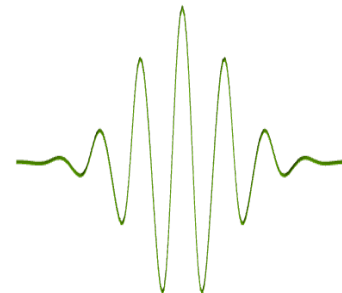


Scale invariance

Singularity exponent $h(\mathbf{x})$ degree of singularity / regularity of signal s around point \mathbf{x}

$$\mu_S[\Psi_r(\mathbf{x})] \sim r^{\dim(\mathbf{x}) + h(\mathbf{x})} \quad d\mu_S \sim \|\nabla s\| d\mathbf{x}$$

μ is a measure defined from gradient modulus and $\Psi_r = \Psi(\mathbf{x}/r)$ is a wavelet kernel



Ocean Circulation

Singularity and local predictability:

$$\mu_s [\psi(\mathbf{x})] = A(\mathbf{x}) \|\psi\|^{h(\mathbf{x})} + o(\|\psi\|^{h(\mathbf{x})}) \quad d\mu_s = |\nabla s| d\mathbf{x}$$

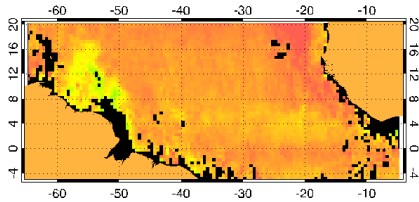
Derived fields, fusion:

$$d\mu_s = S(\mathbf{x}) d\mu_r$$

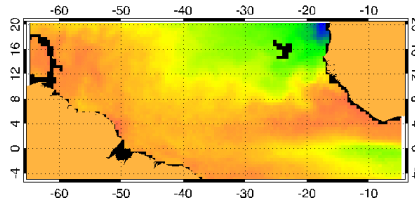
Ocean Circulation

Singularity and local predictability:

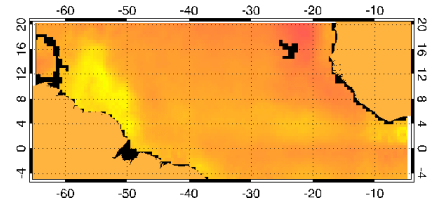
$$\mu_s [\Psi(\mathbf{x})] = A(\mathbf{x}) ||\Psi||^{h(\mathbf{x})} + o(||\Psi||^{h(\mathbf{x})}) \quad d\mu_s = |\nabla s| dx$$



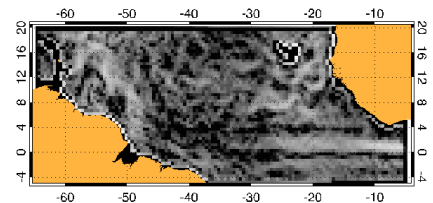
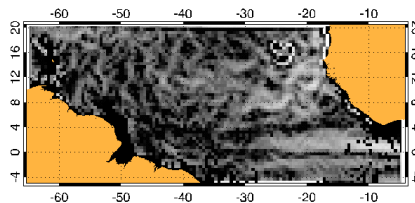
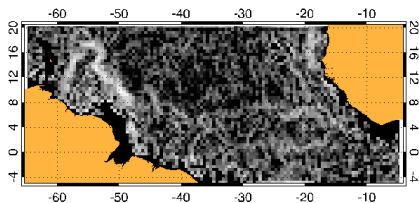
salinity



temperature



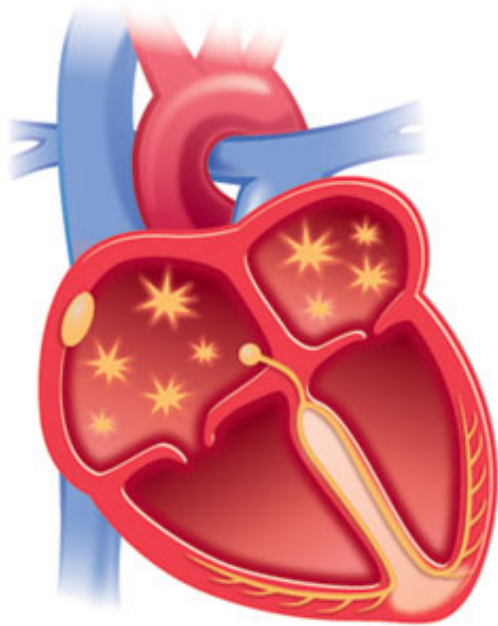
salinity fused



Derived fields, fusion:

$$d\mu_s = S(\mathbf{x}) d\mu_r$$

Atrial fibrillation



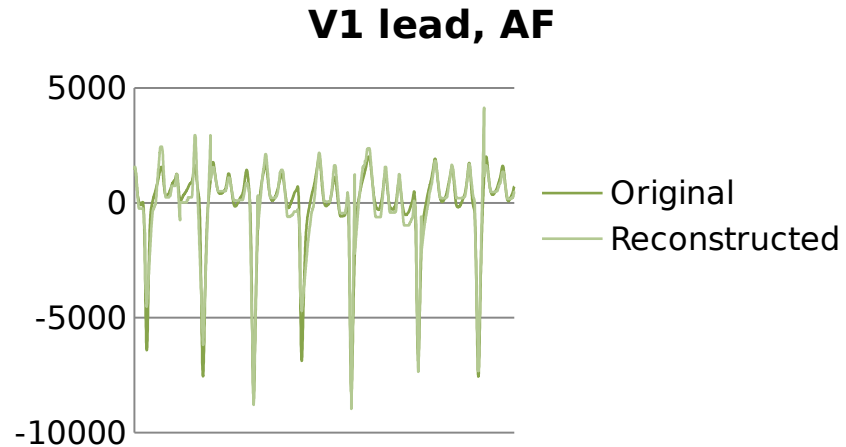
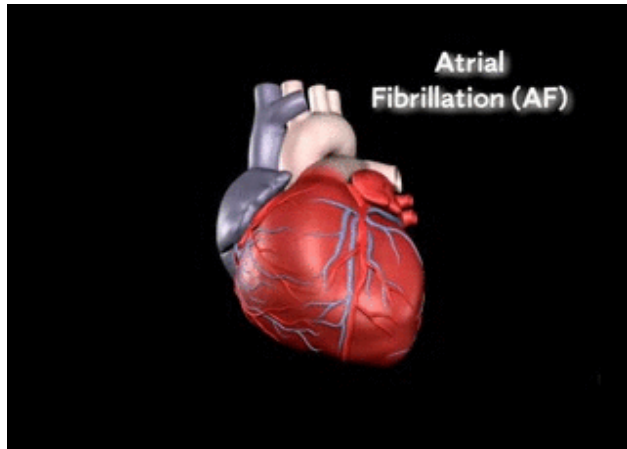
The most extended arrhythmia

Ventricles still pump but:

- Can lead to heart failure
- Forms blood clots (trombosis) that can provoke embolism

Signal well reconstructed from most singular manifold

$$\hat{s}(\mathbf{f}) = (i \mathbf{f} / f^2) \cdot F(\nabla s \delta_{UP})$$

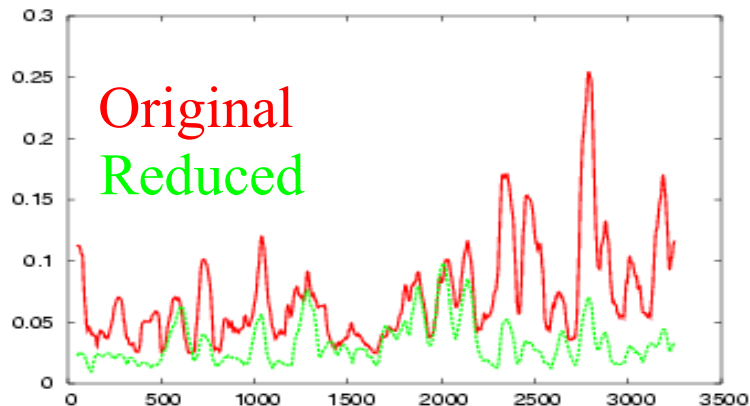


signal s , frequency \mathbf{f} , most singular manifold δ_{UP}

Signal well reconstructed from most singular manifold

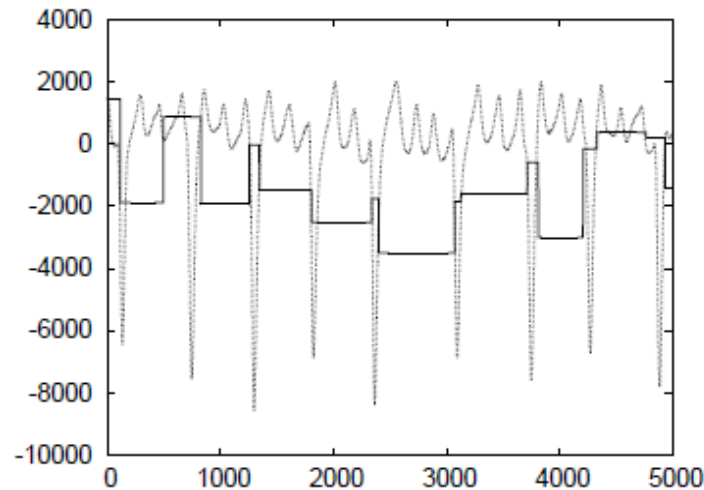
Fast dynamics: MSM orientation defines a reduced signal

Slow dynamics: modulation over the reduced reconstruction



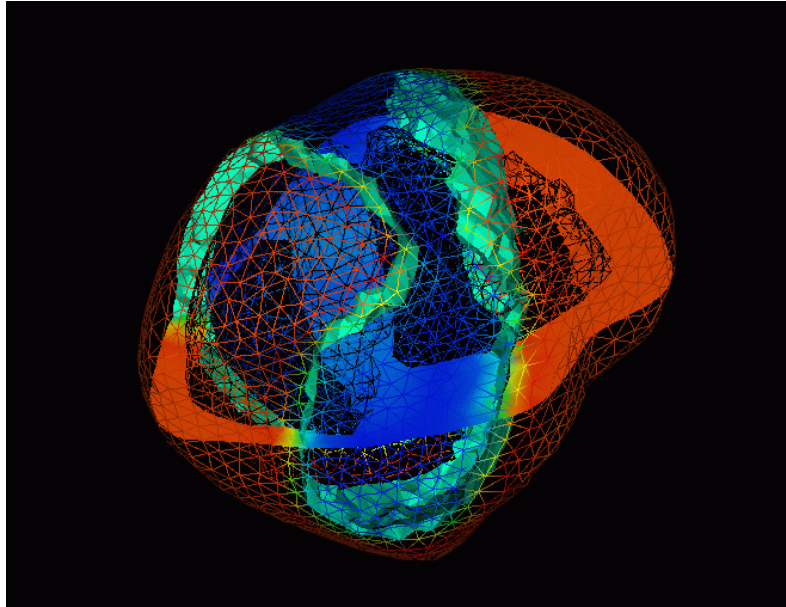
Source field, slow dynamics

$$SF(\mathbf{x}) = d\mu_s / d\mu_r$$



Radon-Nikodym derivative between original signal and reduced signal diff measures

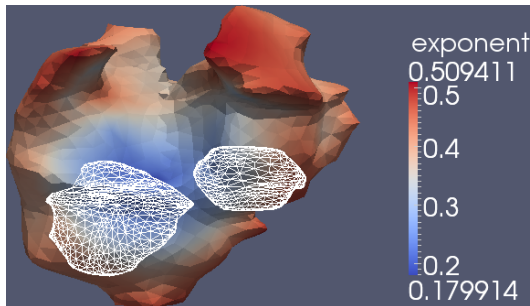
Epicardial mapping of electric potential



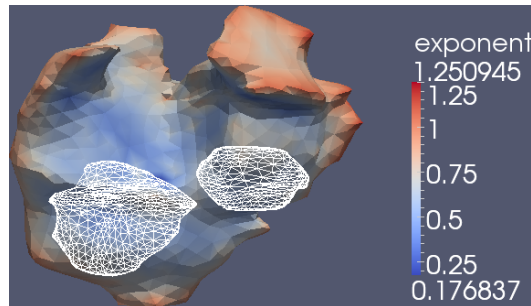
CardioInsight™ projects on the heart surface (inverse-problem), real time

Singularity exponent epicardial maps

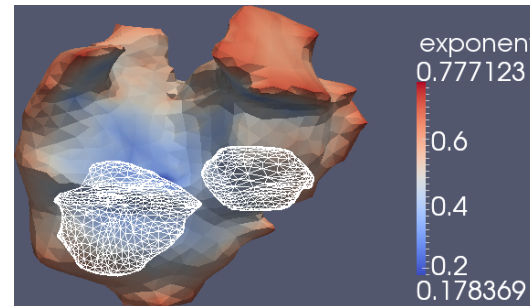
space



time



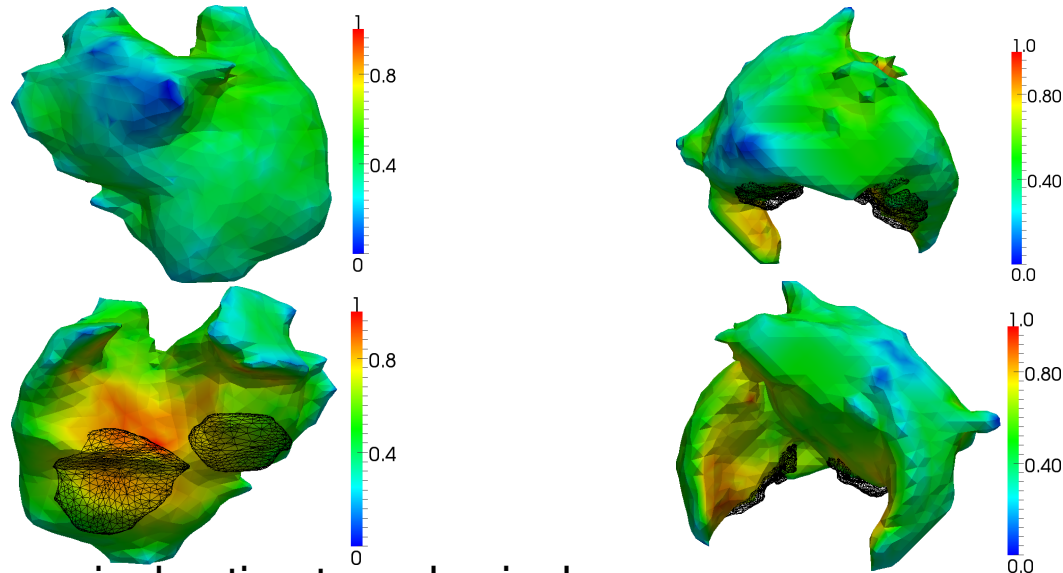
space-time



versatility in the domain considered

$$\mu_S[\Psi_r(x)] \sim r^{\dim(x) + h(x)}$$

Source field in atrial flutter and fibrillation



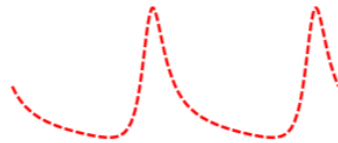
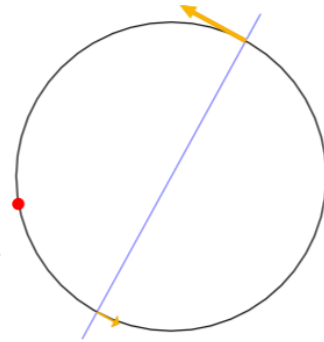
robust numerical estimate – physical non-neuristic, sharp descriptor

A basis for smooth anharmonic signals

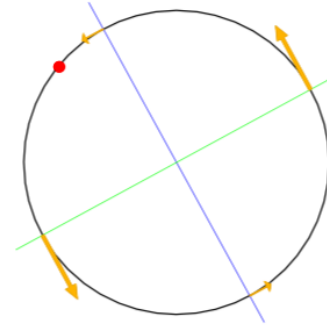
P. Hanusse has developed a nonlinear signal-processing technique centred on the characterization of anharmonicity (as smooth perturbations of harmonicity) in periodic signals. Such characterization is rather sparse in multiple physiological systems.

Illustration of the concept

$$\frac{d\phi}{dt} = \frac{1+r^2+2r\cos(\phi)}{1-r^2}$$



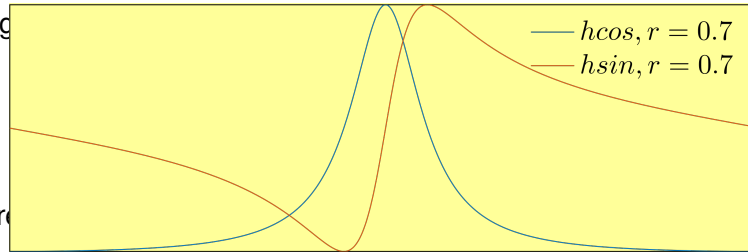
$$\frac{d\phi}{dt} = \frac{1+r^2-2r\cos(2\phi)}{1-r^2}$$



First (left) and second (right) breakings of rotational symmetry framing the notion of *anharmonic* trigonometry

Details

Generally, let $x(t)$ be a periodic sig



First symmetry breaking can be re

$$h\cos : (t, r) \rightarrow \frac{(1 + r^2) \cos(t) - 2r}{1 + r^2 - 2r \cos(t)}$$

$$h\sin : (t, r) \rightarrow \frac{(1 - r^2) \sin(t)}{1 + r^2 - 2r \cos(t)}$$

where r is the anharmonicity parameter.

Extension

The higher orders require these generalizations:

$$\text{hpcos}_n : (t, r) \rightarrow \sum_{k=1}^{\infty} \frac{\cos(kt)r^{k-1}}{k^n}$$

$$\text{hpsin}_n : (t, r) \rightarrow \sum_{k=1}^{\infty} \frac{\sin(kt)r^{k-1}}{k^n}$$

Order 1 is still treatable and corresponds to phase equations that are fractions of trigonometric polynomials.

$$t(\phi) = \phi + \sum_{k=1}^n a_k \text{hpsin}_1(\phi - p_k, r_k) - b_k \text{hpcos}_1(\phi - p_k, r_k)$$

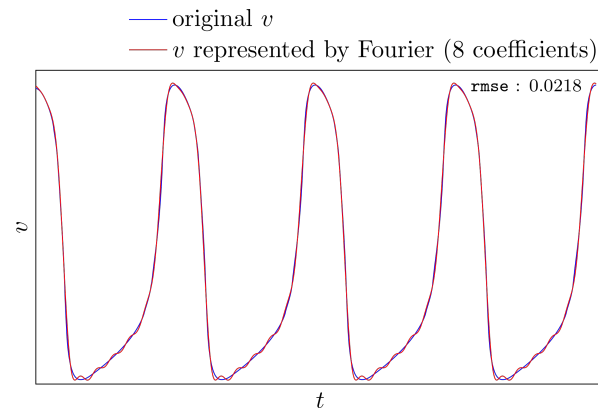
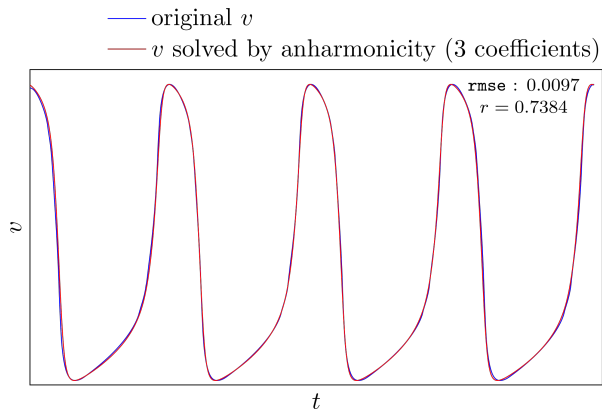
$$p\text{cos}_1(t, r) = -\frac{1}{2} \ln(1 + r^2 - 2r\cos(t))$$

$$p\text{sin}_1(t, r) = \tan^{-1}\left(\frac{r\sin(t)}{1 - r\cos(t)}\right)$$

where we re-use n as the polynomial order since we already fixed 1 for the family of functions. Typical real-world systems are well described at low orders.

Analysis of FitzHugh-Nagumo action potentials

$$\begin{aligned}\dot{v} &= v(v - \alpha)(1 - v) - w, \\ \dot{w} &= \varepsilon(v - \gamma w),\end{aligned}$$



Only 3 anharmonic terms reconstruct a nearly perfect representation of the signal

In contrast, with 8 Fourier terms the signal is ill-represented and plagued with Gibbs phenomena

The presented decomposition thus appears far more compact than Fourier for FitzHugh-Nagumo impulse trains

Thanks for your attention!