The Effect of the Existence of Defective Items in Assembly Operations

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ABSTRACT. Quality is a principle issue in production management (PM). No process is perfect and the production of defective items is unavoidable. Very few studies regard the effect of the existence of defective items (EEDI) in production processes. Further, quality has been studied in isolation to high extent, of other PM domains. In this study, defect rates together with the assembly ratios of the bill of material are embedded in process charts. This facilitates the analysis of the EEDI in assembly operations and enables to quantify them. Apparently, defect rates grow dramatically in assembly operations due to the mutual effects of the assembly's components. Hence prior quality assurance effort is motivated

Keywords: Assembly, Bill of materials, Defect rate, Product structure, Quality.

1   Introduction

Feigenbaum (1991, page 47) minted the term "*hidden plant*": "the proportion of plant capacity that exists to rework unsatisfactory parts, to replace product recalled from the field, or to retest and re-inspect rejected units." He estimated that *hidden plant*s amount "to 15% to as much as 40% of productive capacity". Here, means to quantify these figures and more important to compare alternatives are provided.

2   Defect rates and input/output ratios

Let p_i denote the average defect rate (DR) of operation/activity i. Note that defects due to **common**, chance, or random causes are considered, not quality deterioration due to *assignable* causes as in Kim and Gershwin (2008). If activity i is performed on Q_i units, the mean number of acceptable units is only $(1- p_i)Q_i$. This easily extends to serial processes. If Q_0 units enter a serial process of n operations, the mean number of acceptable units at the end is (e.g., Freiesleben, 2005):

$$Q_n = Q_0 \prod_{i=1}^{n} (1 - p_i)$$.(1)

Figure 1 portrays a serial process chart (e.g., Francis, McGinnis and White, 1992), where each node represents an activity. Each defective item divides its production process into three segments: prior to damage, between damage and detection and after detection. Detected defective items can be removed, thereby save the costs associated with, and the capacity required for future operations. A defective item can either be scrapped, used as it is for lower price, reworked or repaired. The last two cases involve costs and require capacity, additional to the regular capacity and costs, while in the first cases capacity is wasted and income is lost, which is equivalent to cost increase. In any event, larger quantities should be processed to compensate for the poor quality.
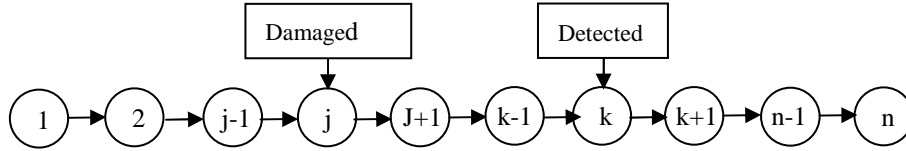

Fig. 1. A defect item divides the production process into three segments

Further, production mangers know how many end-items are needed. From these figures, order-quantities are calculated backward, as in material requirements planning (MRP; e.g., Nahmias, 2009). Whenever defective units are not used as intended, more units should be produce to replace these units. A reworked unit is just as an additional one with, perhaps, additional preparation activities, and repair requires additional repair capacity. Thus, (1) should be re-written as in (2), where p_i is the fraction of defective units that are not repaired:

$$Q_j^{in} = Q^{out} \bigg/ \prod_{i=j}^{n} (1 - p_i)$$ (2)

This calculation accounts for all items that will be damaged, and not repaired, in operation j and subsequent operations up to the last activity, n. Furthermore, this is the minimal quantity – larger quantity maybe processed in activity j if defective items from preceding operations have not been removed earlier. Suppose $j = n$-19; i.e. there are 20 operations to go, including j, and all share the same defect rate of 1%. Then, $Q_j^{in} = Q^{out} / 0.99^{20} = 1.2226 Q^{out}$. Namely, 1226 units will be processed, knowing that only 1,000 of them will be useful!

3   Assembly operations

Products, however, are, usually, not structured serially, as in Figure 1, but in tree structures – a tree for each product. Figure 2 portrays a *product structure* and a bill of material (*BOM*) (e.g., Nahmias, 2009), which tell us which components are assembled and the assembly ratios: each assembly consists of 4, 2 and 1 units of component 1, 2 and 3, respectively. All these components should of course be conforming, but how can defective component be accounted for?
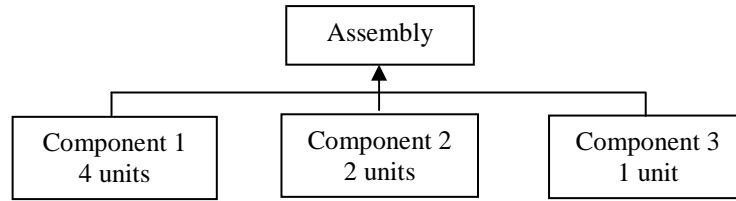
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                    ┌─────────────┐
                    │  Assembly   │
                    └──────▲──────┘
        ┌──────────────────┼──────────────────┐
┌───────────────┐  ┌───────────────┐  ┌───────────────┐
│  Component 1  │  │  Component 2  │  │  Component 3  │
│   4 units     │  │   2 units     │  │    1 unit     │
└───────────────┘  └───────────────┘  └───────────────┘
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Fig. 2. A product structure

An assembly conforms only if all its components do or if it contains redundant components. Redundant components increase the load on the production system even more than defective items – they are added whether needed or not, and in order to simplify the analysis no redundancy is assumed herein.

3.1   Assembly's actual defect rate

The requirement of more than one component to be conforming creates mutual effects between different components – not all the components should be defective – a single defective component may suffice to disqualify a whole assembly!

Accordingly, the defect rates of assembly operations depend not only on its self-defect-rate, pA, but on the defect rates of its components, too. Consequently, actual defect rate should be calculated for each assembly operation. Let K be the number of component types in an assembly, and mk, the assembly ratio of type k component in this assembly from the BOM. Then, the actual defect rate, P^a_A of the assembly is:

$$p_A^a = 1 - (1 - p_A)\prod_{k=1}^{K}\left(1 - p_k\right)^{m_k} \tag{3}$$

Consider for example the assembly of Figure 2. Suppose the components arrive with defect rates p1, p2, and p3, respectively. An assembly is conforming with probability: (1- p1)4(1- p2)2(1- p3)(1- pA). The complement of this probability is the actual defect rate, P^a_A of the assembly operation. This example demonstrates the dramatic increase of the actual defect rates of assembly operations. If the defect rate of each component is 1% and the defect rate of the assembly operation is 0.1%, then the actual defect rate of the assembly operation is about 7%!

Moreover, this increase is due to the mutual effect among components. If, say, 1,000 assembly units are required, then 1,074 units should be assembled, which contain 4,296 units of component 1, 2,148 units of component 2 and 1,074 units of component 3. As noted, a single defective component suffices to disqualify a whole assembly. Only 11 of the 1,074 units of component 3 are defective, the rest – 63 units are assembled with defective units of other components or a failure occurs during the assembly process. The same holds for the other components. Time and resources are required to diagnose the source of failure of each non-conforming assembly! Additional time and resources are required to fix a defective assembly.

As for the component defect rates; when no defect component arrives at an assembly station, the pk's are all zero, and the multiplication – the Π term in (3) equals 1. This cannot happen in reality and hence, the pk's are strictly positive. How positive? Depend on the defect rate of each operation and on the inspection plan.

3.2   Multiple assemblies

The product in Figure 3 extends the analysis to larger tree-product-structures. It involves two assembly operations, the nodes numbered 10 and 14. The letter in each circle is the operation type and bellow is the defect rate, in percent's, of the corresponding operation. The numbers next to the edges entering assembly operations are the assembly ratios. In order to calculate the actual defect rate of the final assembly, which is numbered 14 in Fig. 3, the actual defect rate of the first assembly, #10, is required.
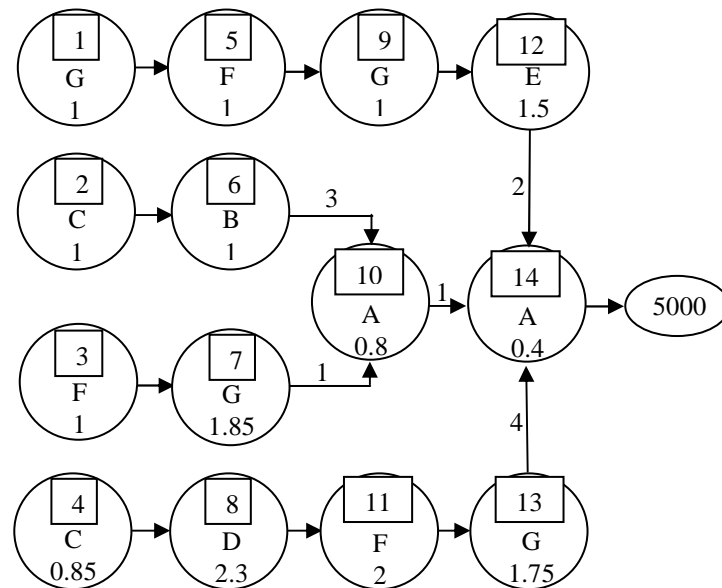

Fig. 3.   A process with two assemblies

Assuming no intermediate inspection, the defect rates arriving at the first assembly from operations 2 and 6 are $100[1-(1-0.99)(1-.99)] = 1.99\%$, and 2.8315% from operations 3 and 7. Hence, the actual defect rate of assembly #10 is $100(1-.9801^3*0.971685*0.992) \approx 9.25\%$. Similarly, the arriving rate on the top branch is 4.425%, and 6.73% on the bottom. Finally, the actual defect rate of the final assembly is $\approx 37.5\%$!!

This example demonstrates a foremost implication of the mutual effects between assemblies' components – the calculations are separated. Defect rates must be calculated forward, first. Then, the quantities are calculated backward, using the assembly ratios of the BOM. If 5,000 end items are required, $8,000 = 5000/(1-0.375)$ units should be assembled. Backward MRP calculations form the three right columns of Table 1, which exposes a much more significant EEDI – the value of quality assurance. Poor quality adds 3 - 7.6% to the required quantities – the ratios between the numbers under perfect inspection to the numbers of a perfect process. 60% are added if no inspection is performed!! Perfect inspection does not exist, and any inspection adds cost and time to the process. A better solution is, of course, process improvements.

Op. #	Op. type	Perfect process	Perfect inspection	No inspection
1	G	10000	10505	16000
2	C	15000	15490	24000
3	F	5000	5208	8000
4	C	20000	21529	32000
5	F	10000	10400	16000
6	B	15000	15335	24000
7	G	5000	5156	8000
8	D	20000	21346	32000
9	G	10000	10296	16000
10	A	5000	5061	8000
11	F	20000	20855	32000
12	E	10000	10193	16000
13	G	20000	20438	32000
14	FA	5000	5020	8000

Table 1. The effect of poor quality – required numbers of units

4   Summary and Conclusions

The contribution of this study is two folds. First, the effect of the interaction between poor quality and assembly operations is explored. This has been achieved via the integration of defect rates with *process charts* and *product structure - BOM*.

The most significant conclusion emanates from Table 1. Even if all stations are in-control and performed as specified, still, many defective items are produced – the HIDDEN FACTORY. Consequently, quality should be considered right from the very beginning – the facility design stages. The numerical data of and BOMs can be easily stored in electronic spreadsheets, which can use it to compare alternatives; e.g., re-placing a station with one of higher quality.

Another conclusion is the effect of the mutual effects among assembly's compo-nents – the rapid growth of the actual defect rates. This, too, should be considered during product and process design.

Finally, the stochastic nature of defects' occurrence should be considered. The cal-culations presented are of means and expected values but individual numbers are random variables. This strongly affects production management an effect which re-quires future research.

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