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## Stochastic subgrid tensor for geophysical flow modeling

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# STOCHASTIC SUBGRID TENSOR FOR GEOPHYSICAL MODELING

Valentin Resseguier, Etienne Mémin, Bertrand Chapron

## Motivations

- Rigorous identification of subgrid dynamics effects
- Taking into account likely small-scale dynamics (stochastic backscatter)
- Forecast of likely distinct scenarios
- Quantification of the dynamics errors:
  - Diagnostic for numerical simulations (mesh refinements, ...)
  - Ensemble data assimilation and forecasts**

## Transport under location uncertainty

### Assumption:

Velocity decomposition:  $v = w + \sigma \dot{B}$  with

- $w$  large-scale velocity
- $\sigma \dot{B}$  uncorrelated in time, divergence-free, correlated in space, Gaussian, inhomogeneous and anisotropic

From stochastic calculus theory, we proved:

$$\frac{D\theta}{Dt} = \partial_t \theta + w^* \cdot \nabla \theta + \sigma \dot{B} \cdot \nabla \theta - \nabla \cdot \left( \frac{1}{2} a \nabla \theta \right) = 0$$

$$w^* = w - \frac{1}{2} \nabla \cdot a$$

$$a = \sigma \sigma^T$$

Drift correction

Multiplicative random forcing

Balanced energy exchanges

## Systematic derivation of random models with the new $\frac{D}{Dt}$

Conservations (mass, linear momentum, ...)

Navier-Stokes

Boussinesq

Uncertainty

QG MU

## SQG under Strong Uncertainty

### Description

- QG assumption under strong uncertainty:
- kills the interior dynamics
  - creates horizontal velocity divergence
  - provides a diagnostic relation

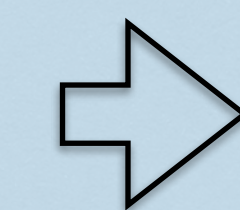
### Modified geostrophic balance

Geostrophic balance

$$f \times u = -\frac{1}{\rho_b} \nabla p'$$

Horizontal Diffusion

$$+ \frac{a}{2} \Delta u$$



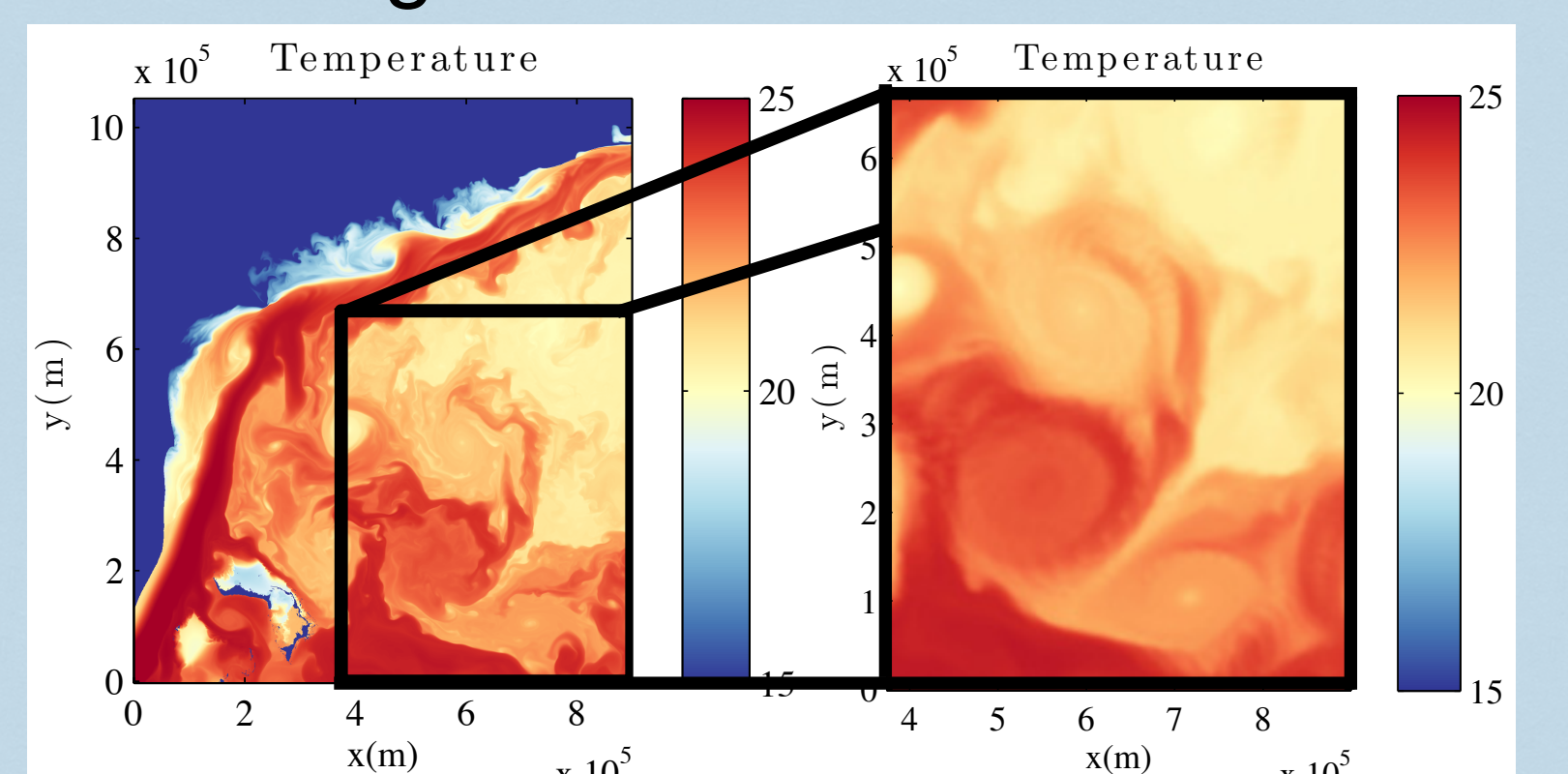
$$\nabla \cdot u \propto \Delta \nabla^\perp \cdot u$$

### Testbed data:

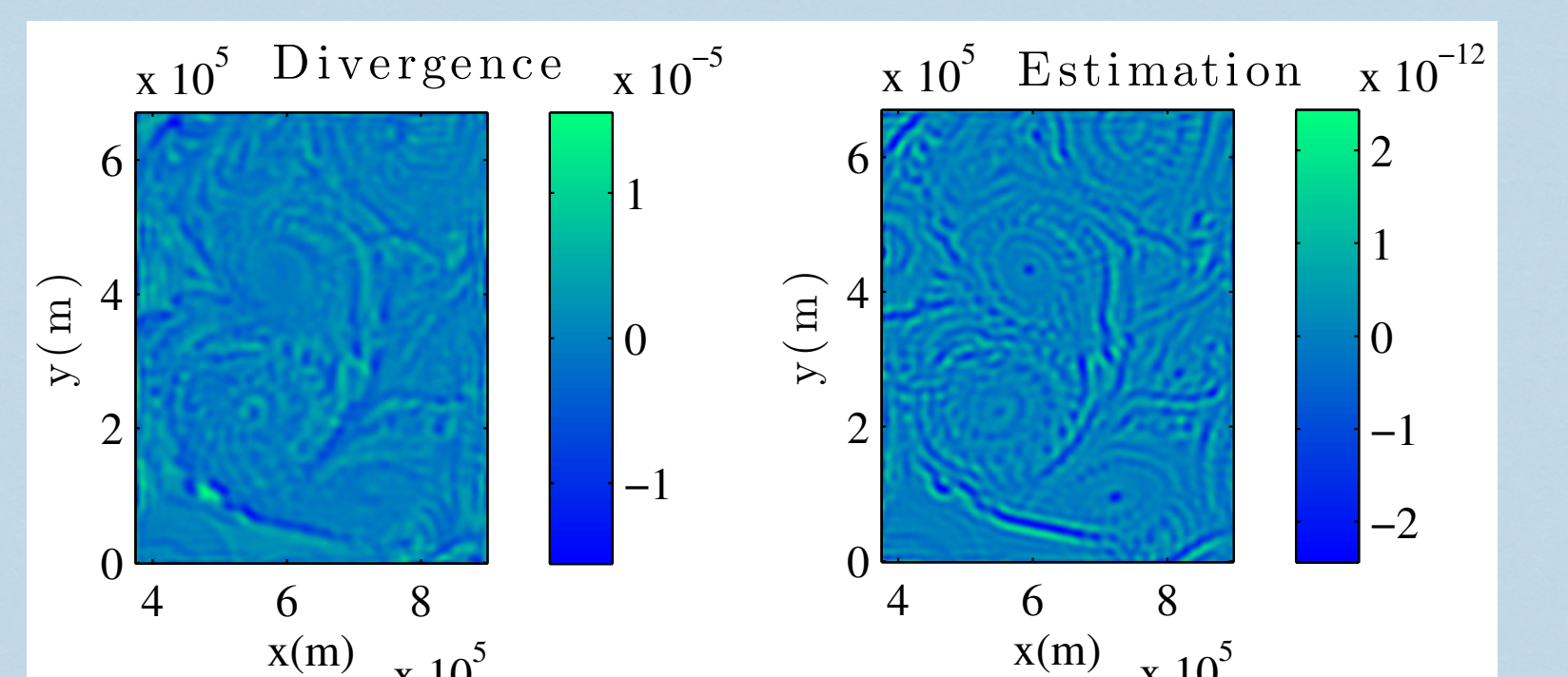
very high-resolution model outputs  
Gula, J., Molemaker, J., and McWilliams J.  
"Gulf Stream dynamics along the southeastern US seaboard."  
*Journal of Physical Oceanography* 45.3 (2015): 690-715.

Original

Filtered



### Results



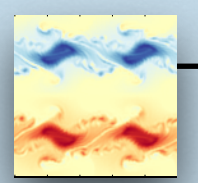
### Conclusion

- Frontolysis / frontogenesis on warm / cold side of fronts
- Estimation of horizontal divergence or diffusive coefficient and subgrid variance
- Strong uncertainty assumption verified *a posteriori* for the Gulf Stream during wintertime at mesoscales

## SQG under Moderate Uncertainty

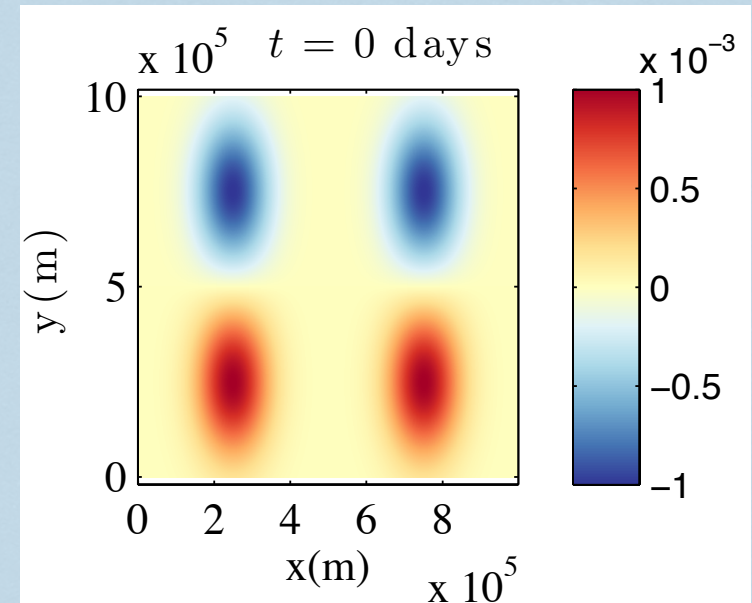
### Description

- Usual SQG relationship and transport under uncertainty of buoyancy
- Good illustration of our stochastic transport
- Simulations done with homogeneous small-scale velocity

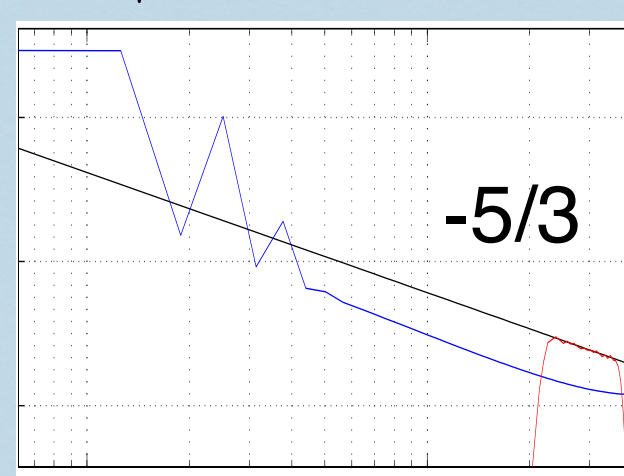


### One realization: better small-scale representation

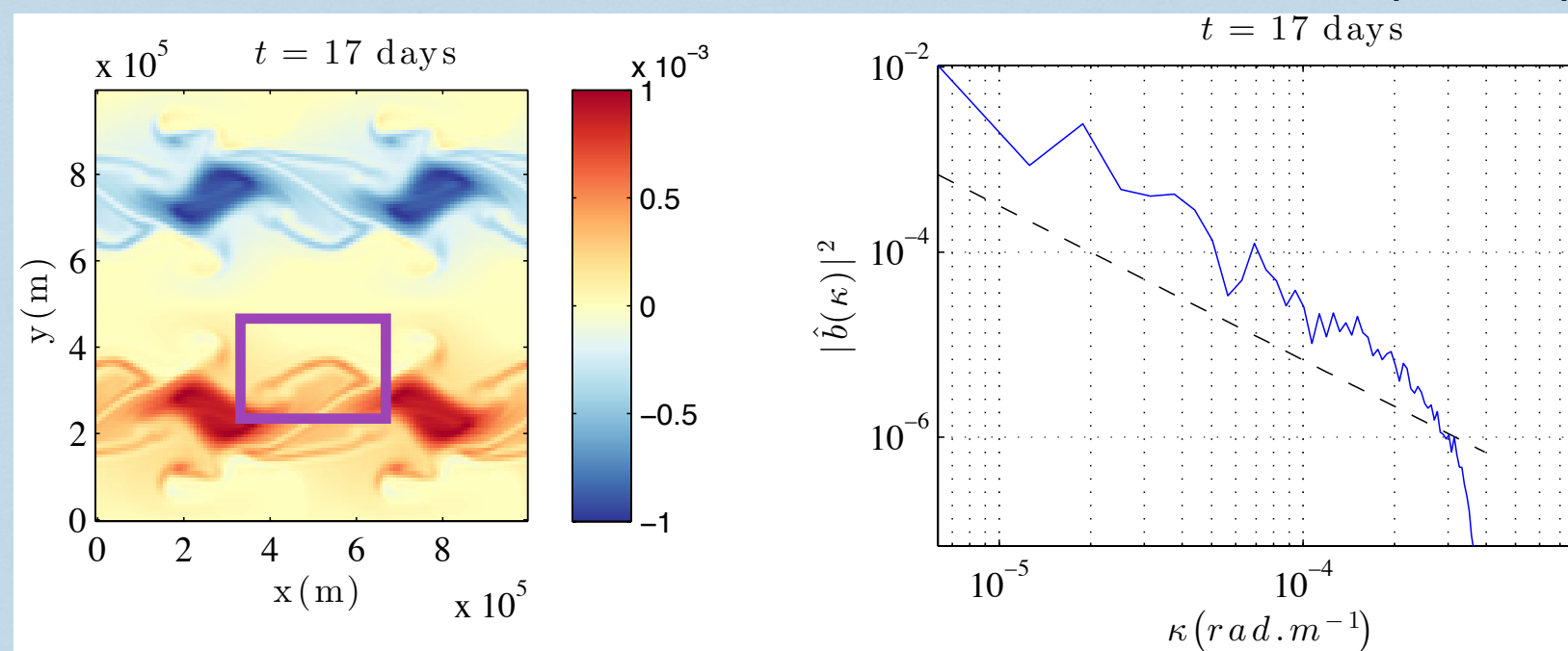
Initial condition



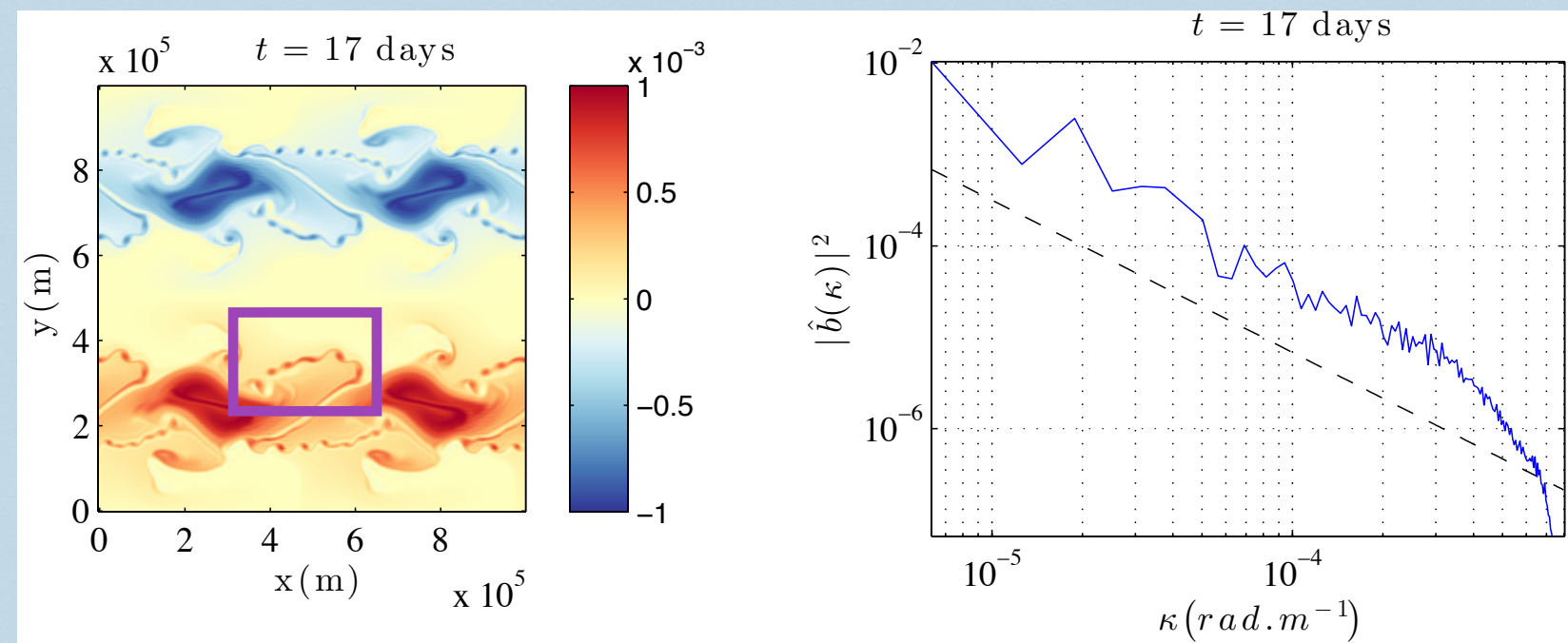
Spectrum of  $w|_{t=0}$  and  $\sigma \dot{B}$



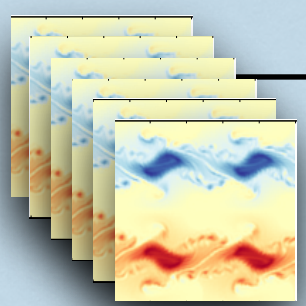
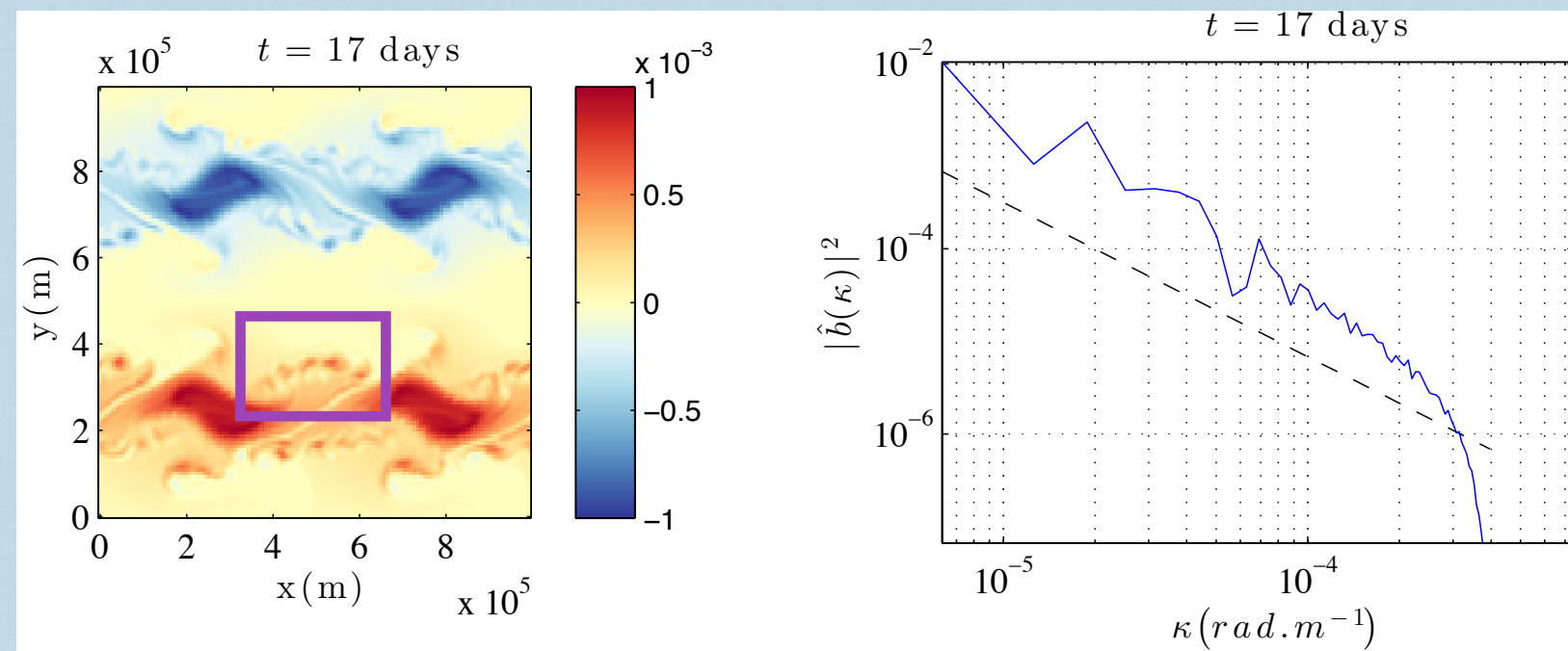
LR deterministic simulation (128<sup>2</sup>)



Reference: HR deterministic simulation (512<sup>2</sup>)

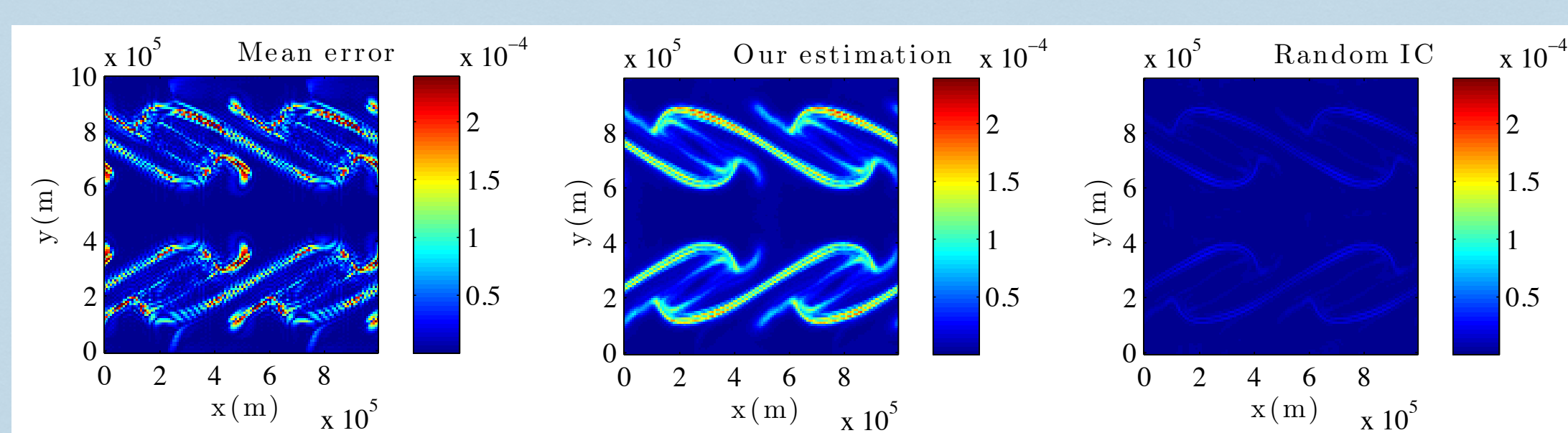


LR stochastic simulation (128<sup>2</sup>)

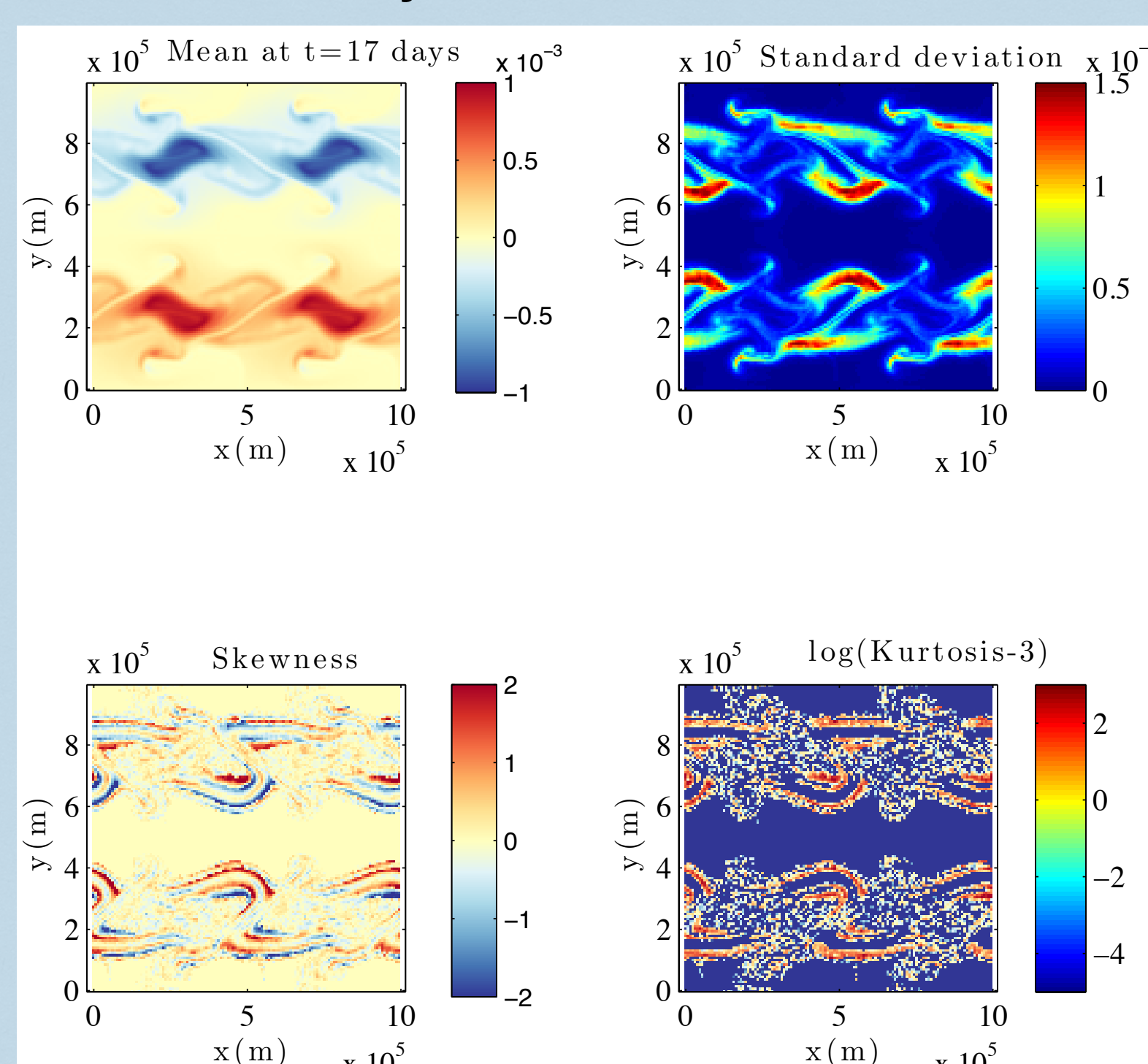


### Ensemble of simulations

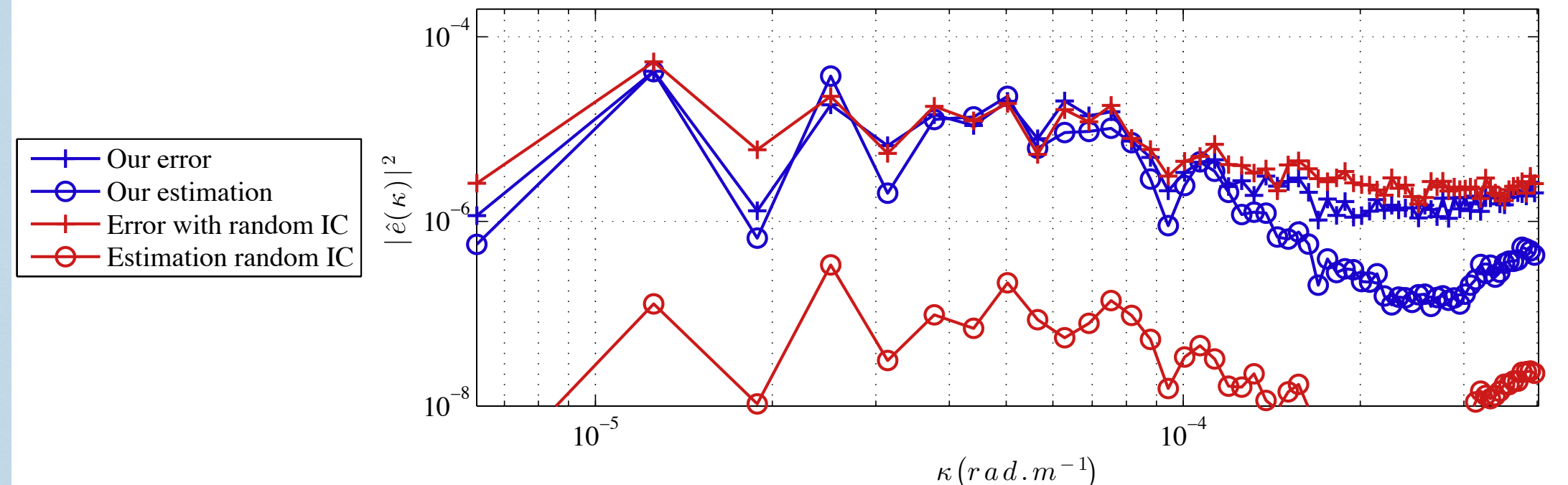
#### Errors estimation



#### Point-wise moments: variability, extreme events, ...



Spectrum of the errors and its estimation at t=13 days



### Conclusion

- Random transport applicable to any dynamics
- Better representation of small scales
- Extreme events
- Good errors estimation in location and amplitude**
- Likely scenarios

code available online: [link from Fluminance website](#) - V. Resseguier