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Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

Valentin Resseguier,
Etienne Mémin,
Bertrand Chapron

Motivations for deriving random fluid dynamics models

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Quantification of modeling errors:
 - especially for Data assimilation: ensemble forecasts
- **Predicting possible distinct scenarios**

Contents

- Chaotic transitions
- Dynamics under location uncertainty
- Ensemble of simulations

Chaotic transitions

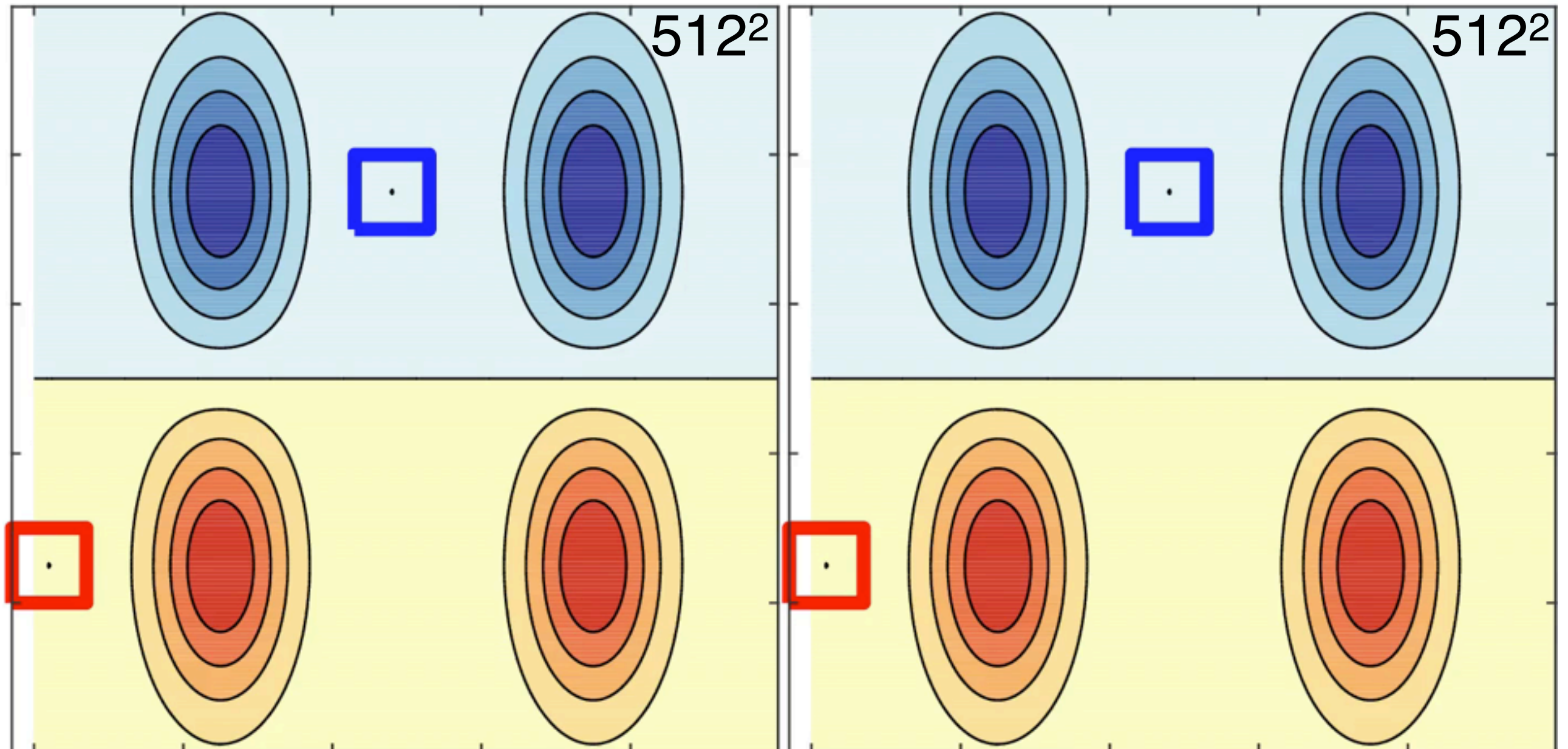
in SQG dynamics

Reference flow: deterministic SQG 512^2

Initial condition 1  Scenario 1

$t = 0$ days

$t = 0$ days

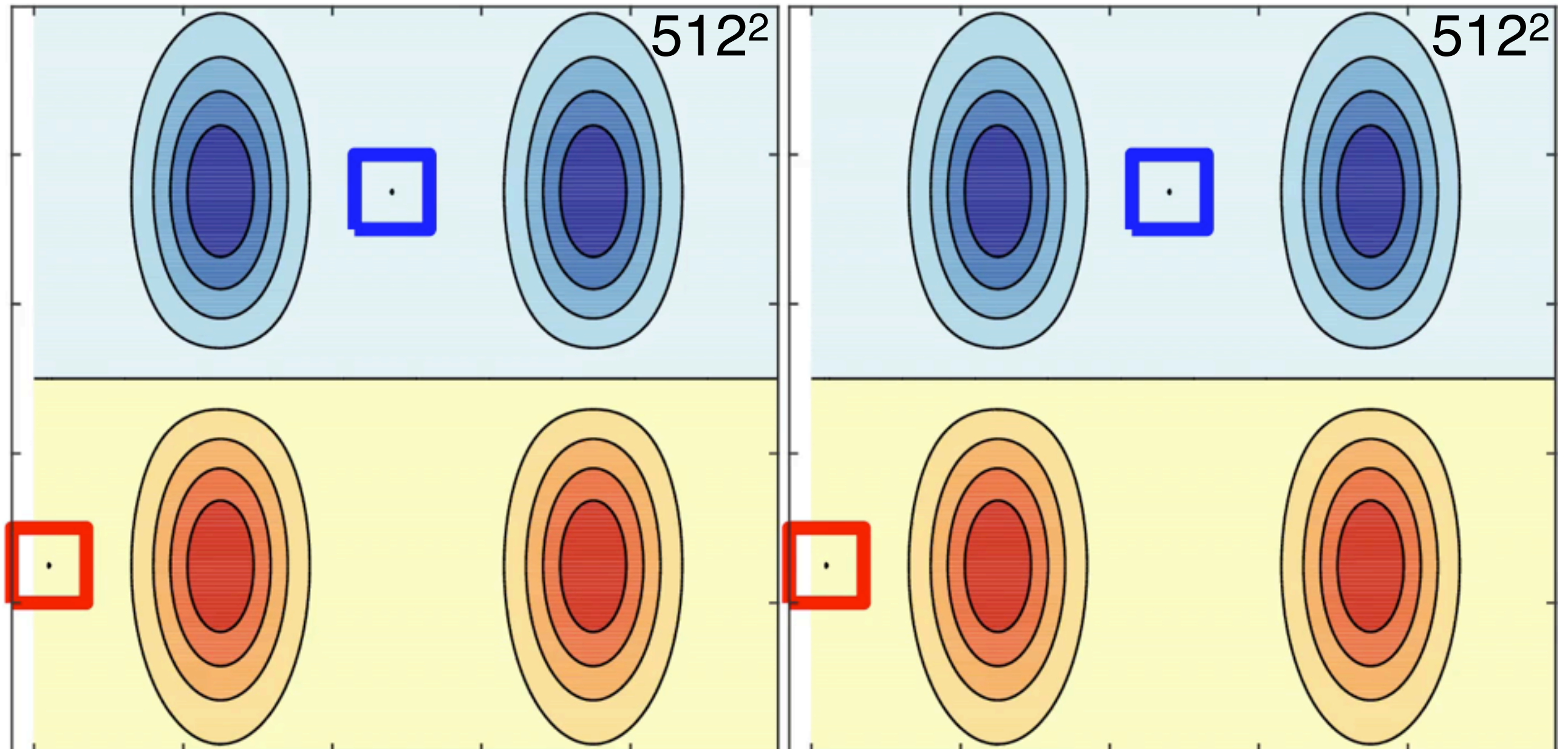


Reference flow: deterministic SQG 512^2

Initial condition 1  Scenario 1

$t = 0$ days

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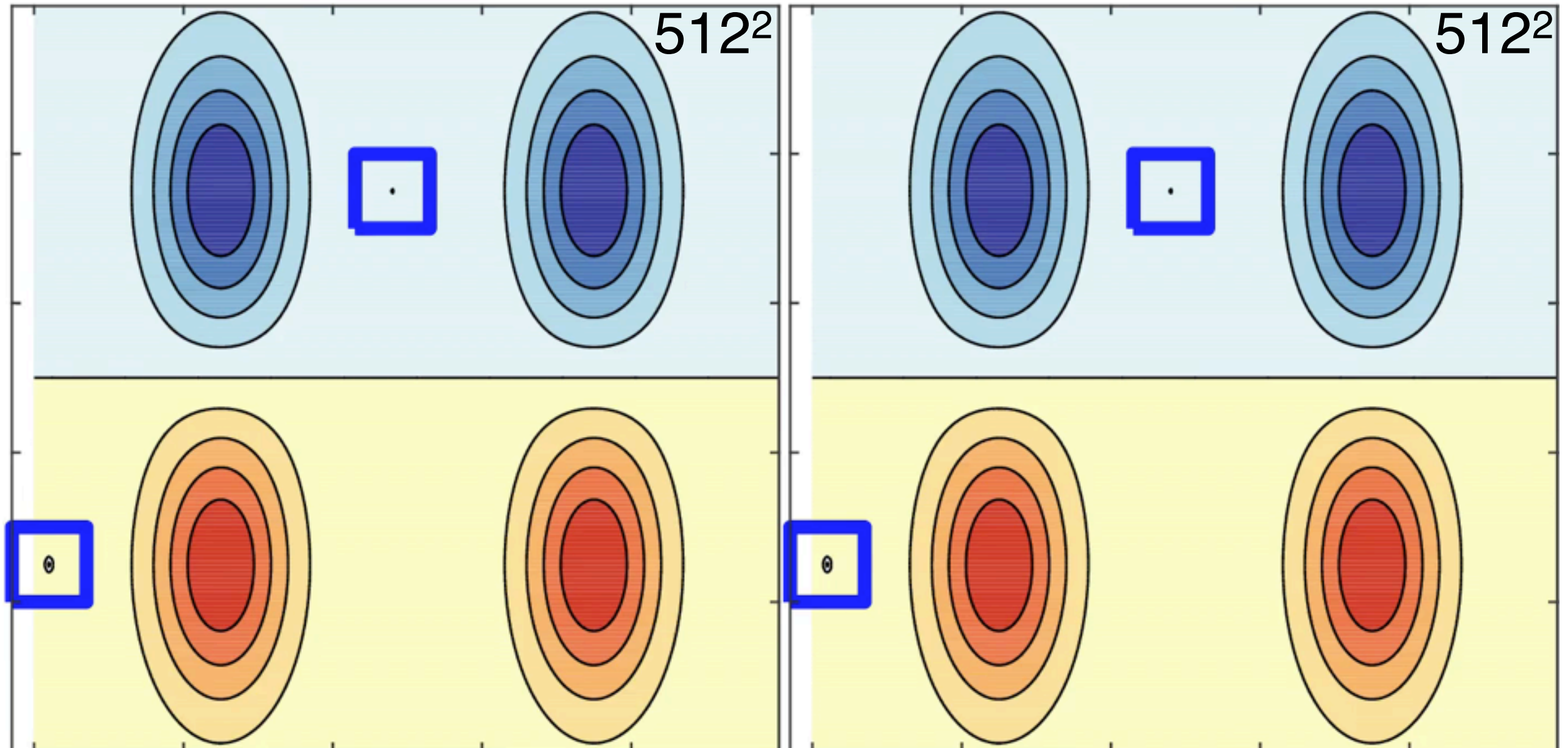


Reference flow: deterministic SQG 512^2

Initial condition 2 \rightarrow Scenario 2

$t = 0$ days

$t = 0$ days

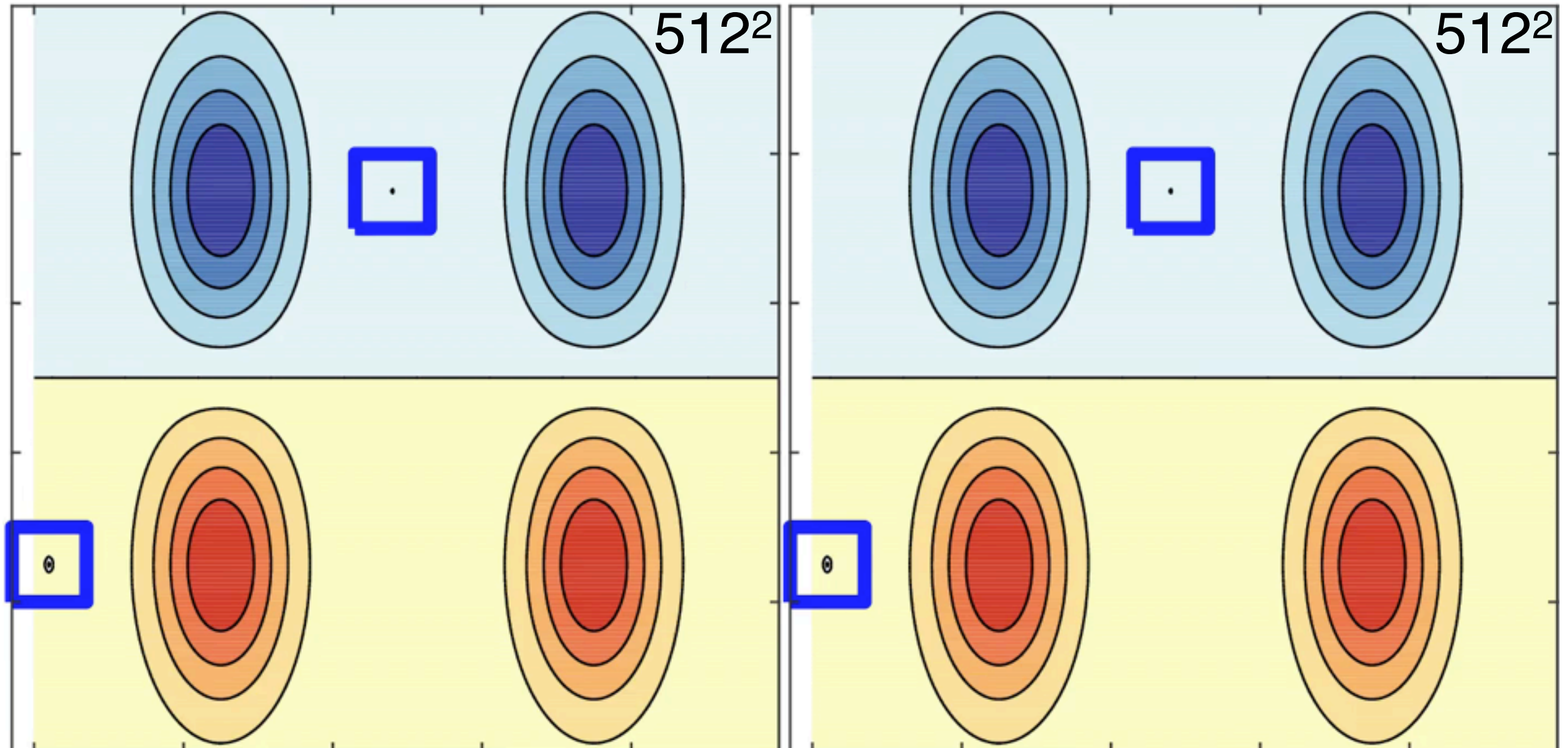


Reference flow: deterministic SQG 512^2

Initial condition 2 \rightarrow Scenario 2

$t = 0$ days

$t = 0$ days



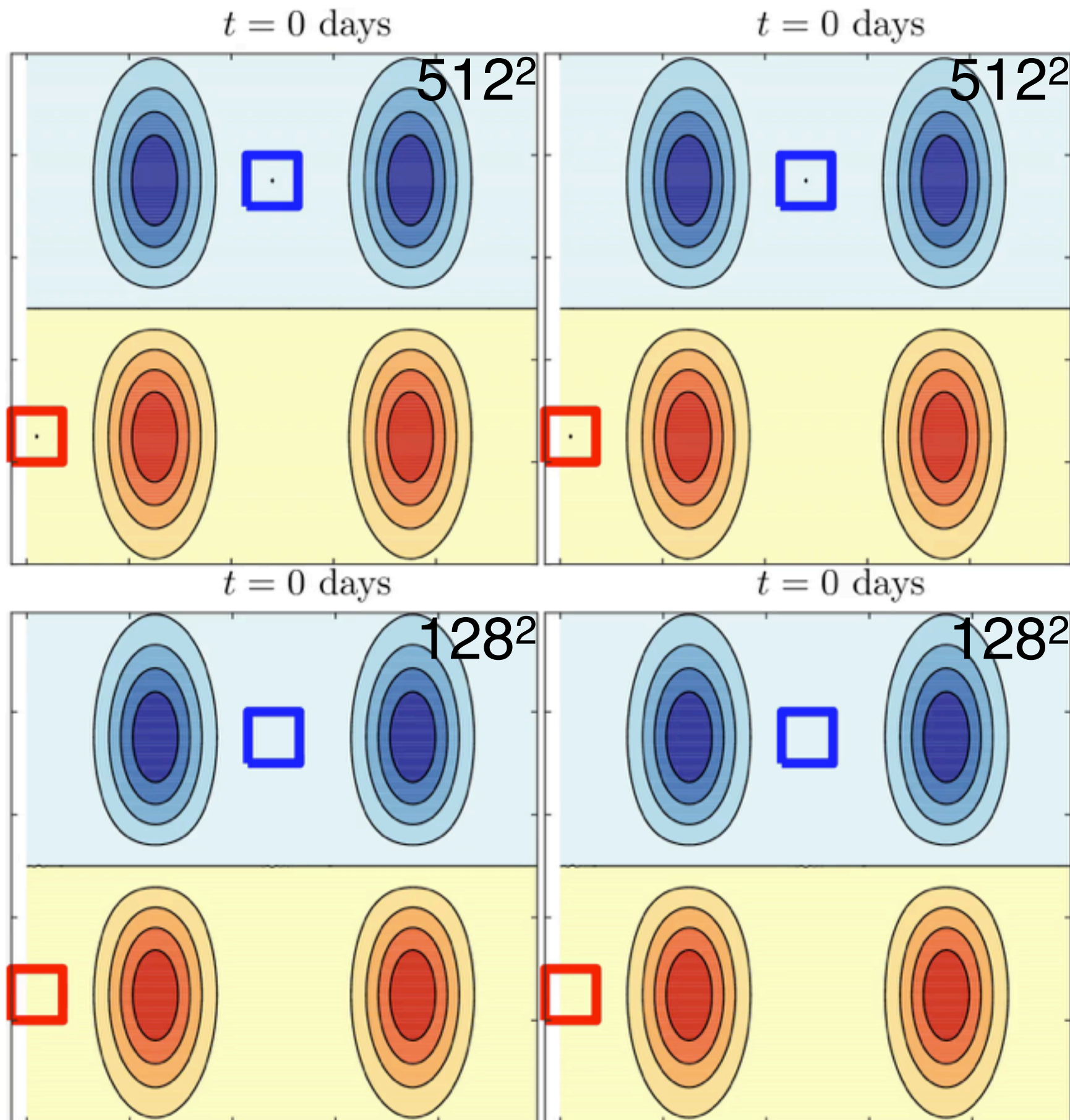
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1



?



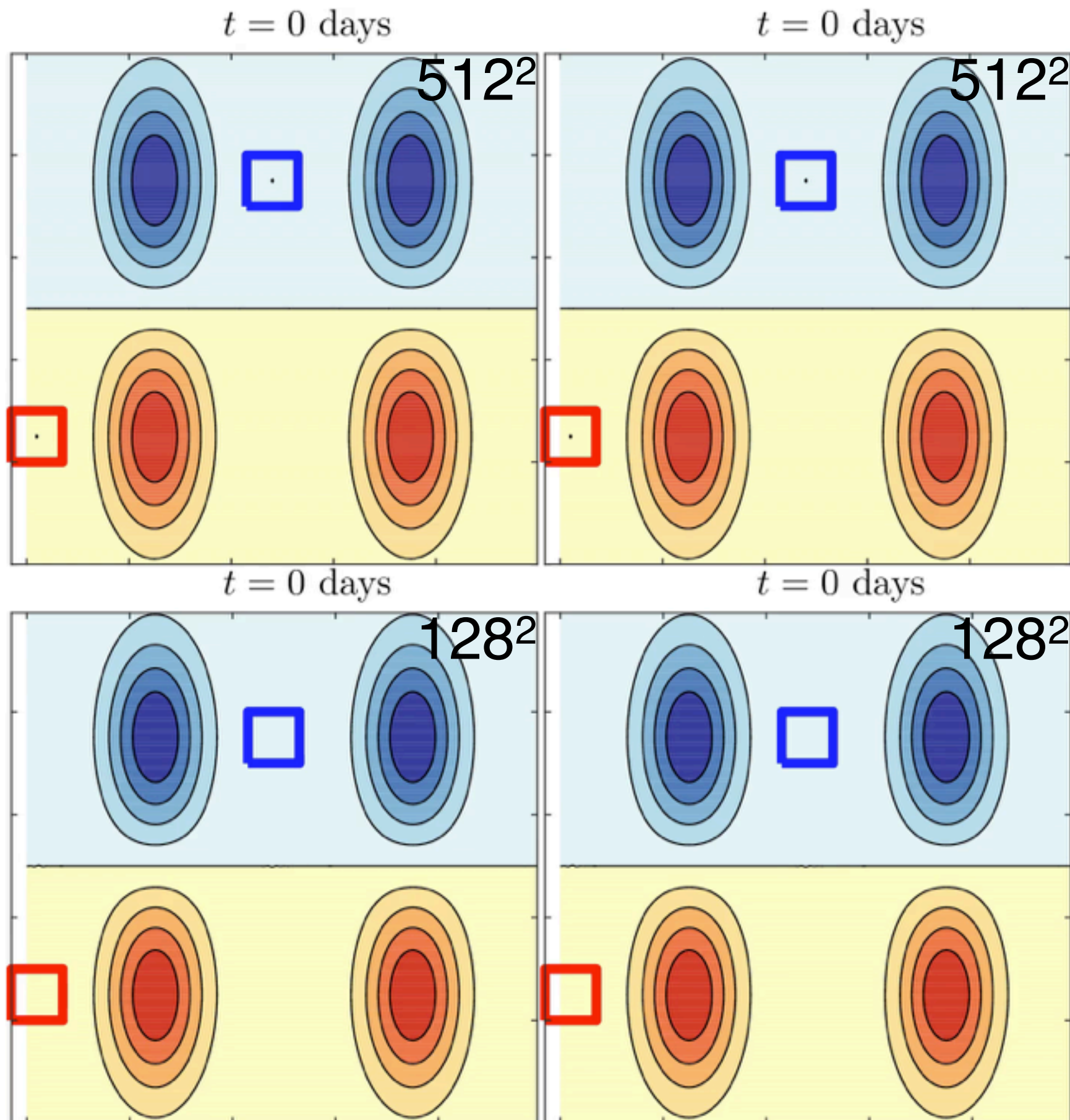
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1

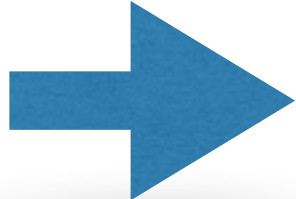
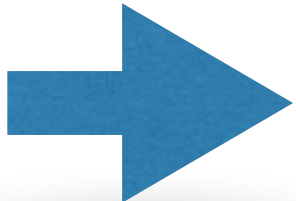
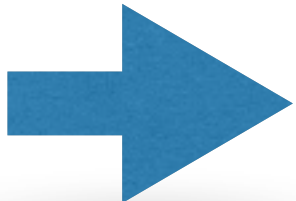



?



Dynamics under location uncertainty

Random equations

- Random initial conditions  Underdispersive + need large ensemble
- Arbitrary Gaussian forcing  Adding energy + wrong phase
- Averaging, homogenization  Assumptions and energy issues
- Adding white random velocity  $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

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Drift correction

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The equation is divided into two main components:

- Advection (Blue Box):** This part contains the terms $\partial_t \Theta$, $w^* \cdot \nabla \Theta$, and $\sigma \dot{B} \cdot \nabla \Theta$.
 - An orange arrow labeled "Drift correction" points to the w^* term.
 - A purple arrow labeled "Multiplicative random forcing" points to the $\sigma \dot{B} \cdot \nabla \Theta$ term.
- Diffusion (Green Box):** This part contains the term $\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$.

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

Drift correction

Multiplicative random forcing

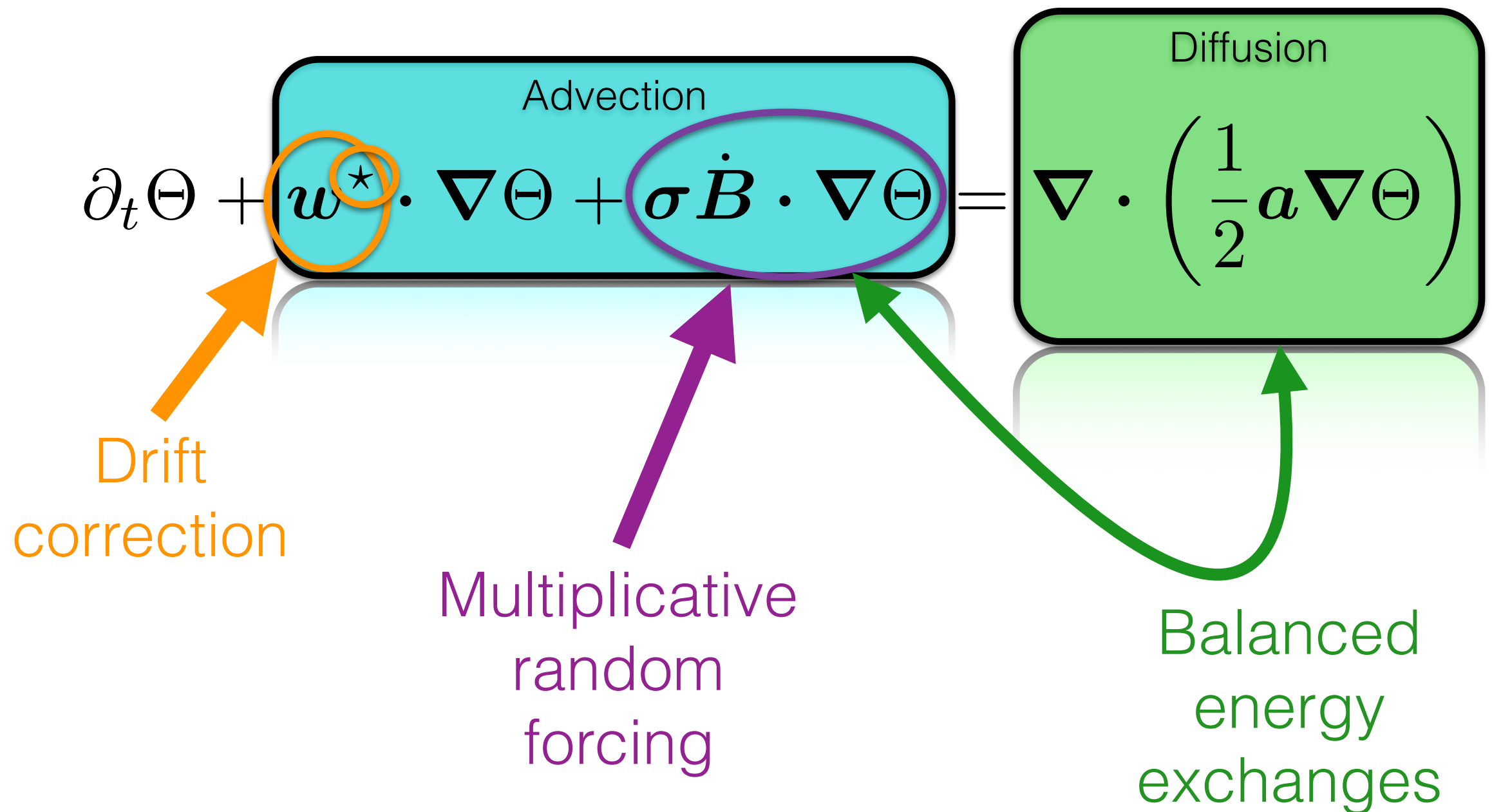
Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The left side of the equation is labeled "Advection" and is enclosed in a cyan box. The right side is labeled "Diffusion" and is enclosed in a green box. The term w^* is circled in orange, and the term $\sigma \dot{B}$ is circled in purple. An orange arrow points from the text "Drift correction" to w^* , and a purple arrow points from the text "Multiplicative random forcing" to $\sigma \dot{B}$. A green arrow points from the diffusion box back to the advection box.

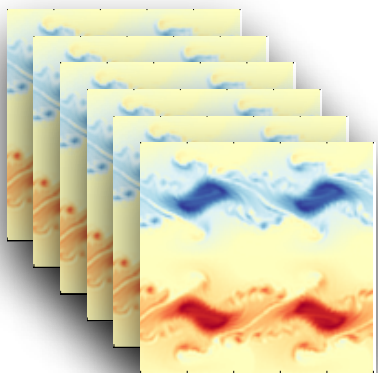
Advection of tracer Θ



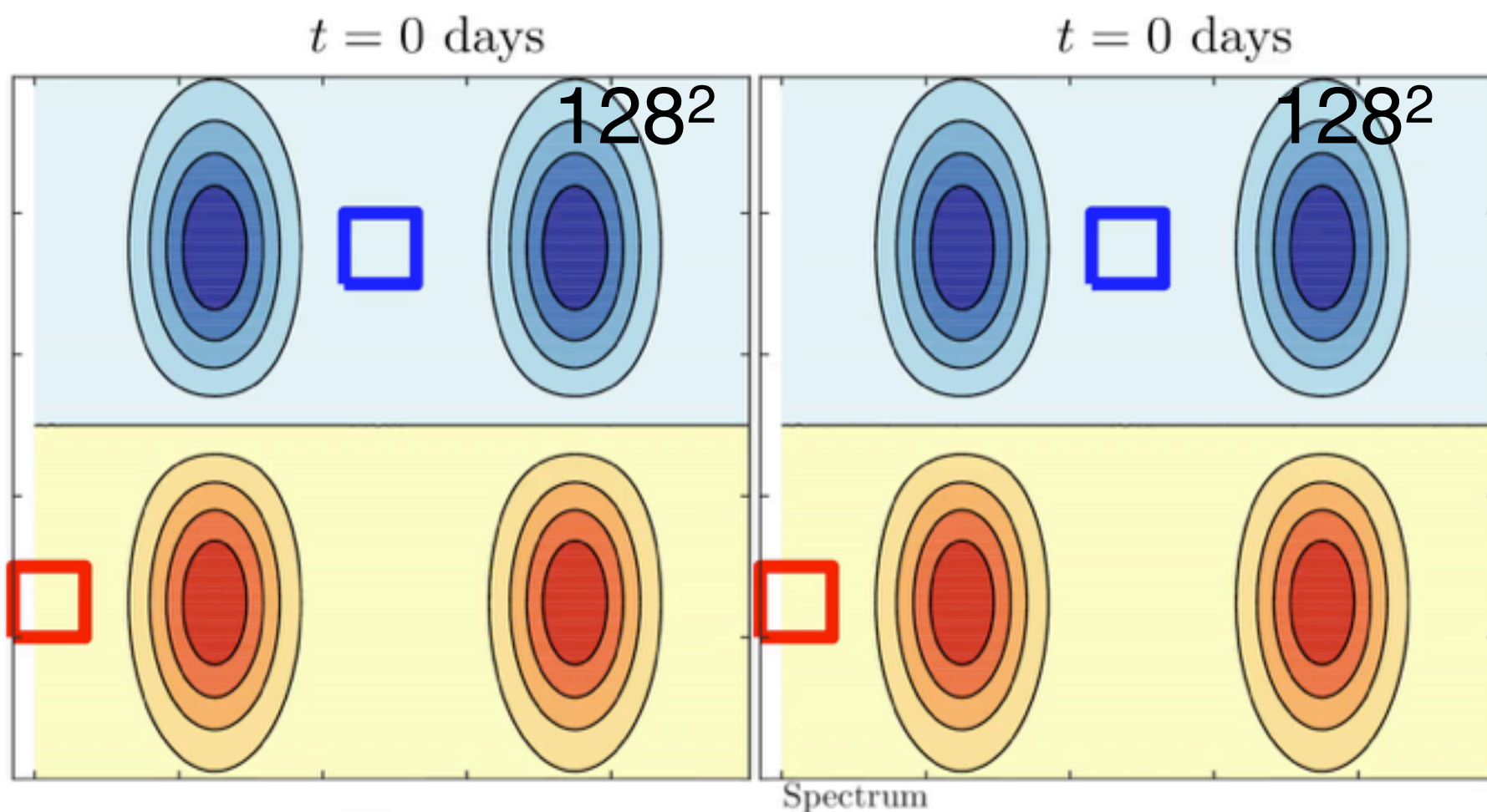
Ensemble of simulations

with SQG MU

Code available online

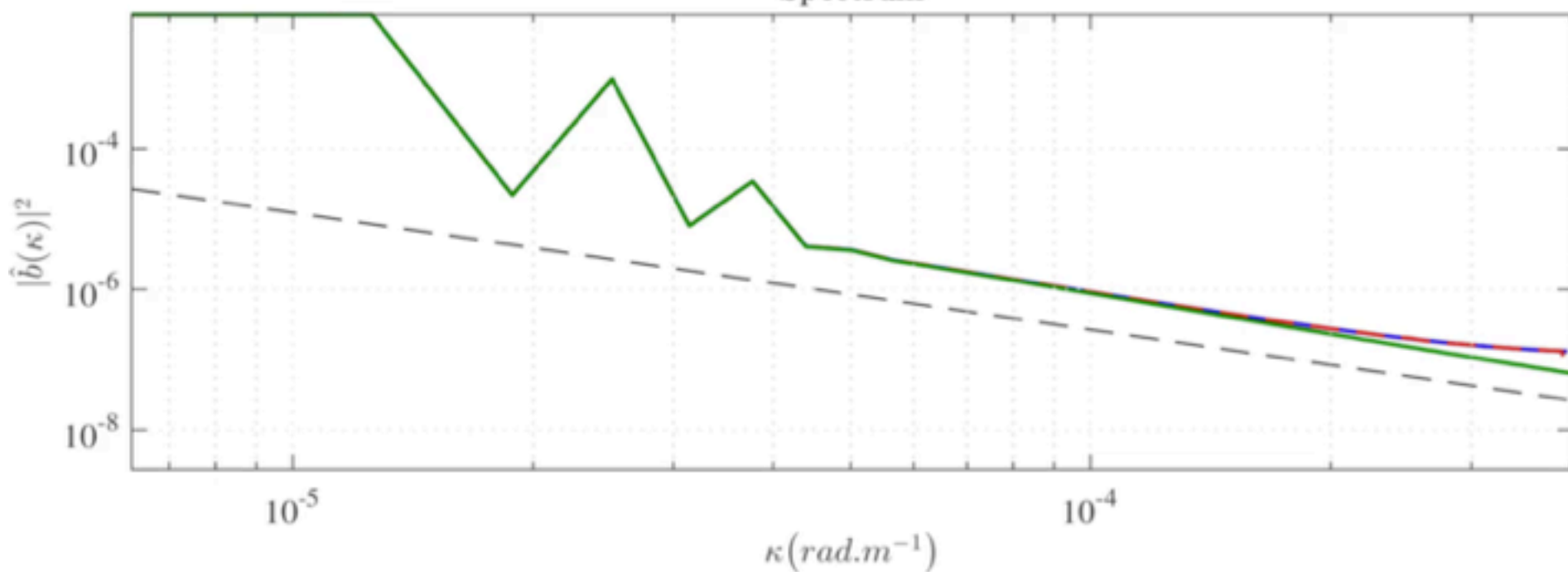


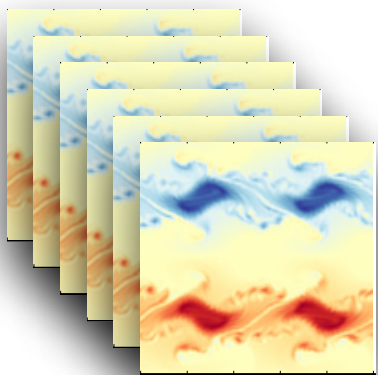
examples of realizations:



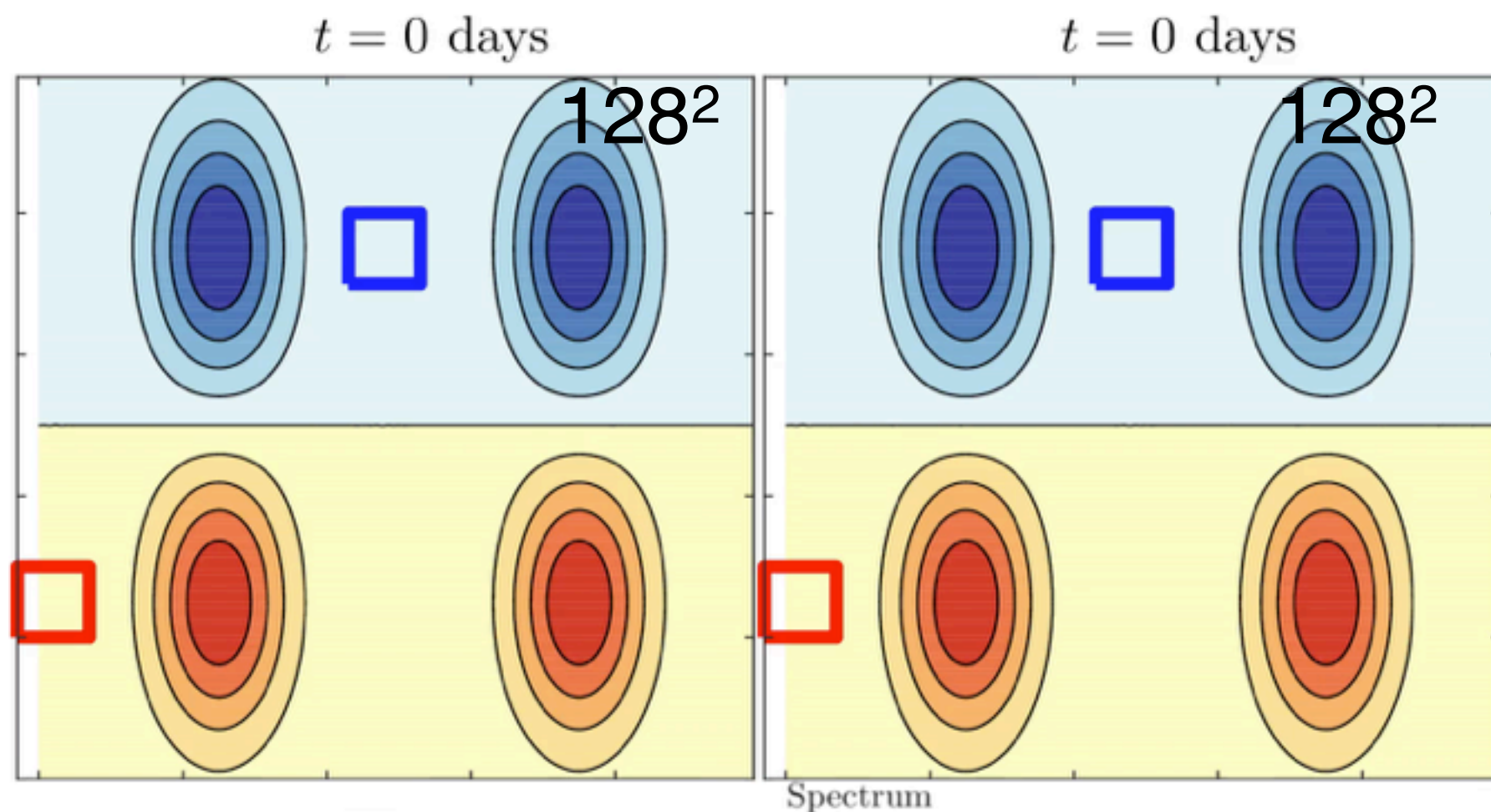
Spectrum

mean²/variance:

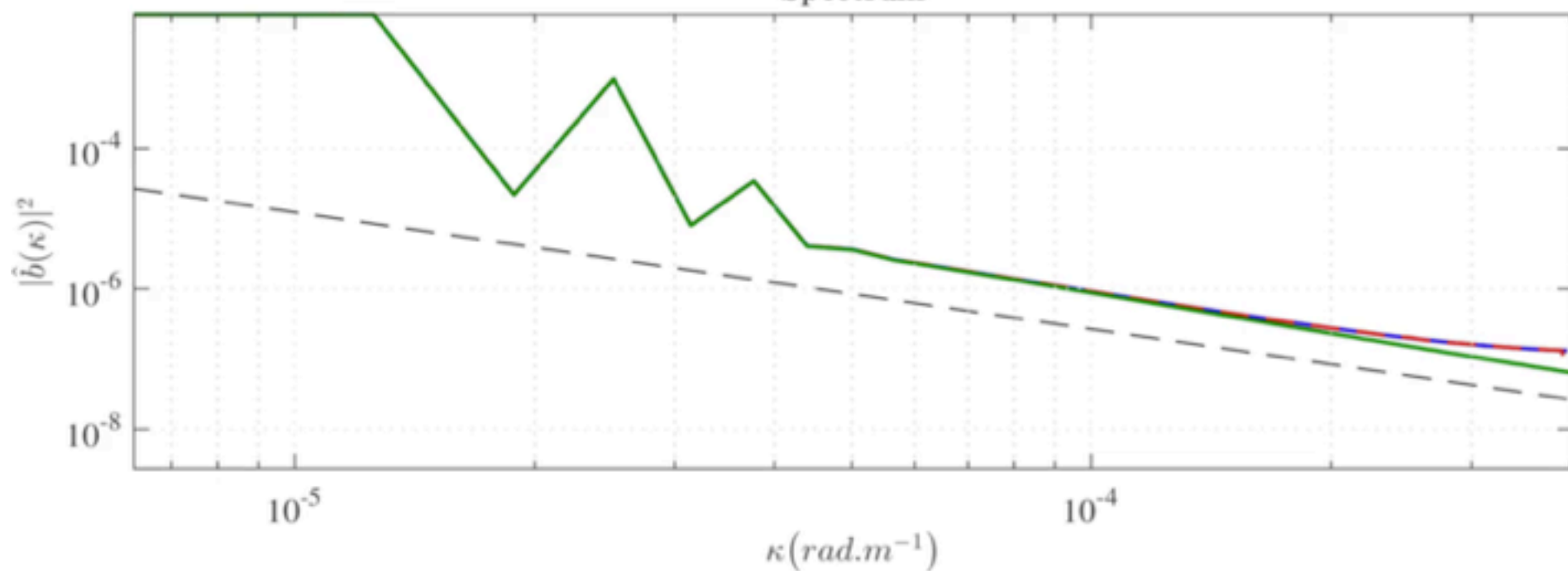


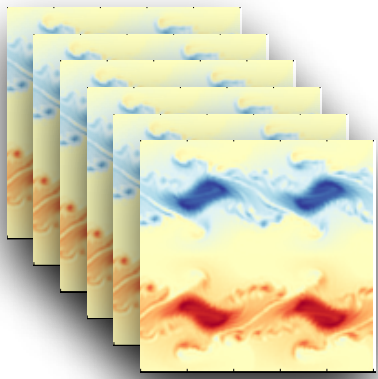


examples of realizations:



Spectrum
 mean²/variance:



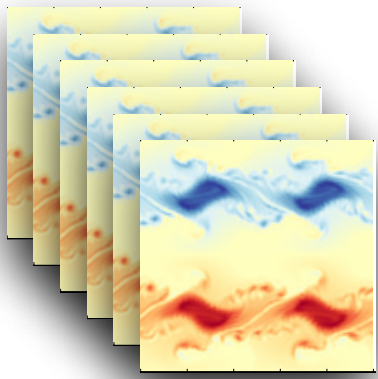


Visualization of the ensemble

At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$



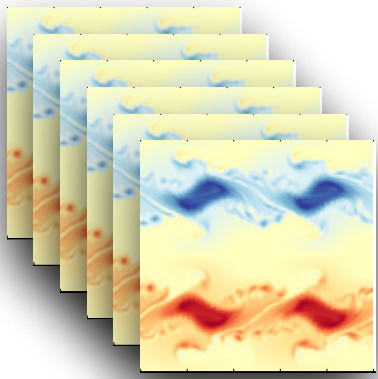
Visualization of the ensemble

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i^{th} realization



Visualization of the ensemble

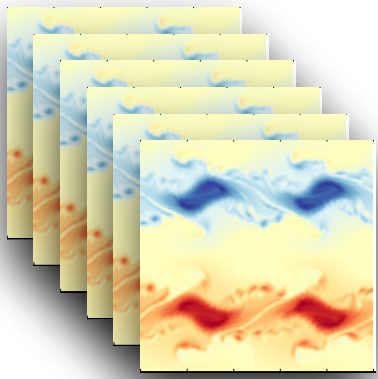
At a fixed time t ,

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i^{th} realization

Mean



Visualization of the ensemble

At a fixed time t ,

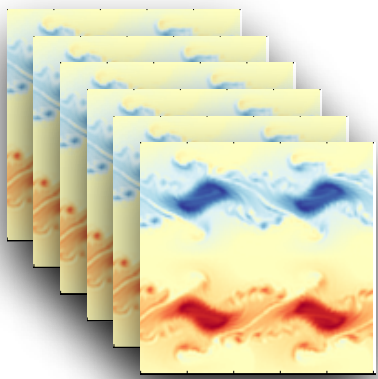
Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization

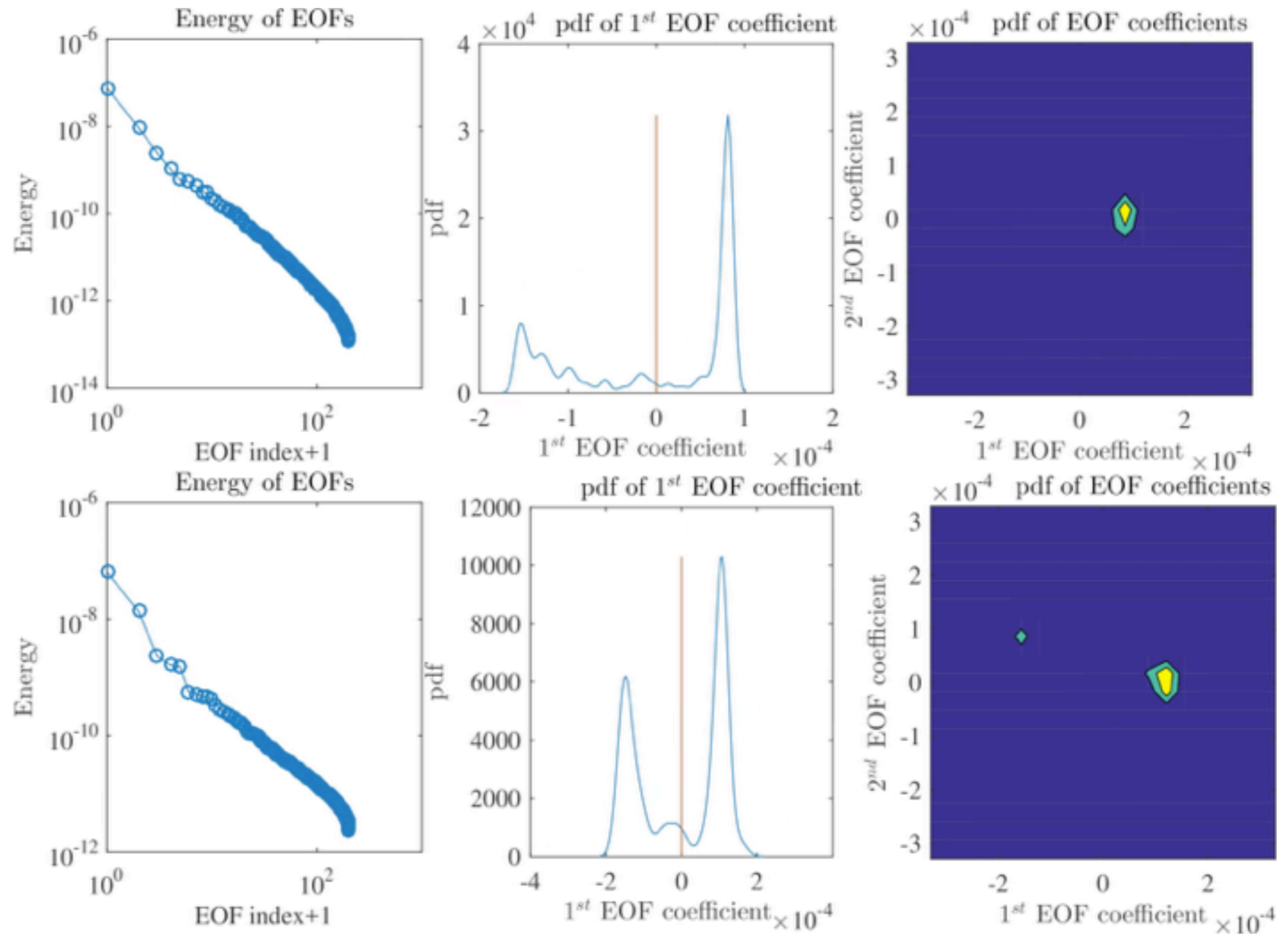
Mean

(random) EOF coefficient

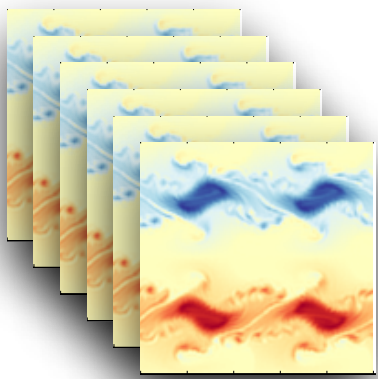


Visualization of the ensemble (200 realizations)

Random initial conditions

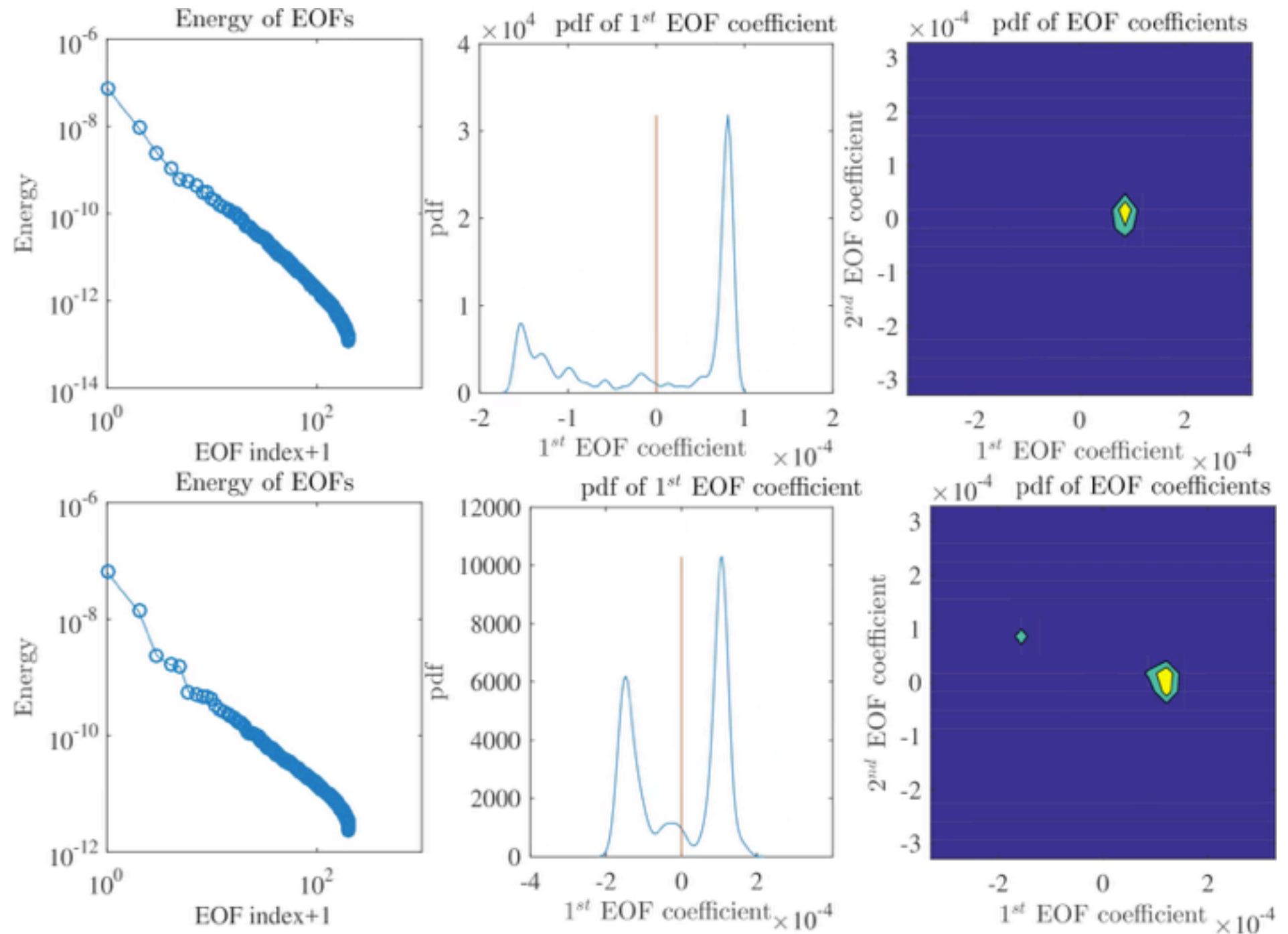


Under location uncertainty

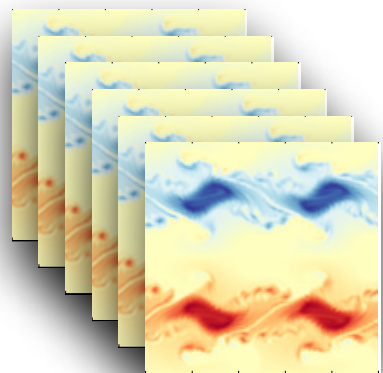


Visualization of the ensemble (200 realizations)

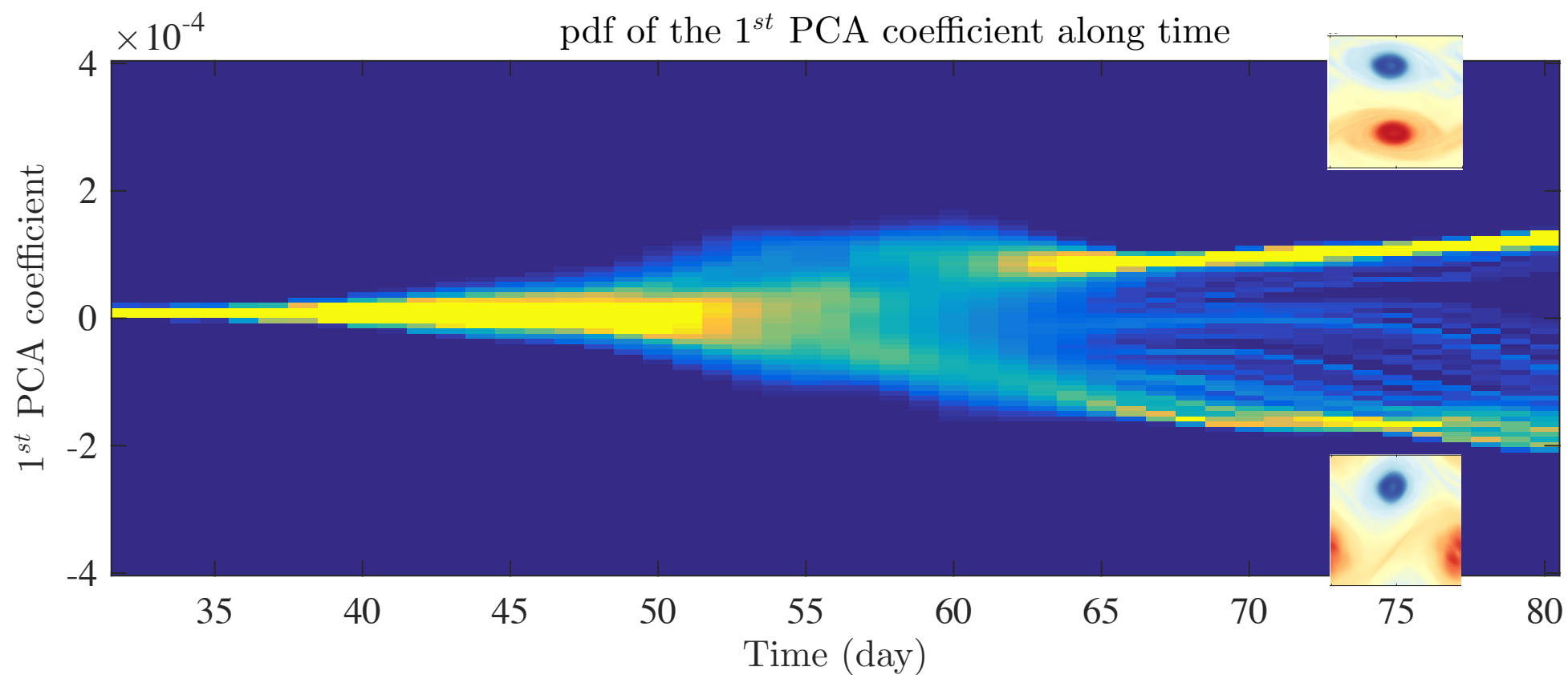
Random initial conditions



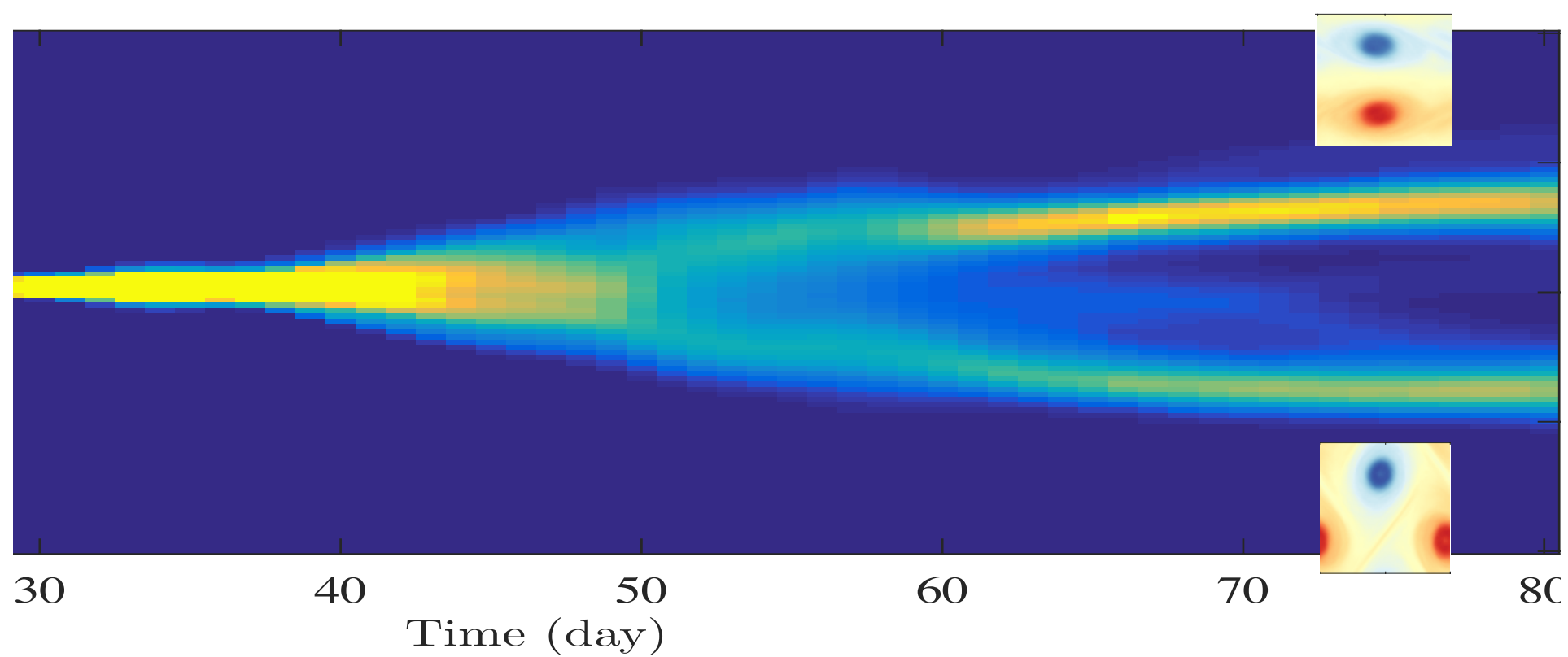
Under location uncertainty



Random
initial
conditions



Under
location
uncertainty



Conclusion

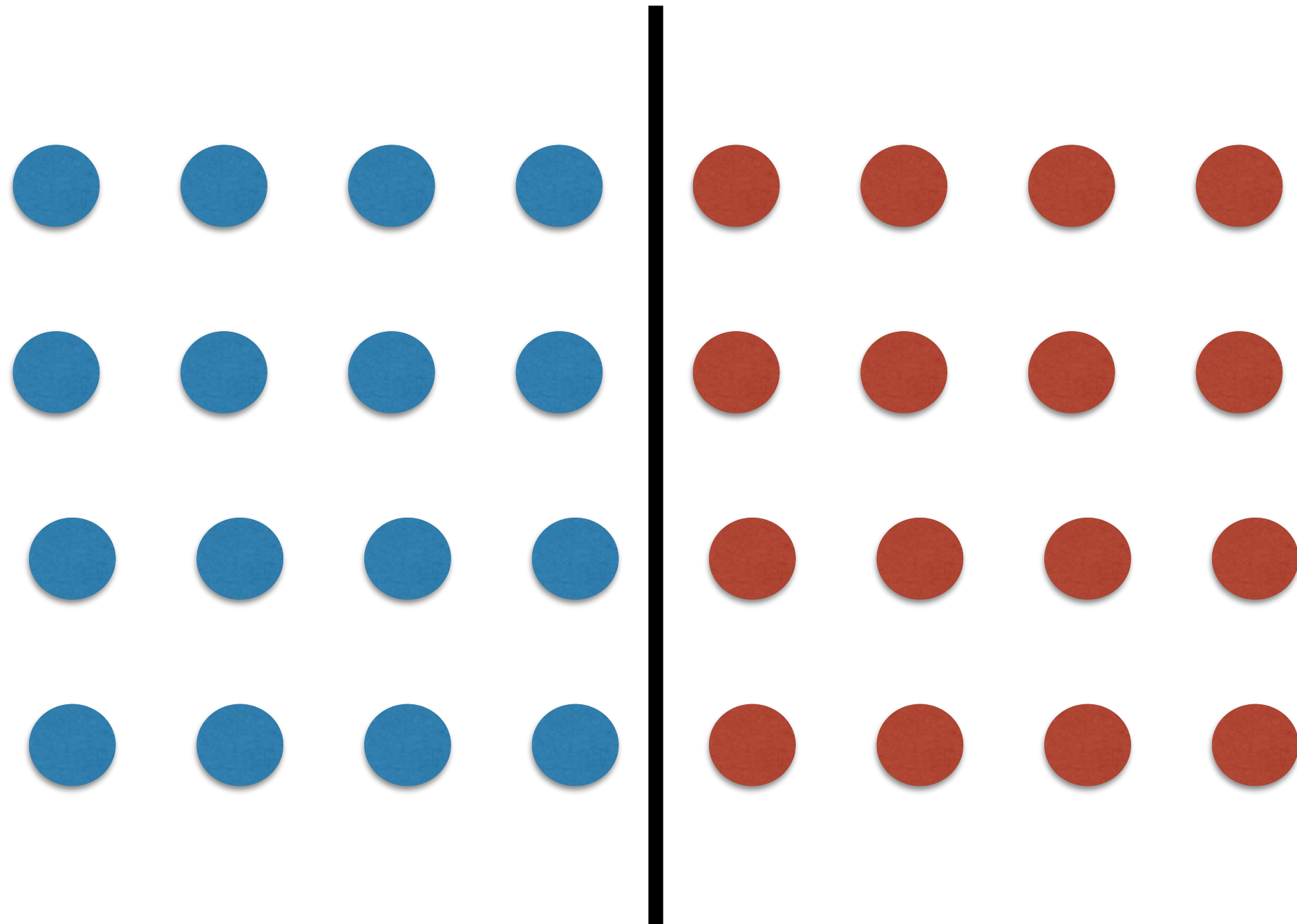
Conclusion

- Random transport applicable to any dynamics
- Link (inhomogeneous) diffusion, drift correction and multiplicative noise
- Predict likely scenarios with few realizations
- Other results:
 - Better small scales
 - Estimate positions and amplitudes of errors
 - Extreme events
 - Additional physical information

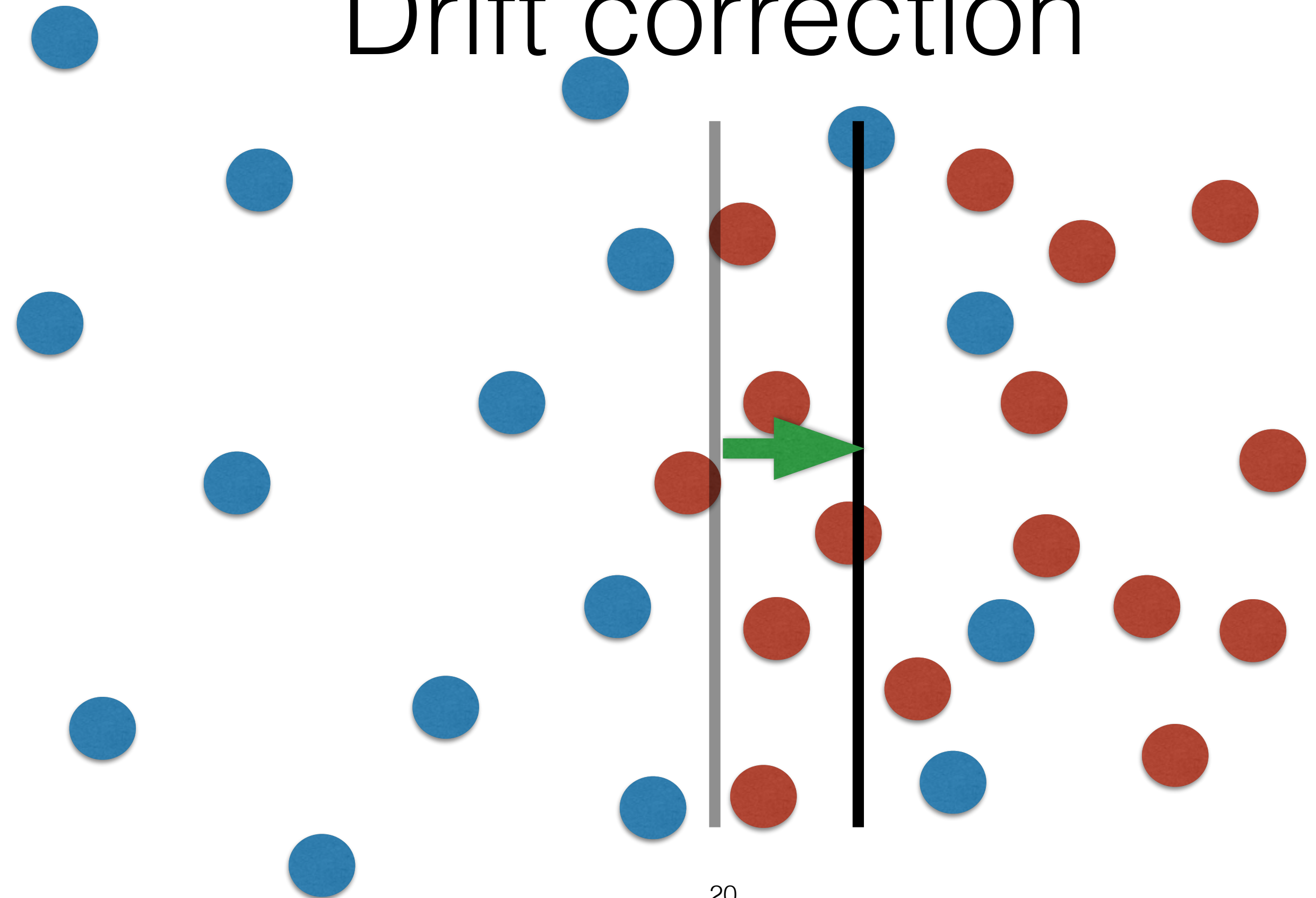
Thank you for your attention

Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

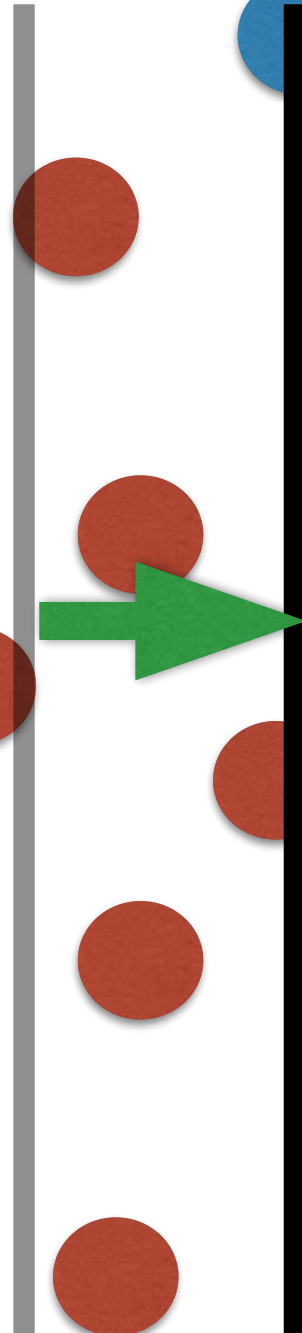


Drift correction



Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot \mathbf{a})^T$$



SQG under Moderate Uncertainty

SQG MU

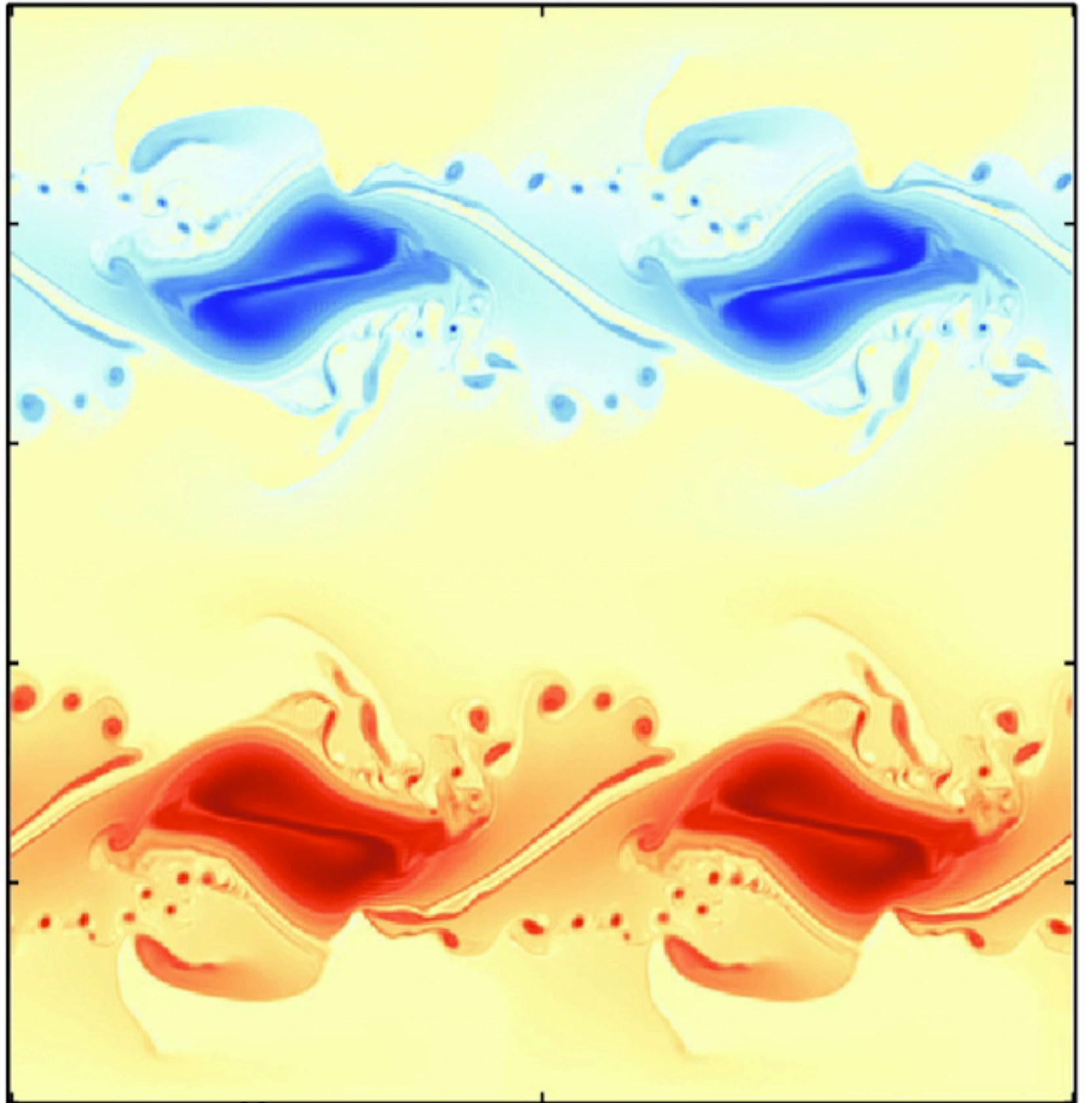
Code available online

$t = 17$ days

Reference flow:
deterministic

SQG

512 x 512



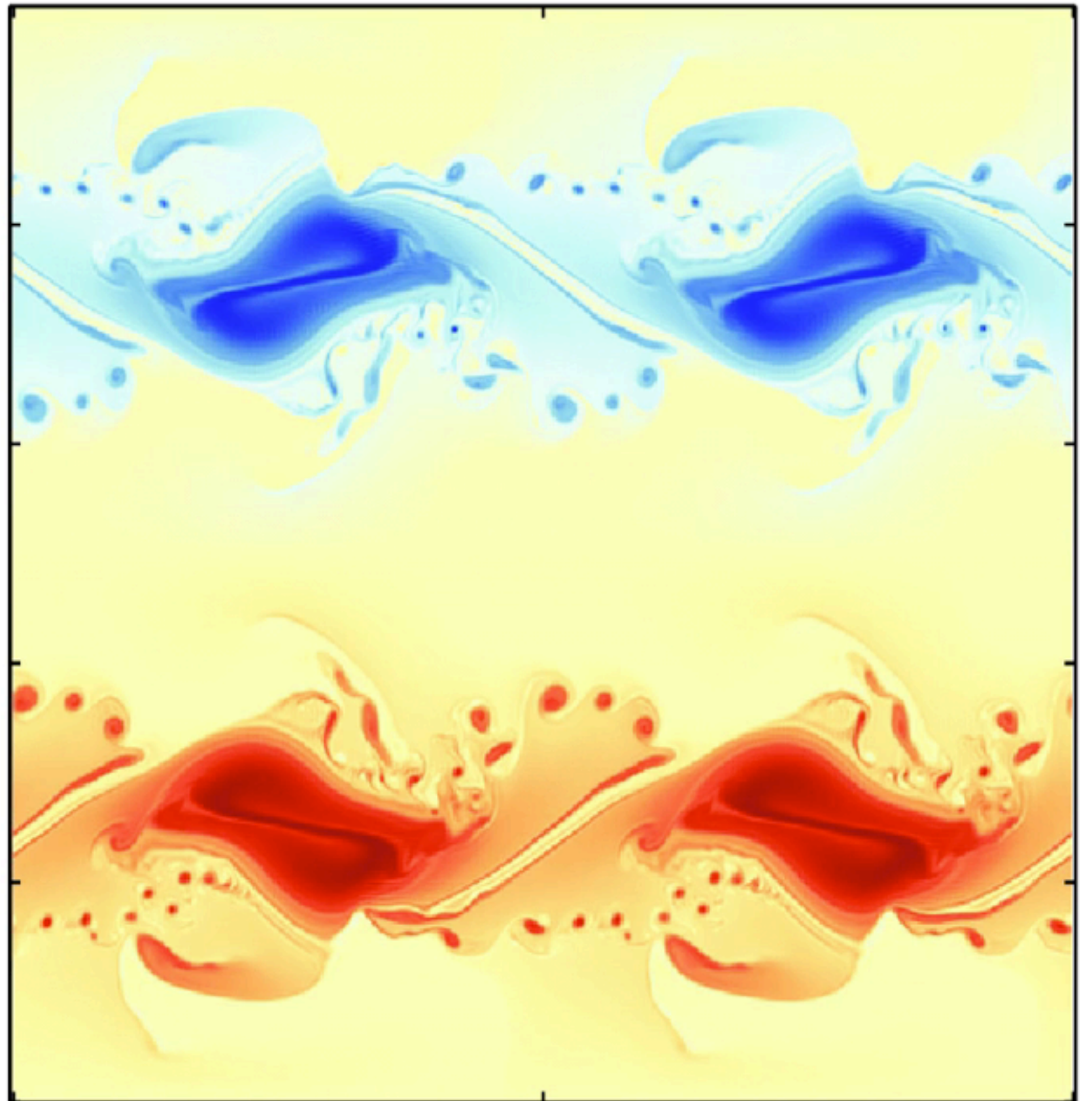
$t = 17$ days

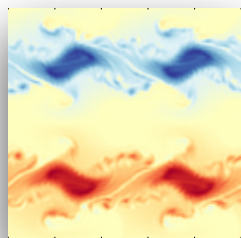
Reference flow:

deterministic

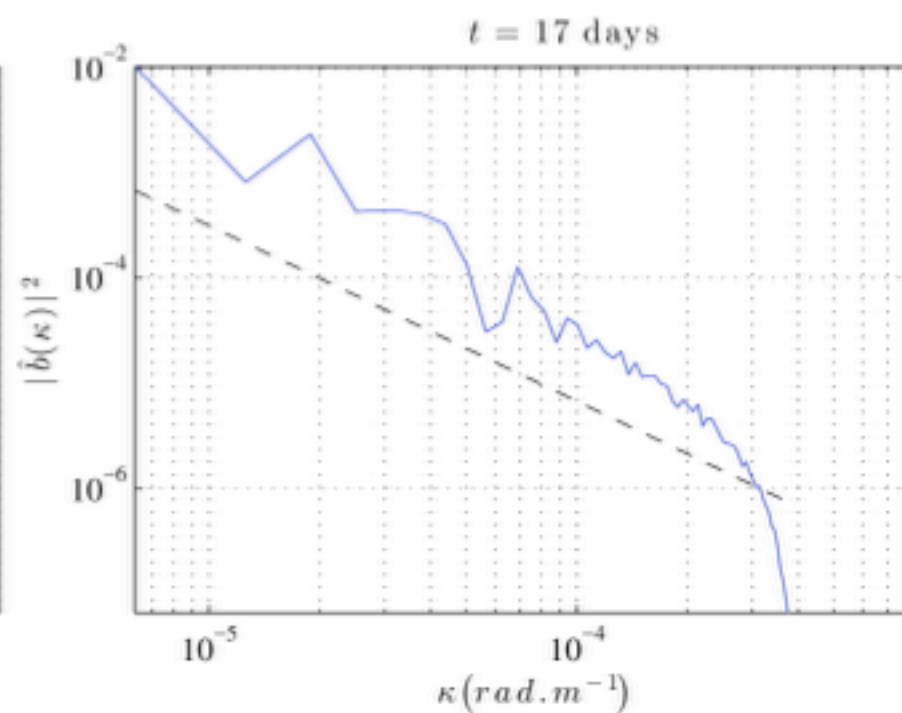
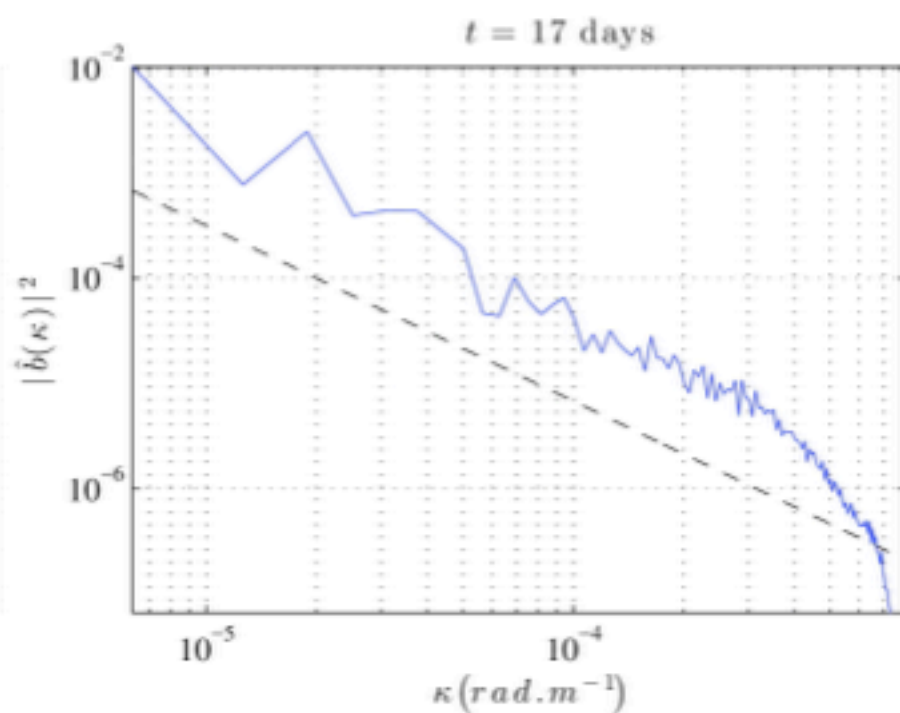
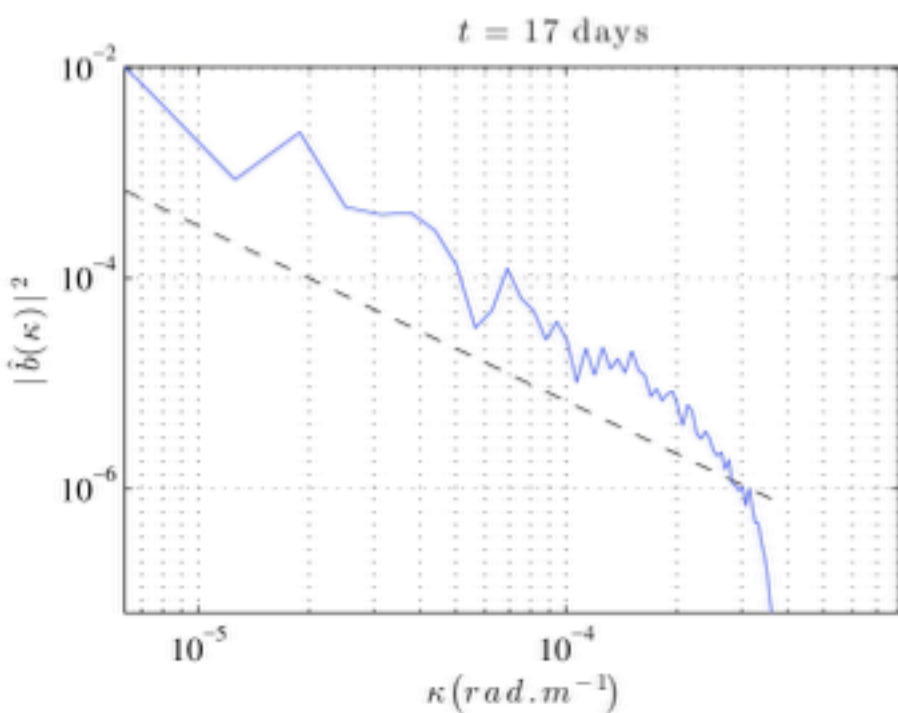
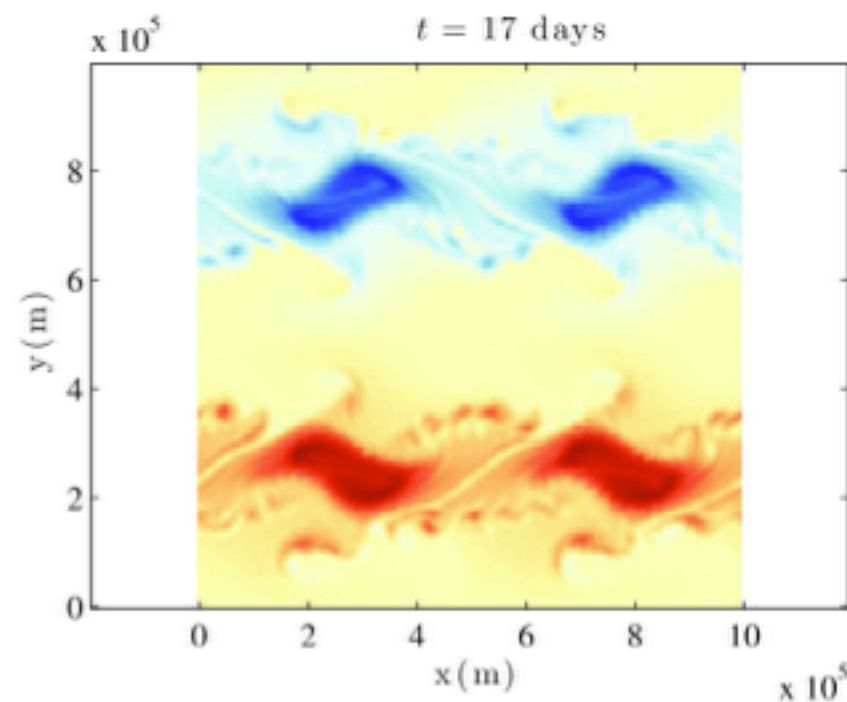
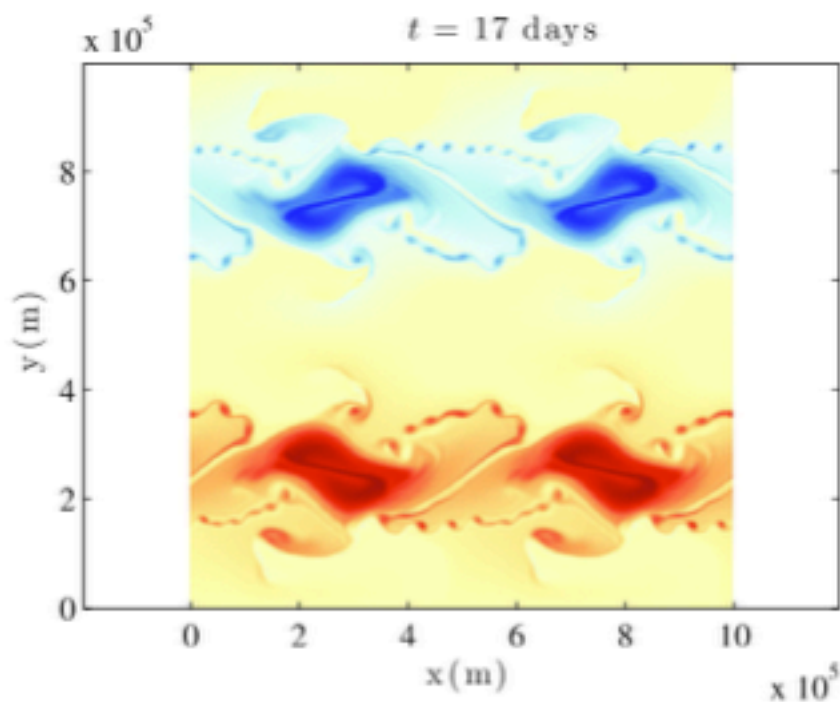
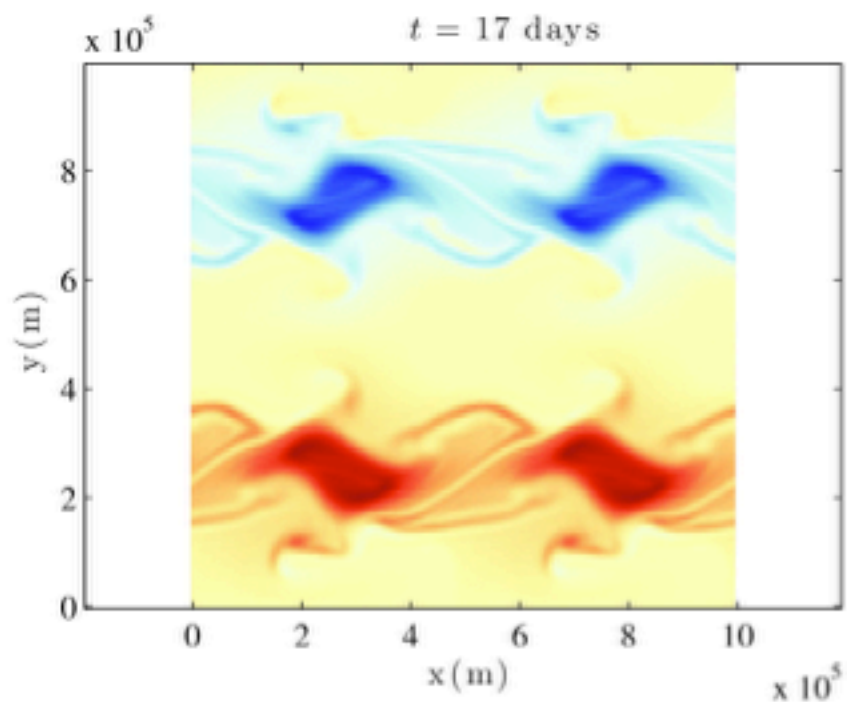
SQG

512 x 512





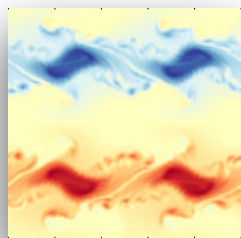
One realization



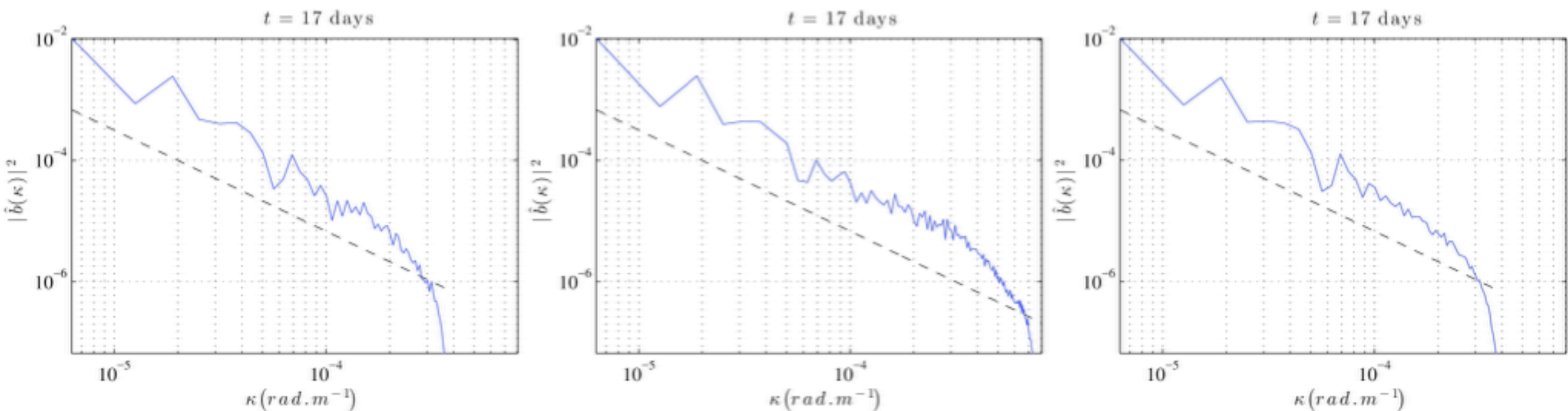
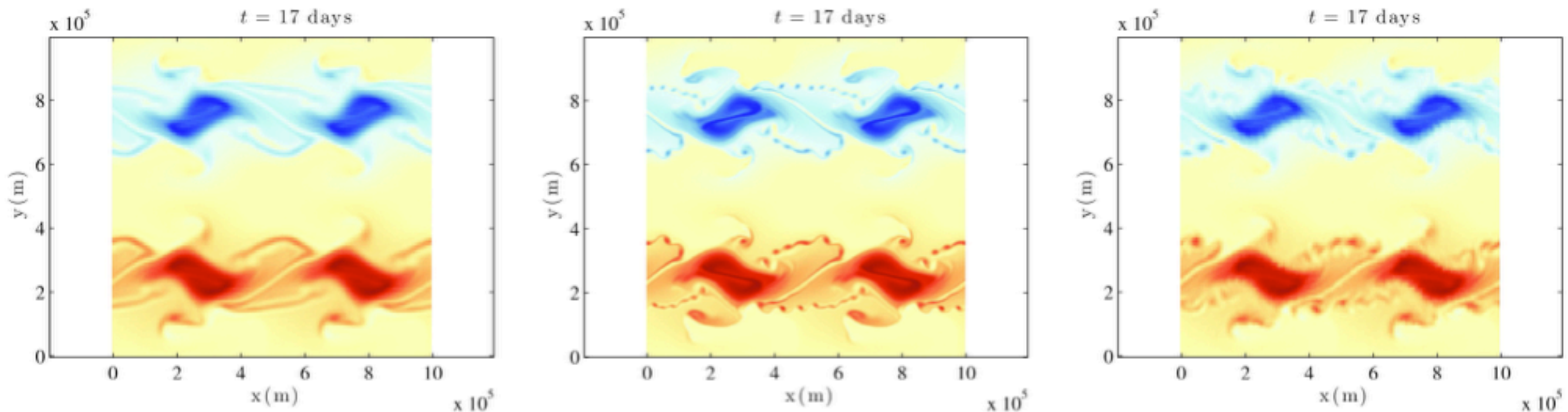
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



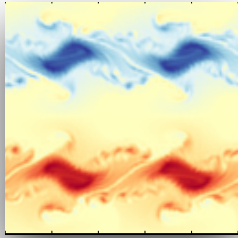
One realization



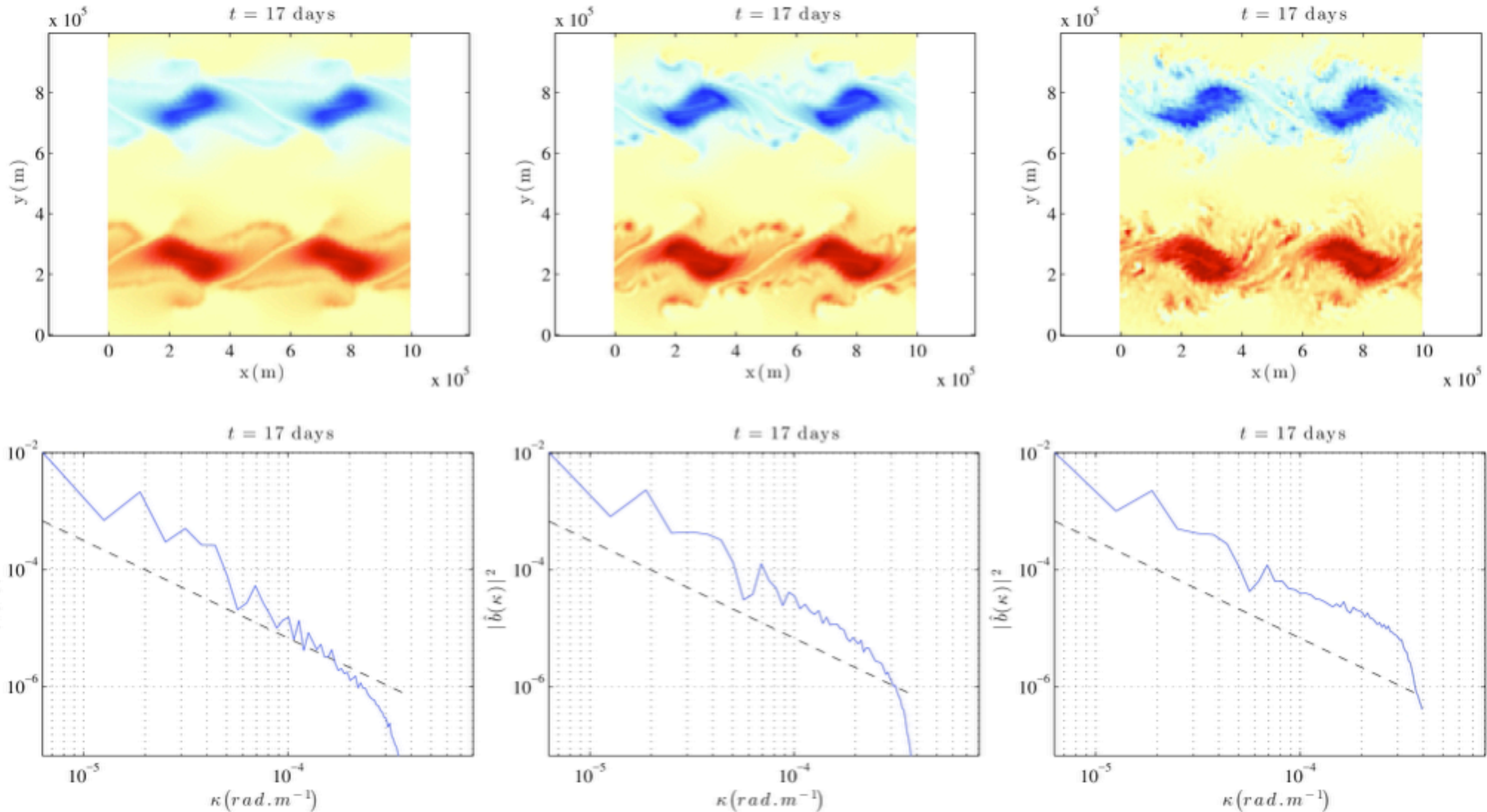
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



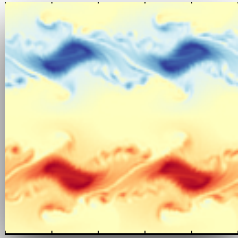
One realization



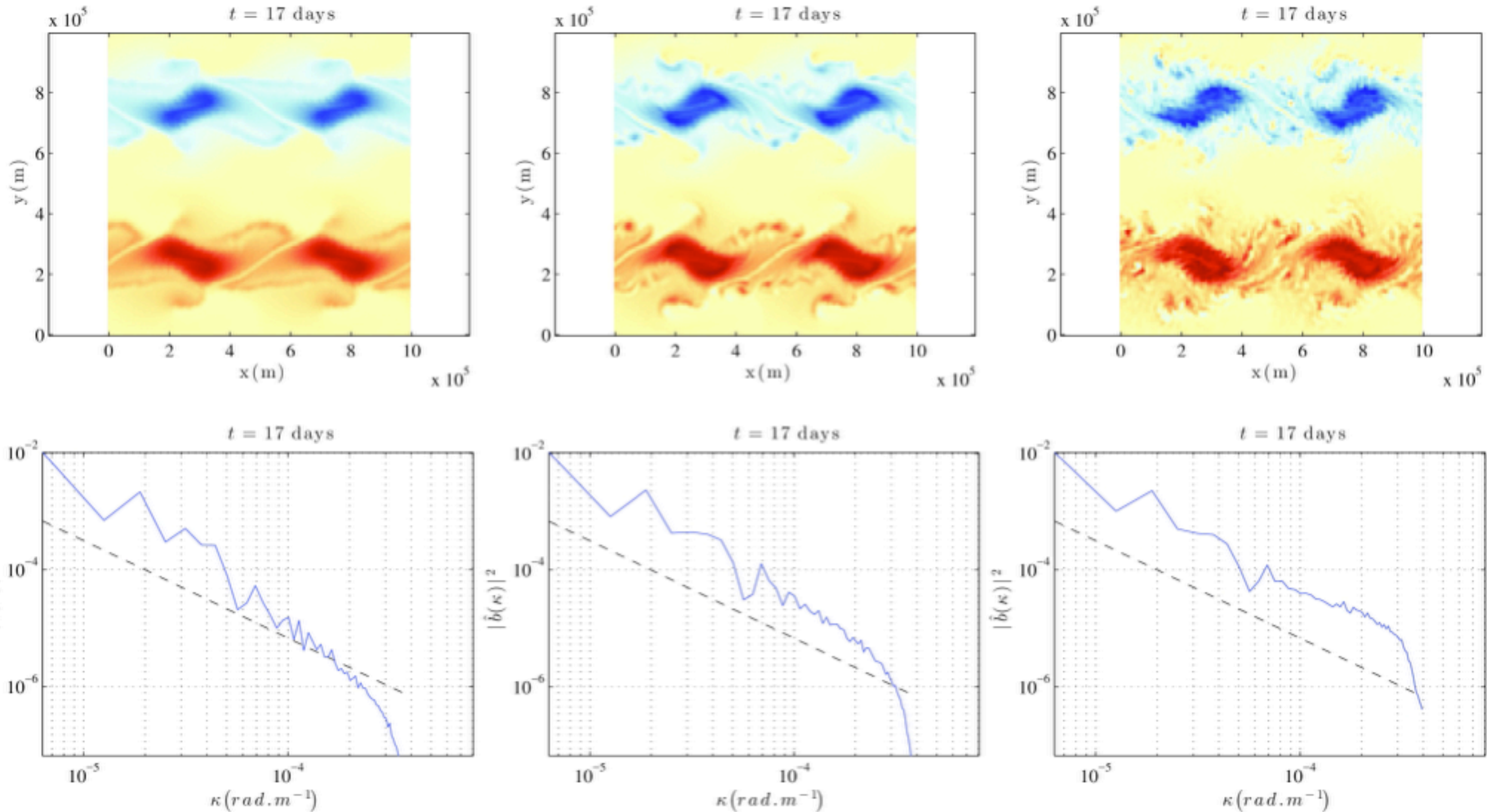
Lower noise

Our model

Larger noise



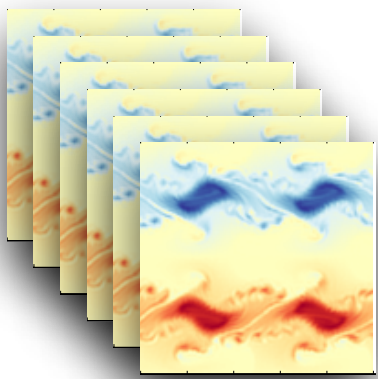
One realization



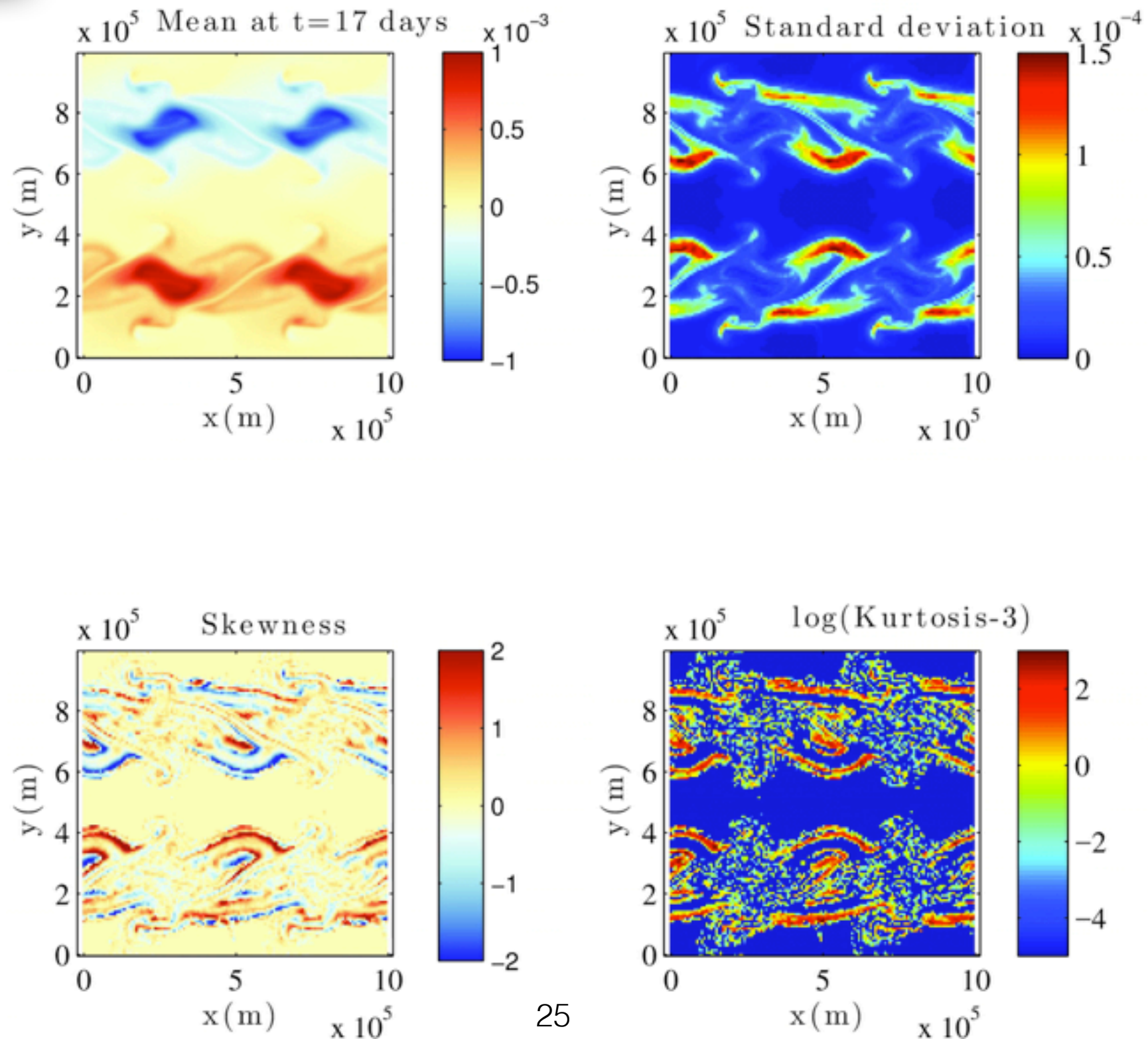
Lower noise

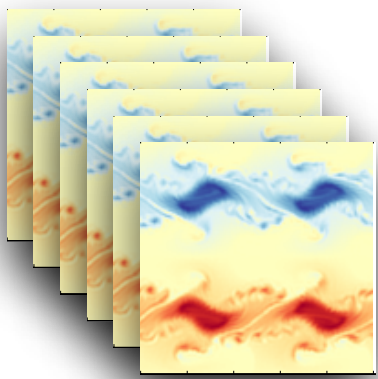
Our model

Larger noise

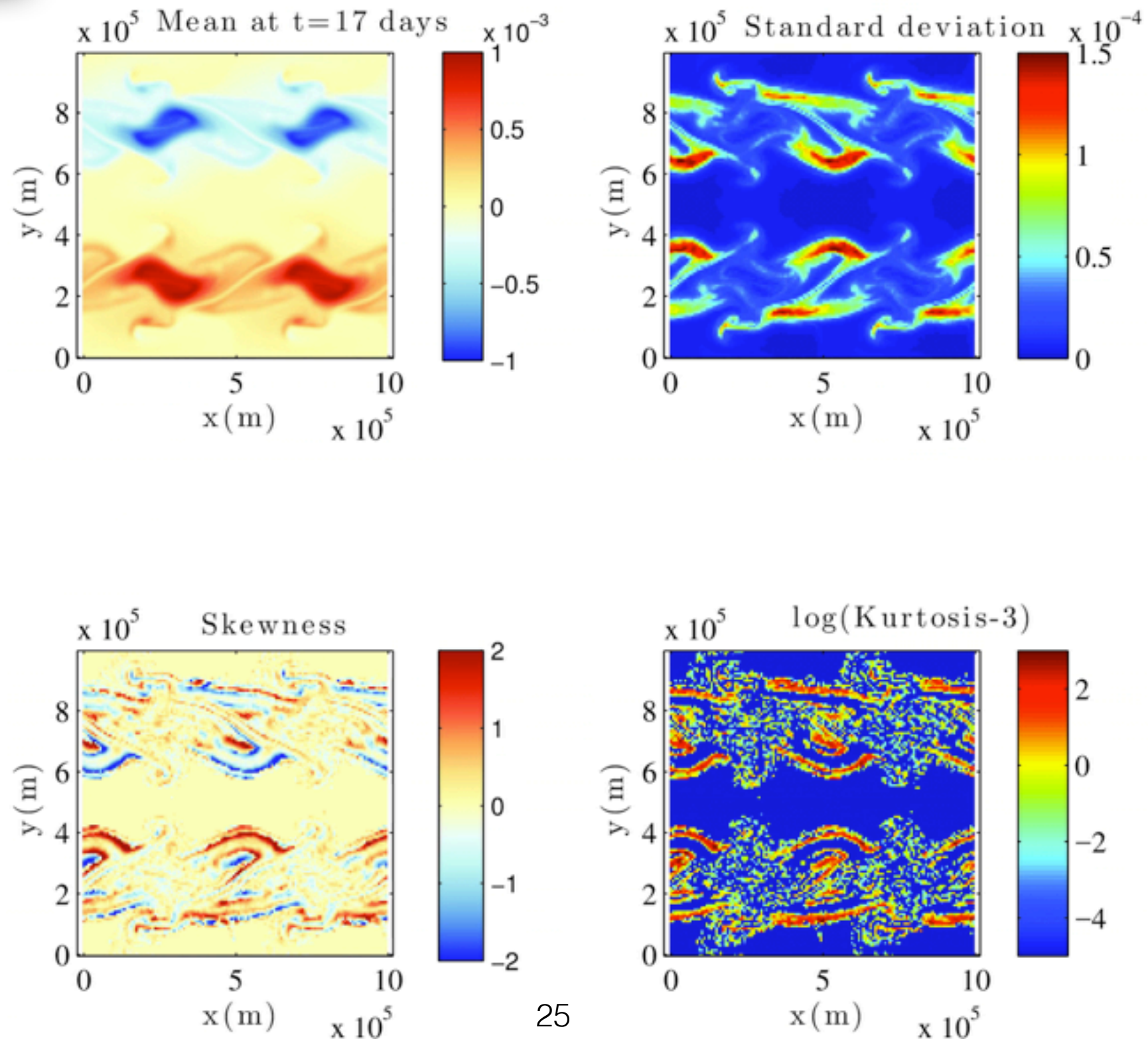


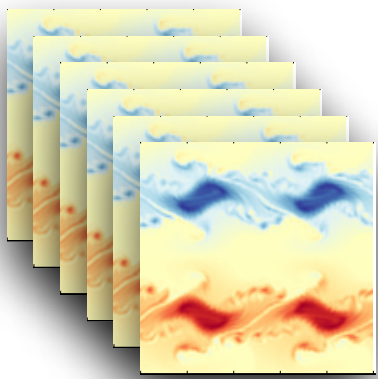
Ensemble



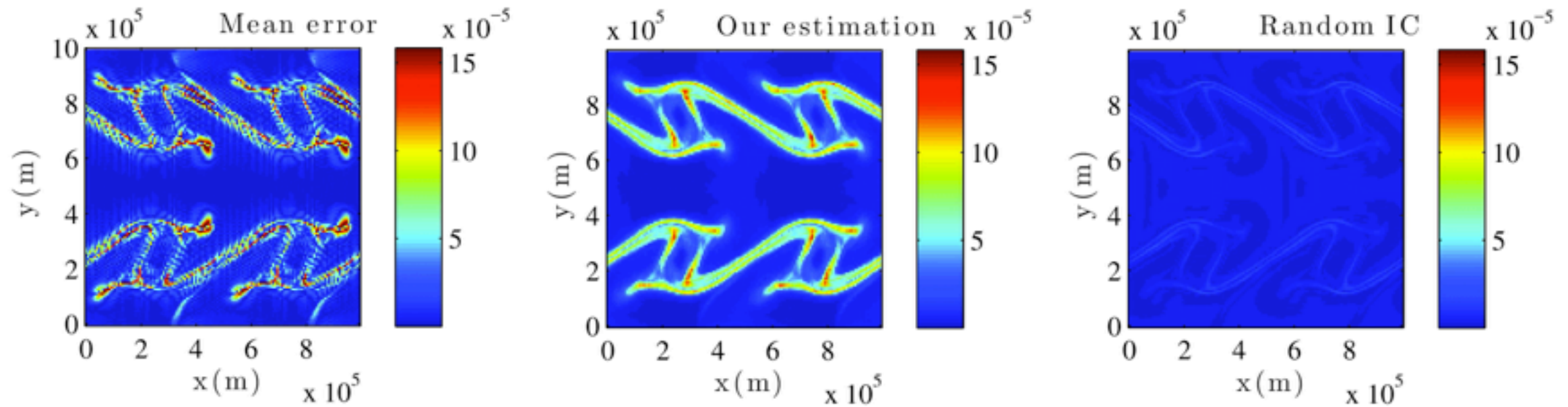


Ensemble

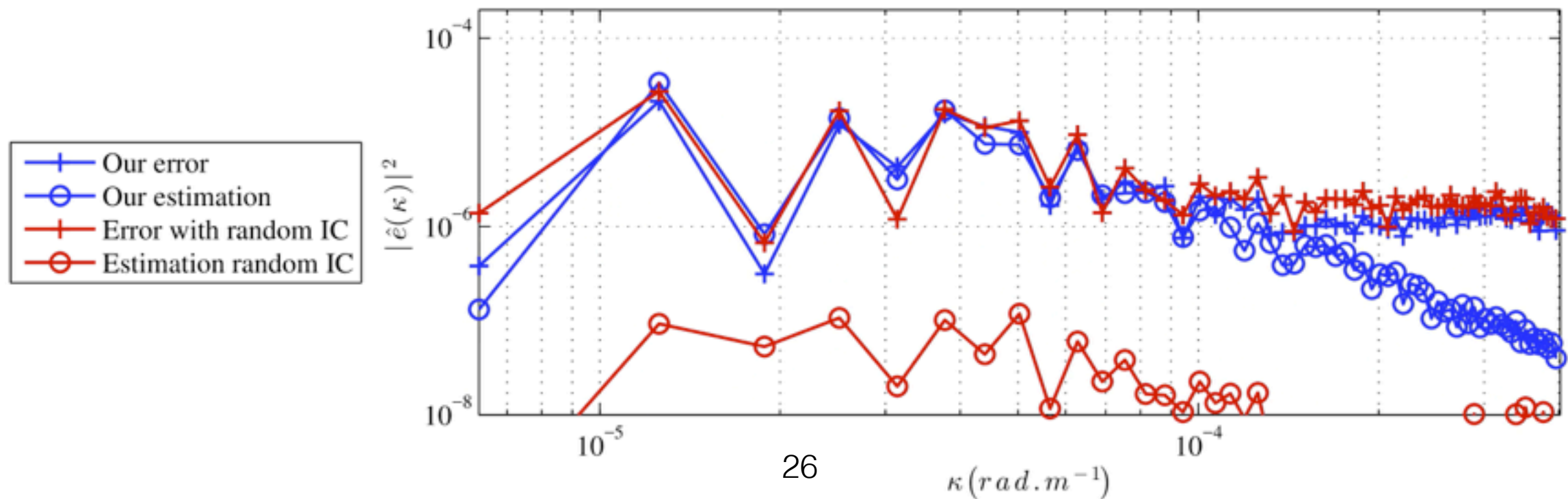


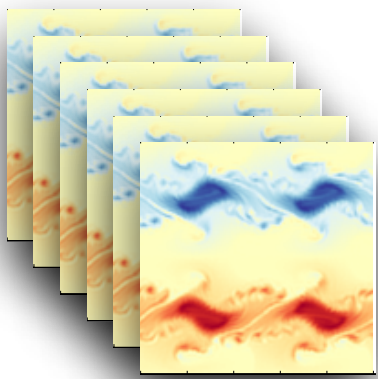


Ensemble

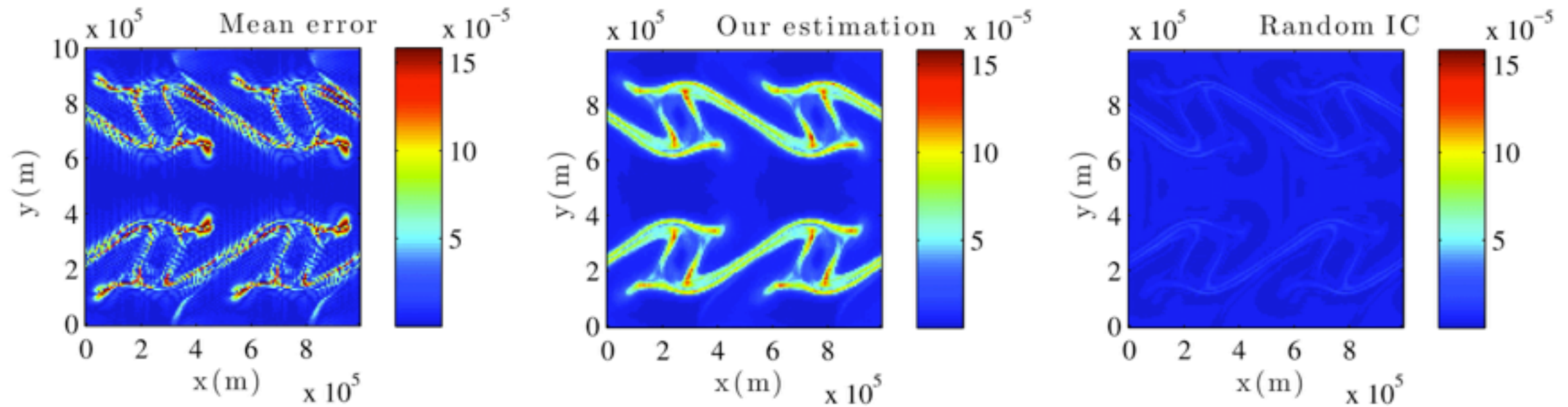


Spectrum of the errors and its estimation at $t=12$ days

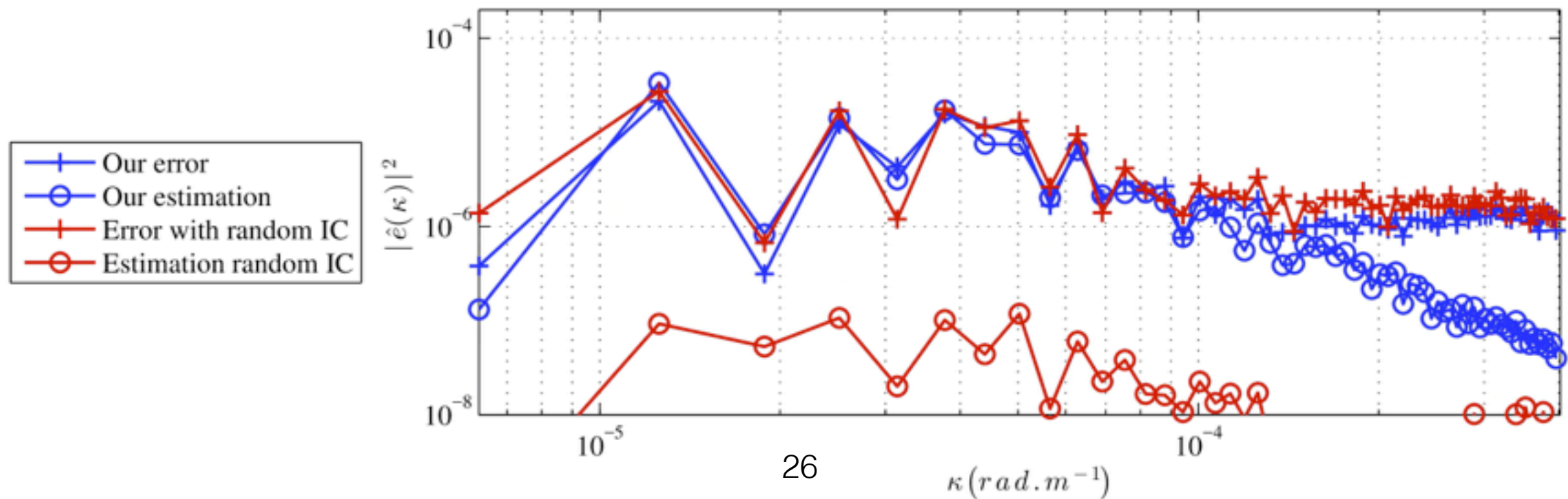




Ensemble



Spectrum of the errors and its estimation at $t=12$ days



Likely SQG scenarios

tracked by SQG MU

SQG under Strong Uncertainty

SQG SU

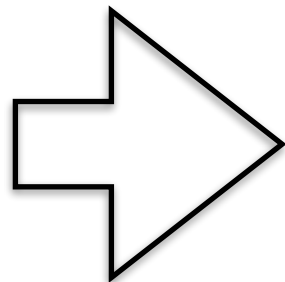
Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Horizontal
Diffusion

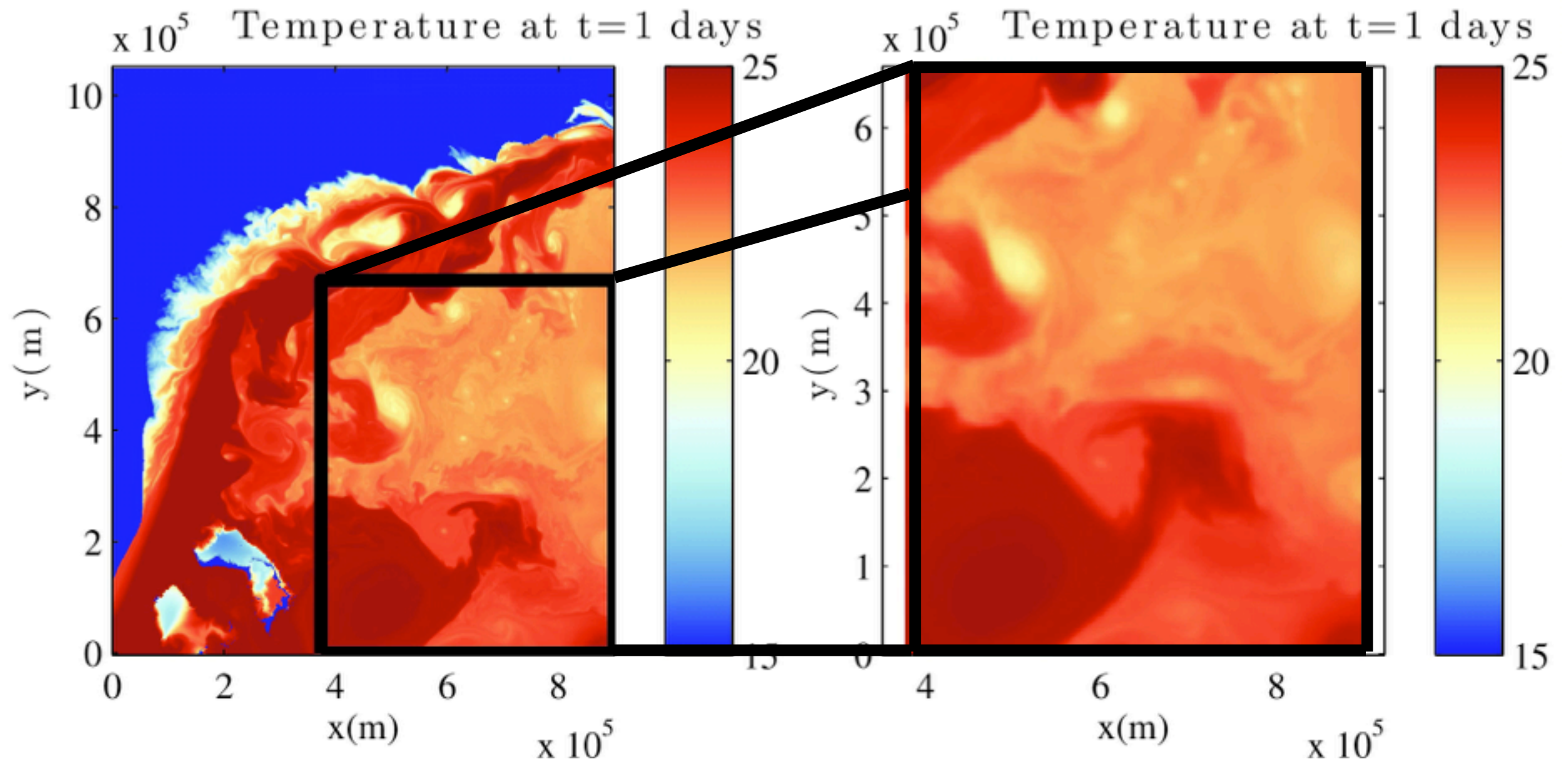
$$+ \frac{a}{2} \Delta \mathbf{u}$$



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

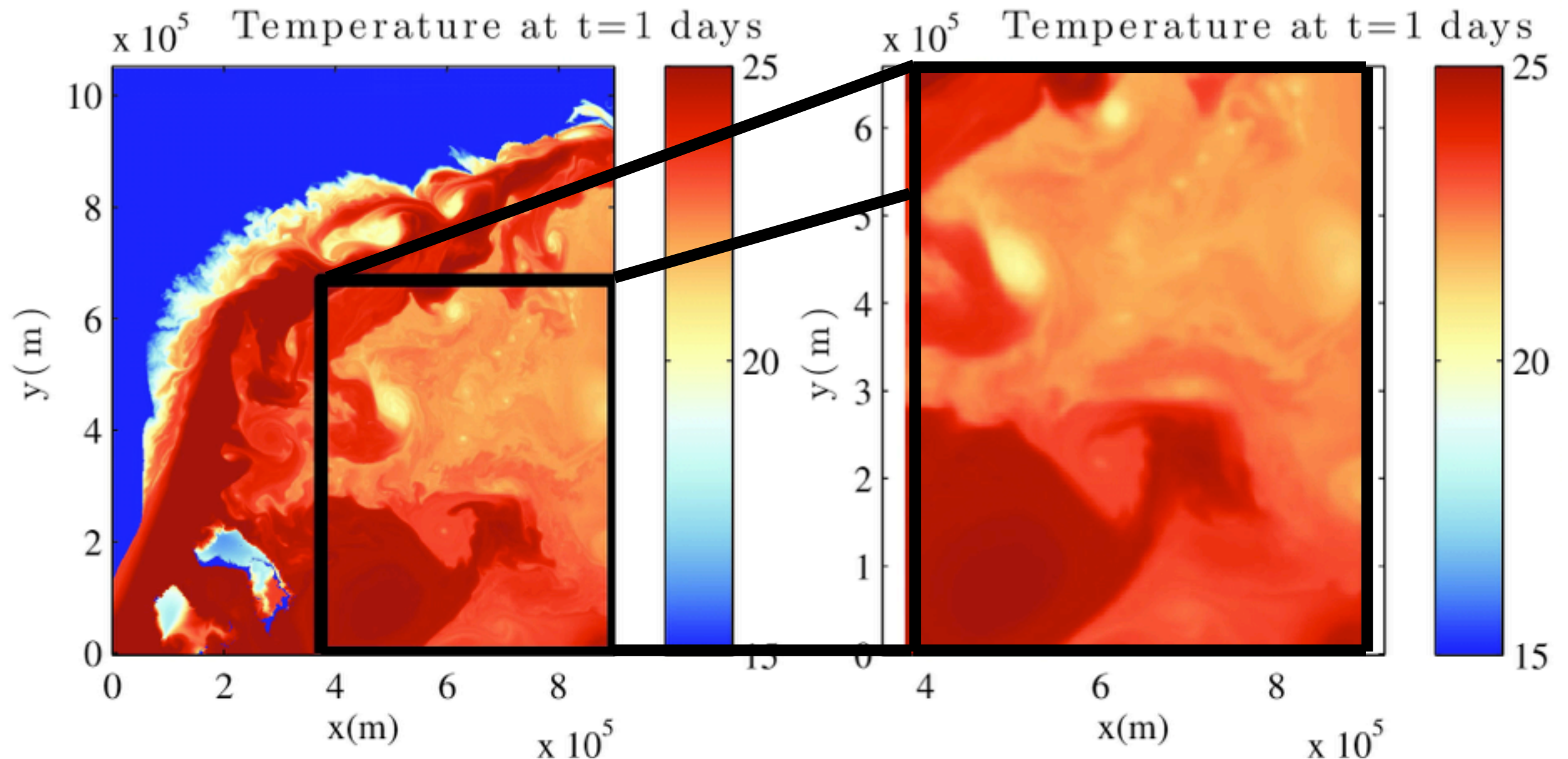
Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

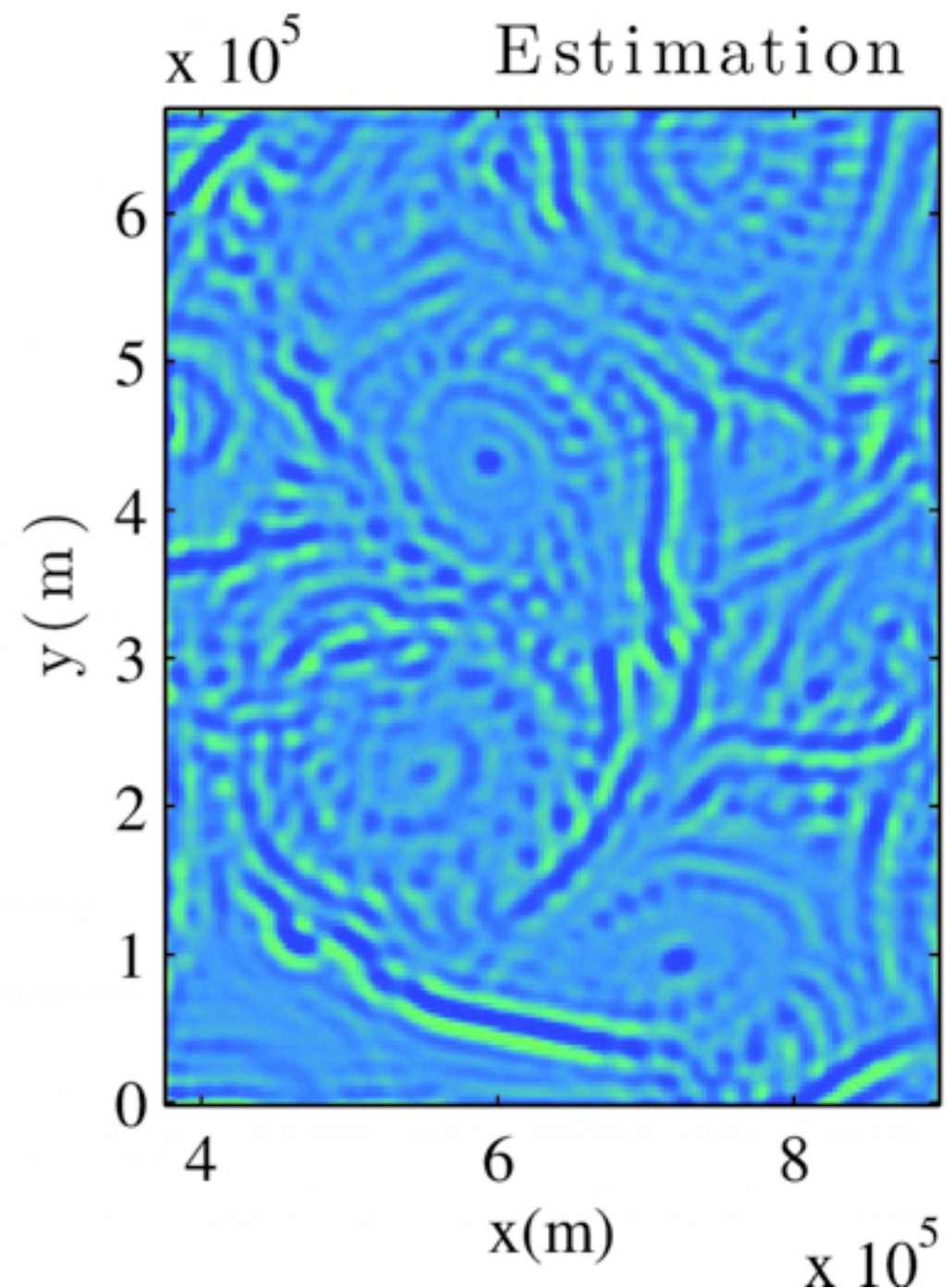
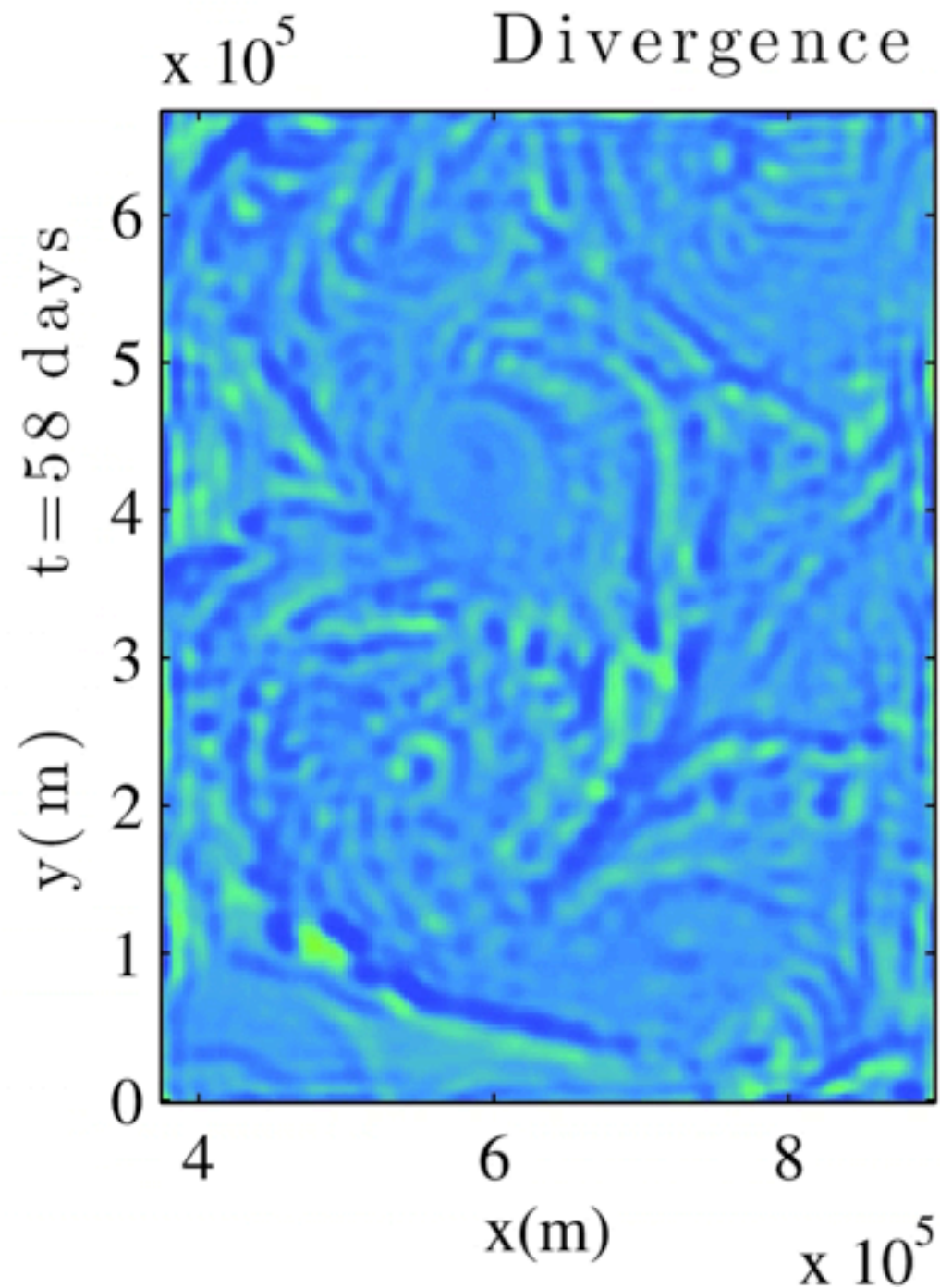


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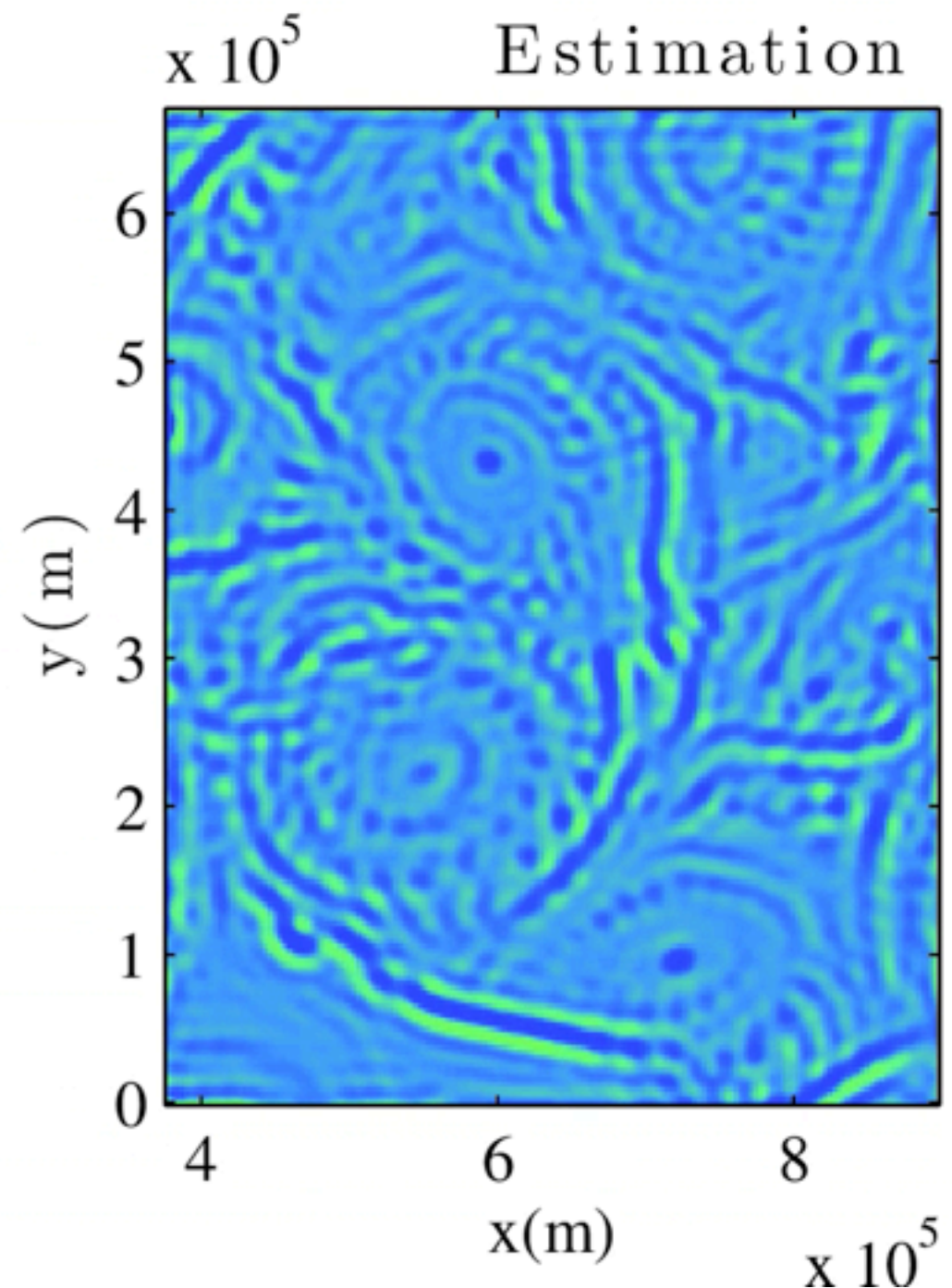
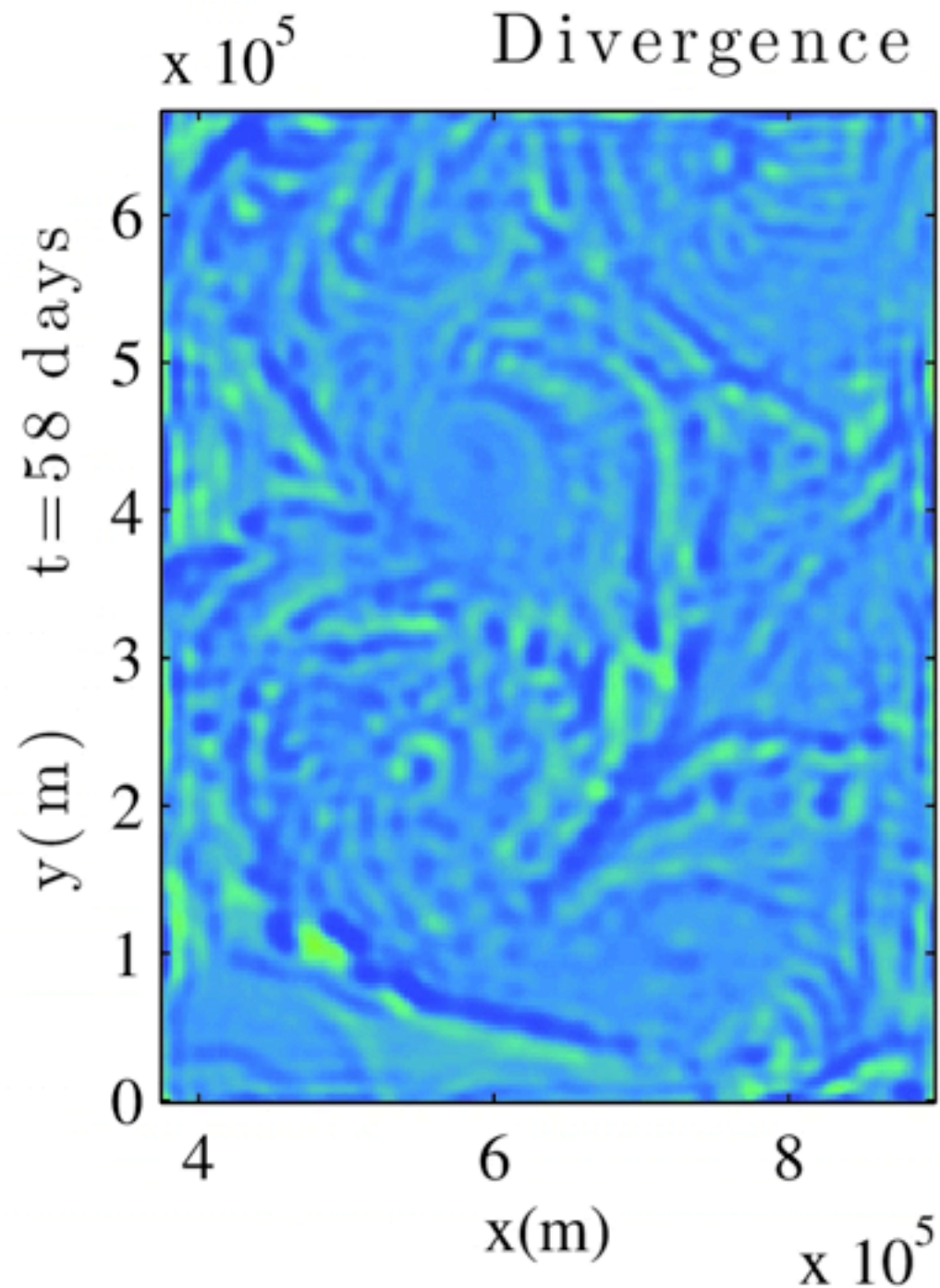
Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
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Spatial test



Spatial test



Spectral test

