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Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

Valentin Resseguier,
Etienne Mémin,
Bertrand Chapron

Motivations for deriving random fluid dynamics models

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Quantification of modeling errors:
 - especially for Data assimilation: ensemble forecasts
- **Predicting possible distinct scenarios**

Contents

- Chaotic transitions
- Dynamics under location uncertainty
- Ensemble of simulations

Chaotic transitions

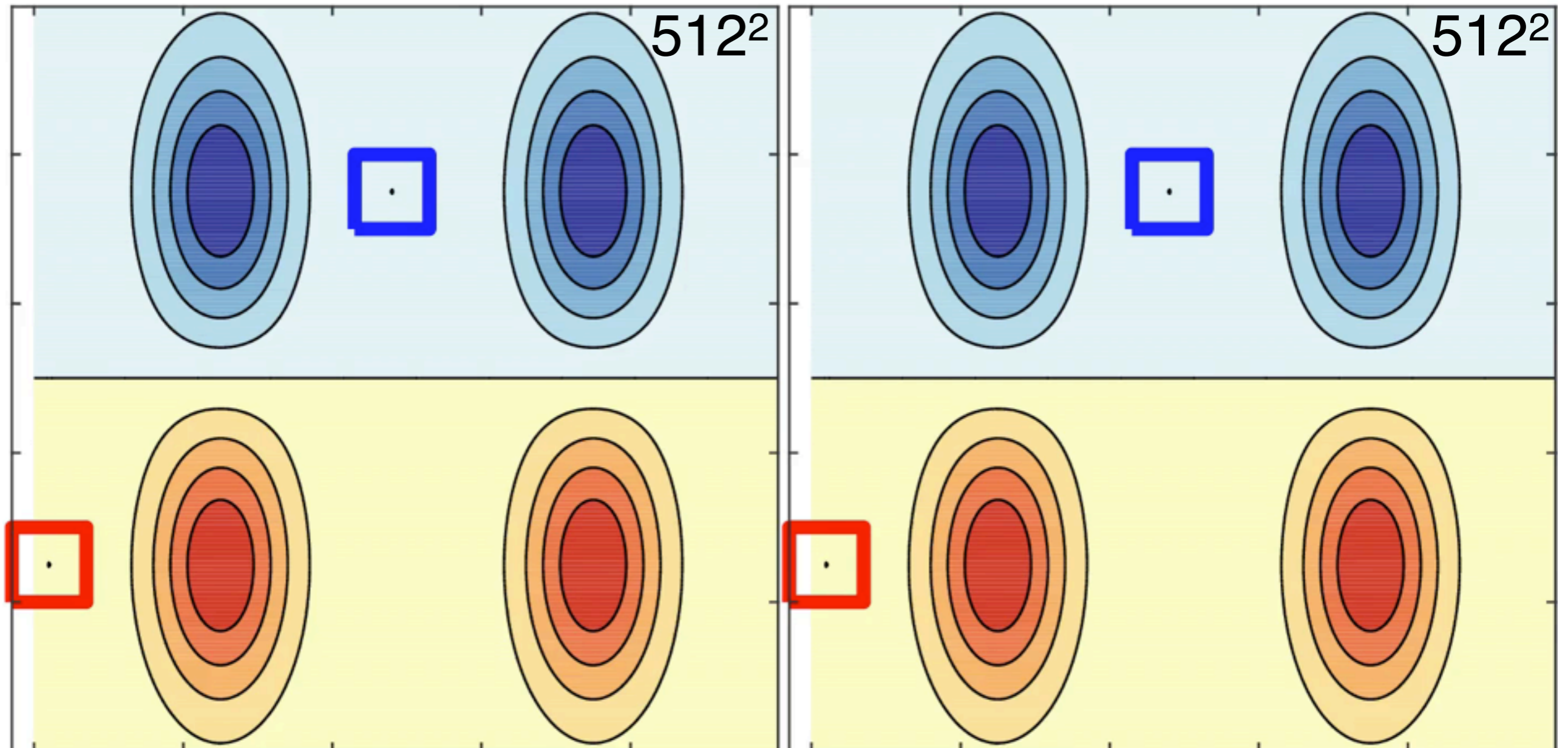
in SQG dynamics

Reference flow: deterministic SQG 512^2

Initial condition 1  Scenario 1

$t = 0$ days

$t = 0$ days

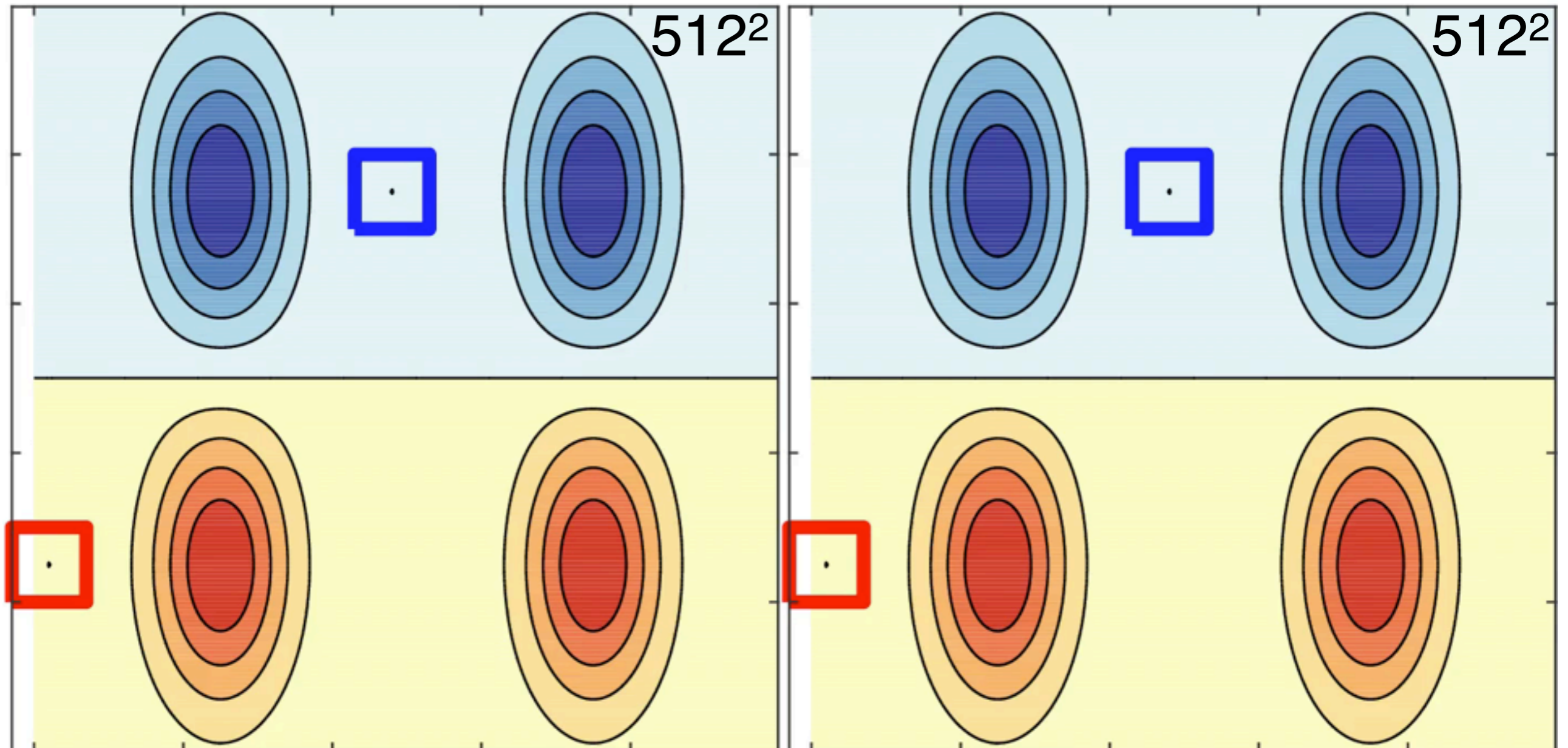


Reference flow: deterministic SQG 512^2

Initial condition 1  Scenario 1

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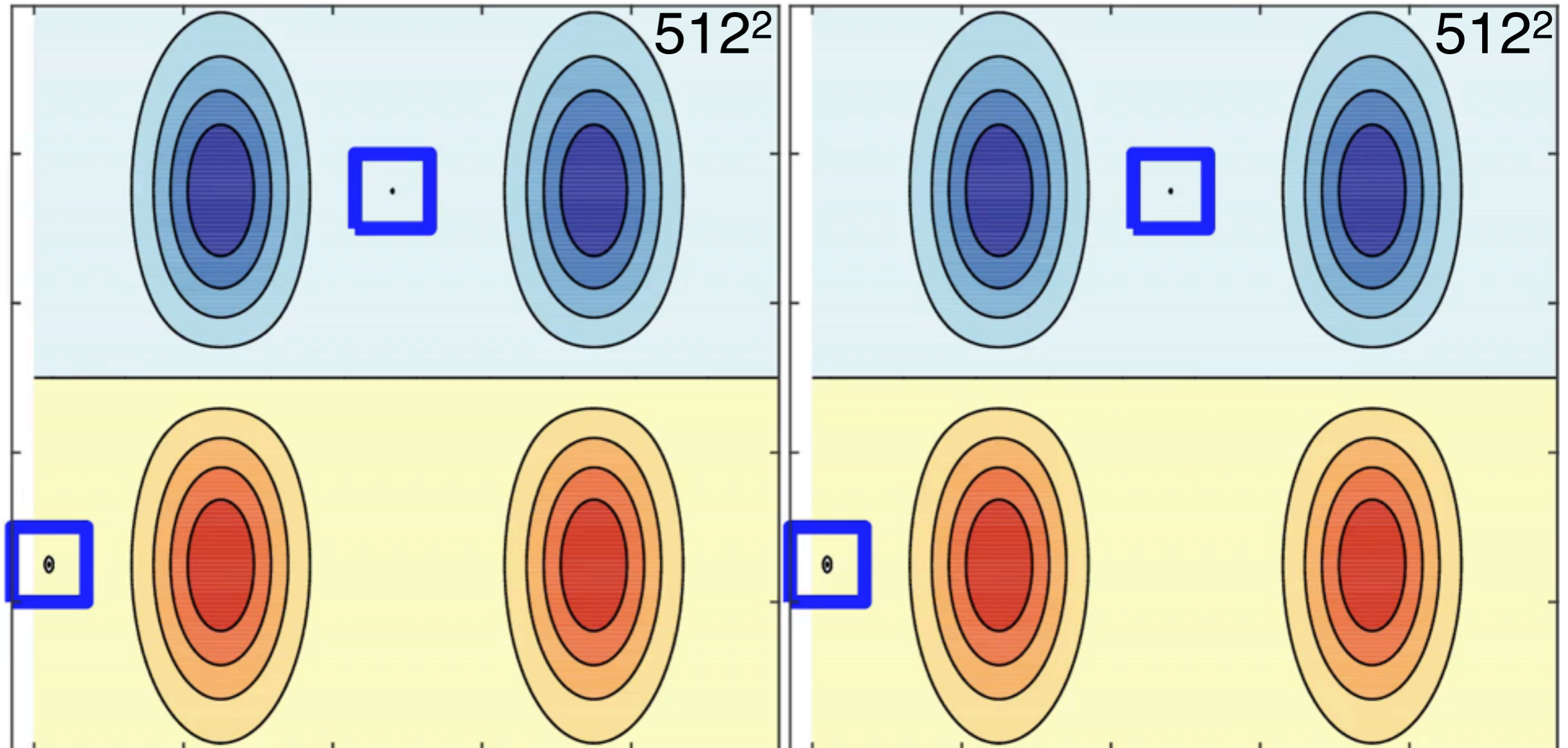


Reference flow: deterministic SQG 512^2

Initial condition 2 \rightarrow Scenario 2

$t = 0$ days

$t = 0$ days

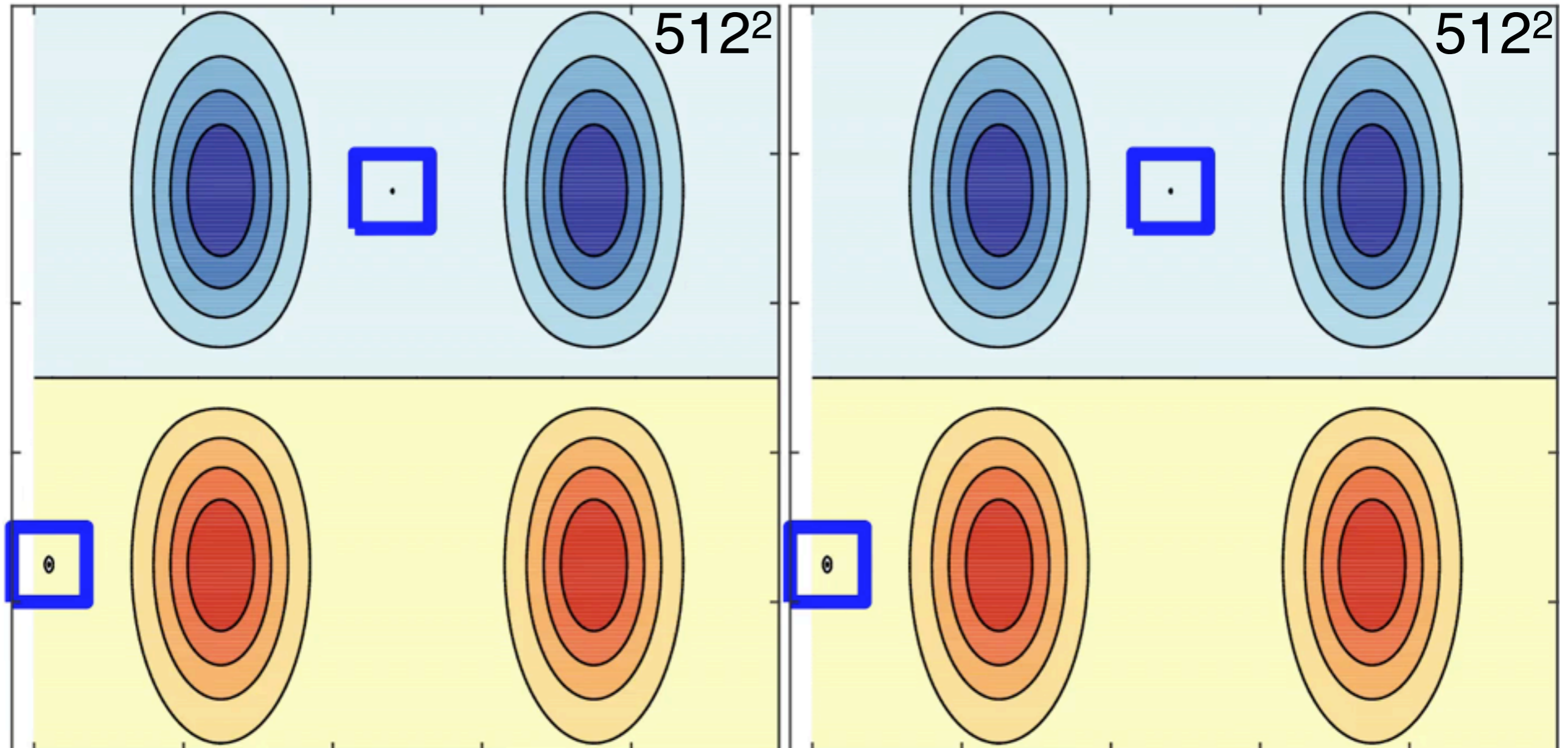


Reference flow: deterministic SQG 512^2

Initial condition 2 \rightarrow Scenario 2

$t = 0$ days

$t = 0$ days



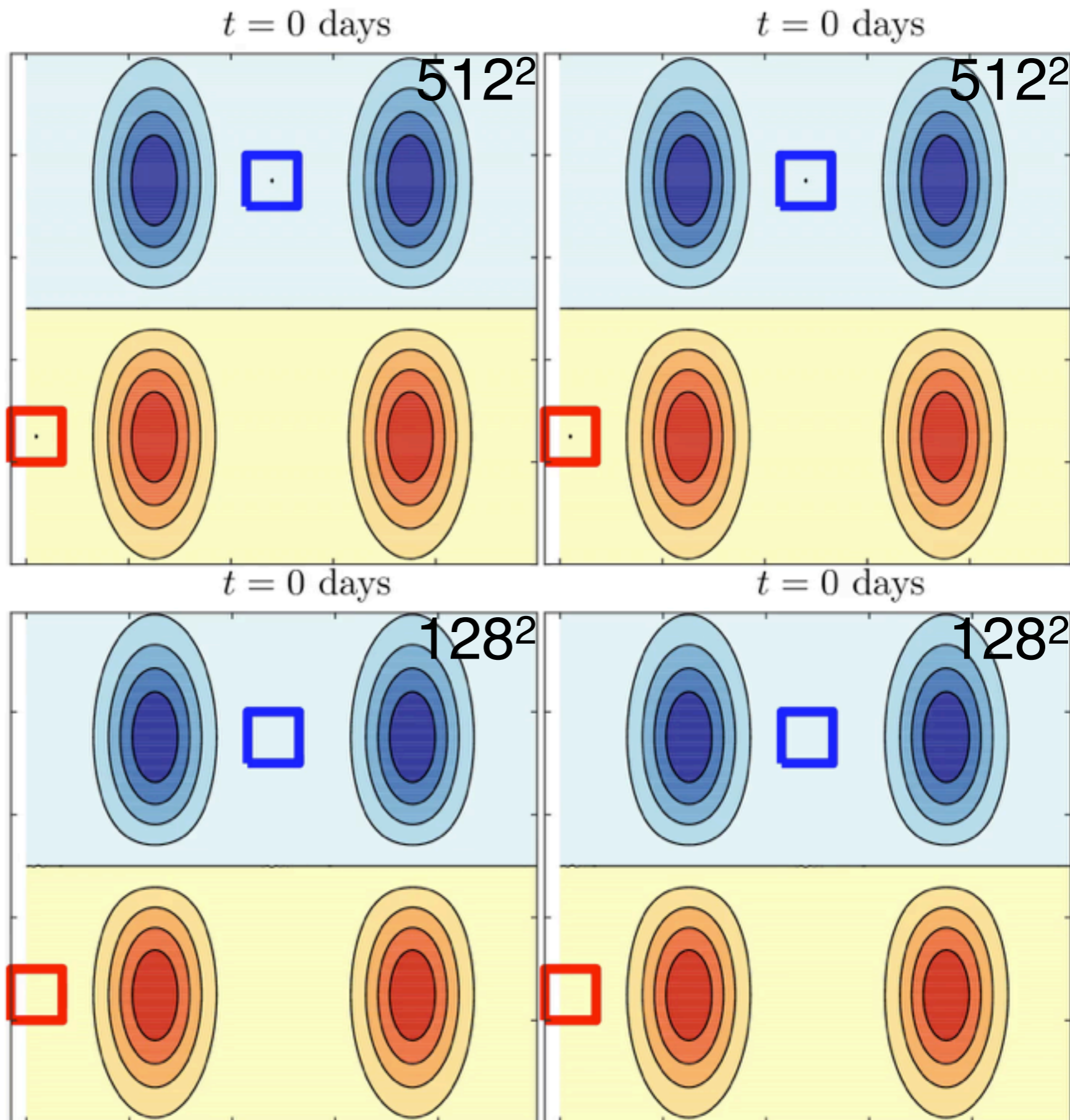
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1



?



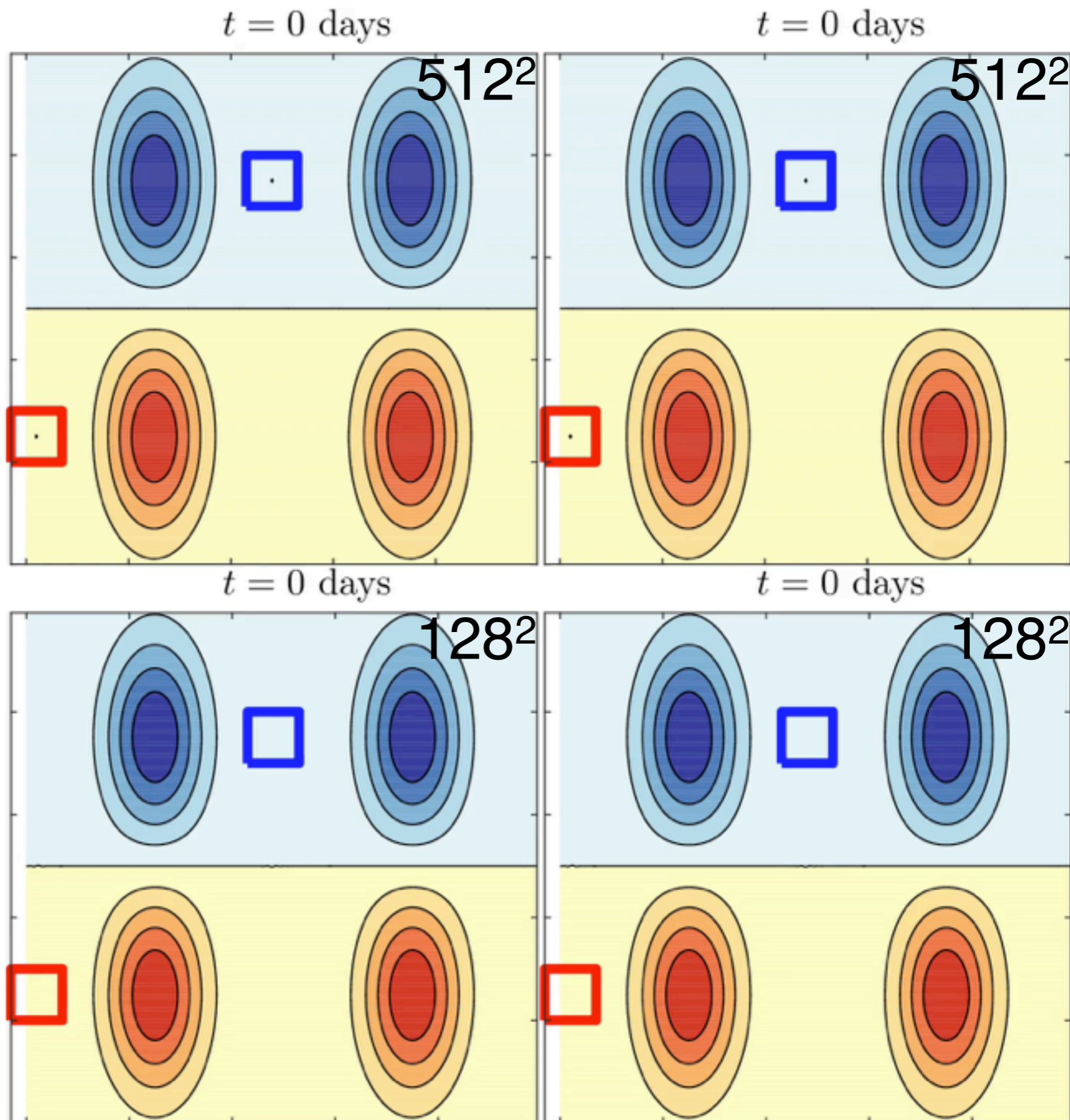
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1

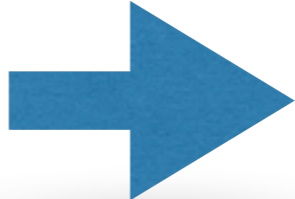
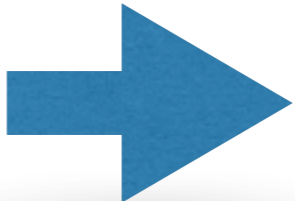
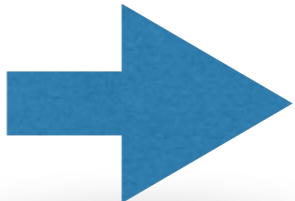



?



Dynamics under location uncertainty

Random equations

- Random initial conditions  Underdispersive + need large ensemble
- Arbitrary Gaussian forcing  Adding energy + wrong phase
- Averaging, homogenization  Assumptions and energy issues
- Adding white random velocity  $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

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Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The left side of the equation is enclosed in a blue rounded rectangle labeled "Advection". The right side is enclosed in a green rounded rectangle labeled "Diffusion".

An orange arrow points from the label "Drift correction" to the w^* term in the advection part. A purple arrow points from the label "Multiplicative random forcing" to the $\sigma \dot{B} \cdot \nabla \Theta$ term in the advection part.

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

Advection of tracer Θ

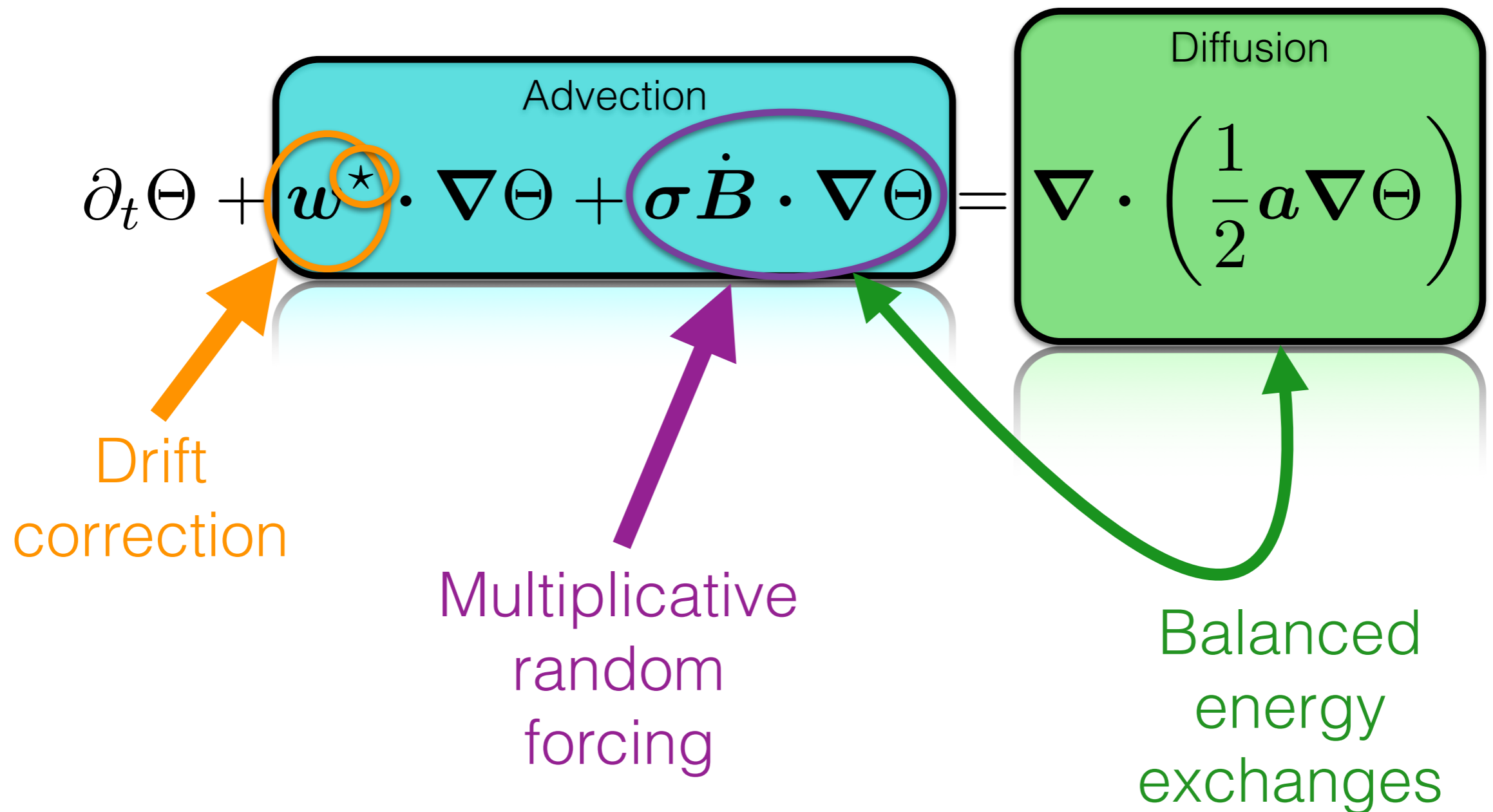
The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The equation is divided into two main components:

- Advection (Cyan box):** This part includes the time derivative $\partial_t \Theta$ and the advection terms $w^* \cdot \nabla \Theta$ and $\sigma \dot{B} \cdot \nabla \Theta$.
 - An orange arrow points to w^* with the label "Drift correction".
 - A purple arrow points to $\sigma \dot{B} \cdot \nabla \Theta$ with the label "Multiplicative random forcing".
- Diffusion (Green box):** This part is represented by the right-hand side of the equation: $\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$. A green arrow points from this box back to the advection terms.

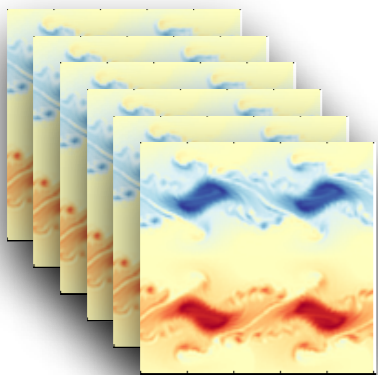
Advection of tracer Θ



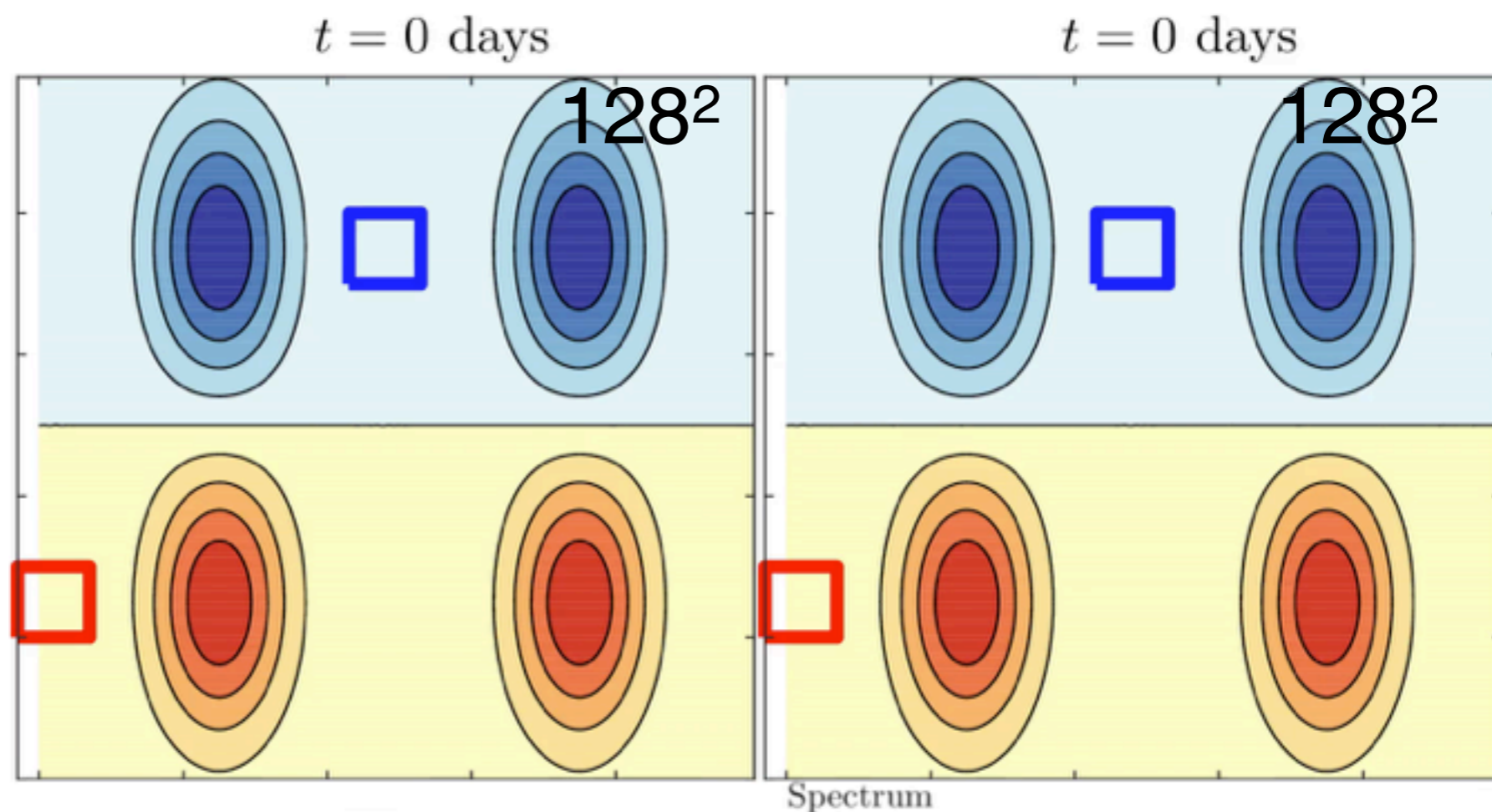
Ensemble of simulations

with SQG MU

Code available online

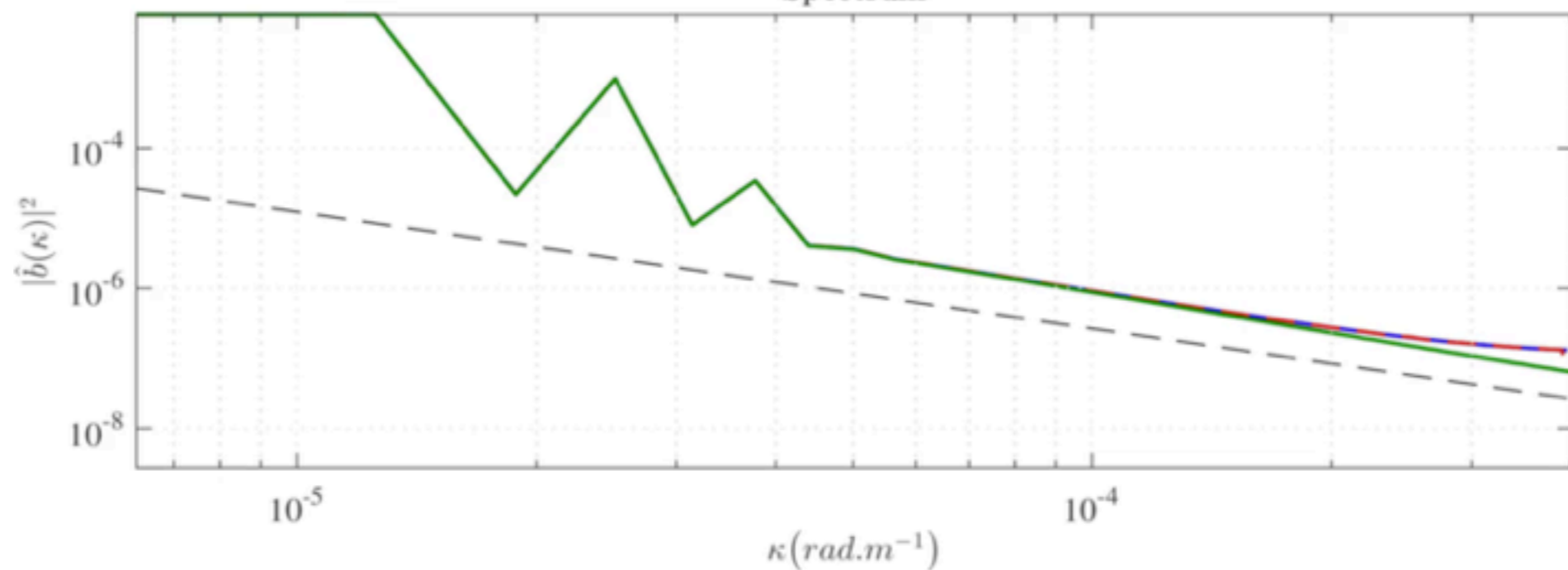


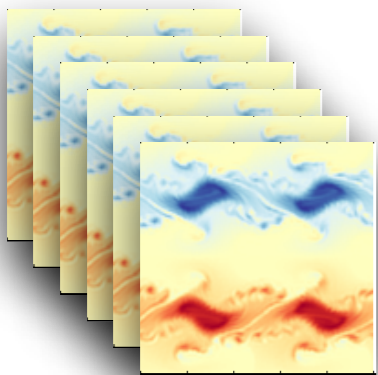
examples of realizations:



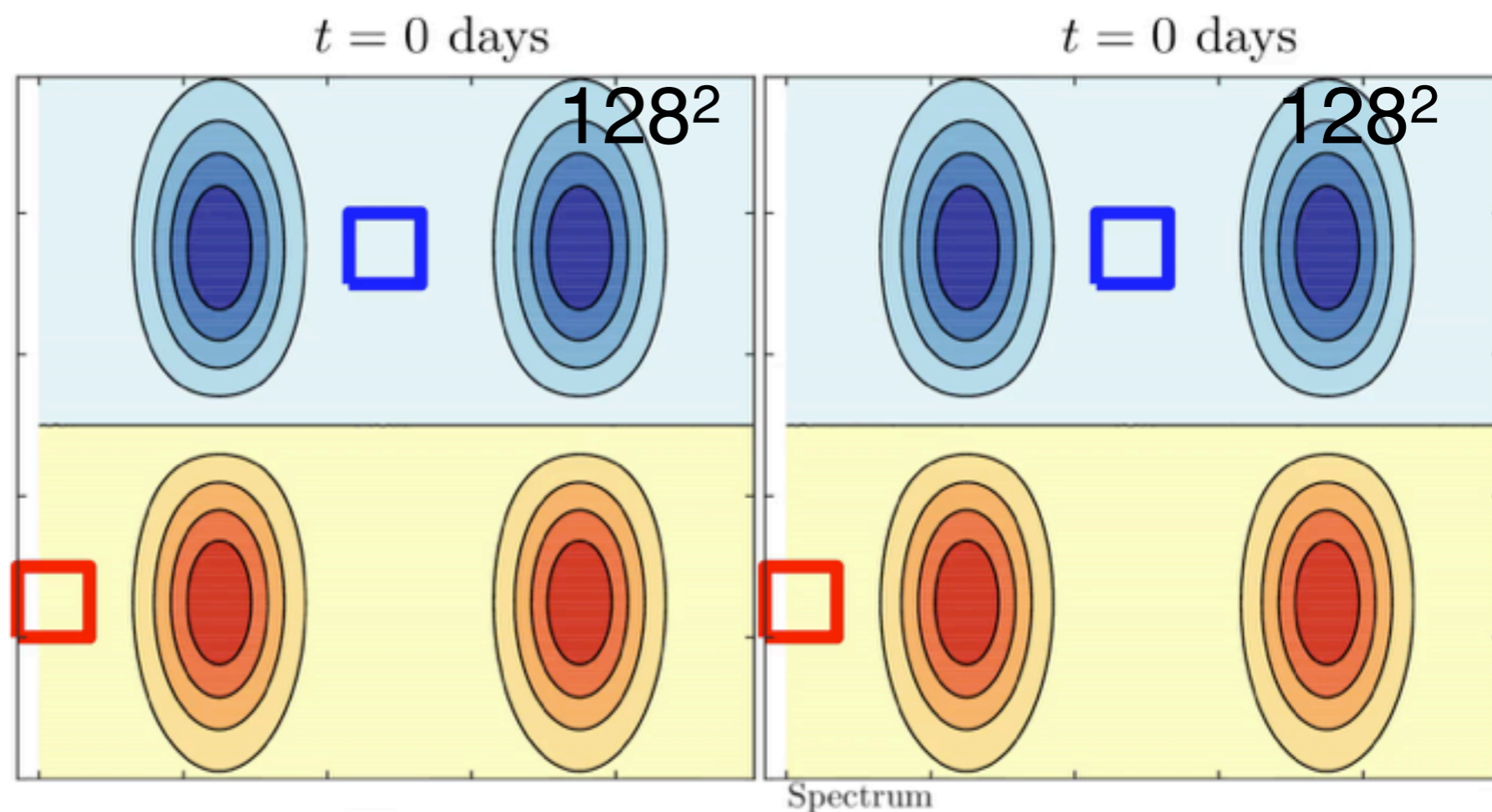
Spectrum

mean²/variance:



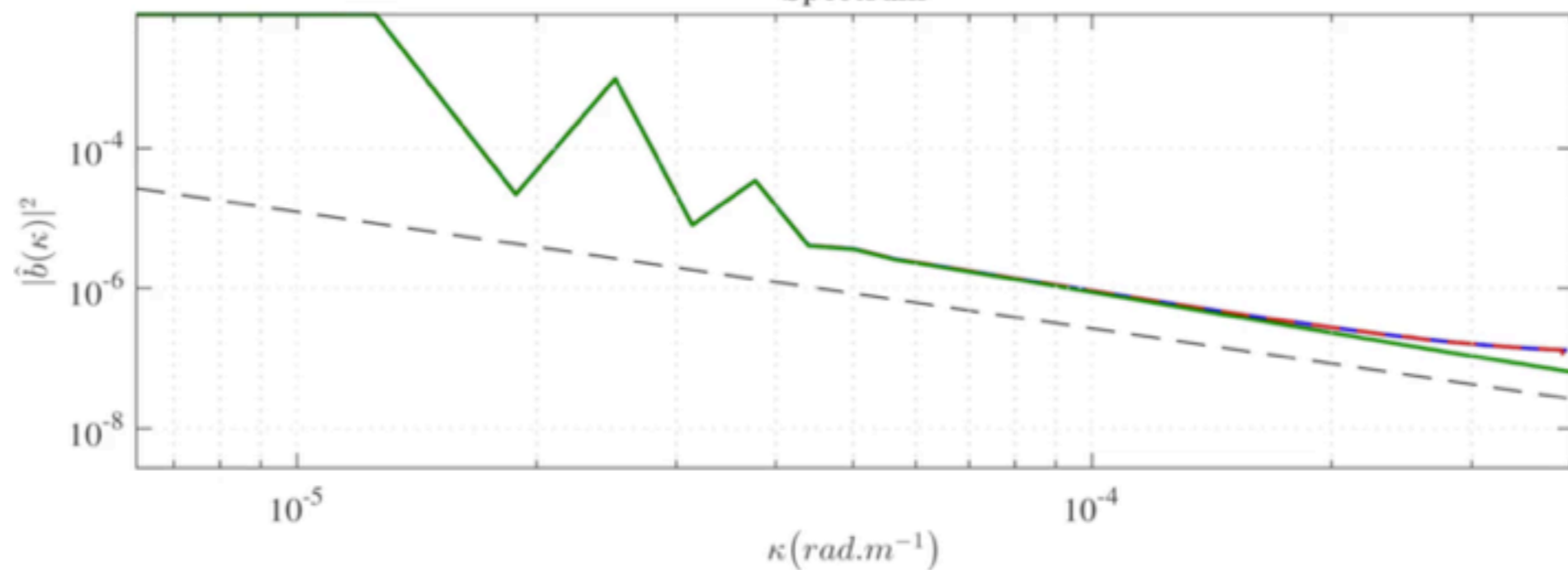


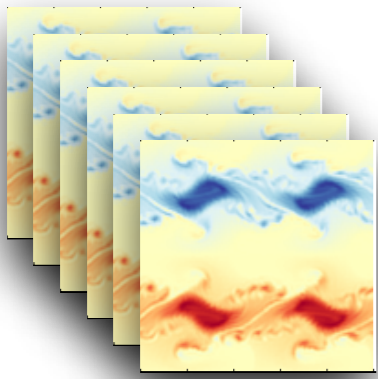
examples of realizations:



Spectrum

mean²/variance:



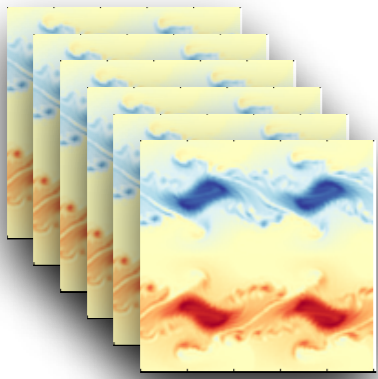


Visualization of the ensemble

At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$



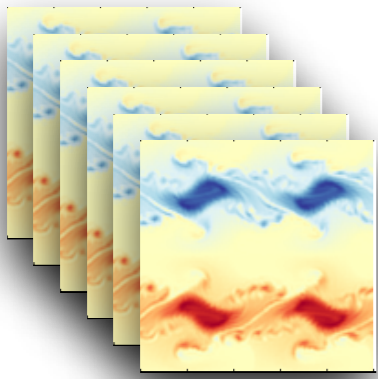
Visualization of the ensemble

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i^{th} realization



Visualization of the ensemble

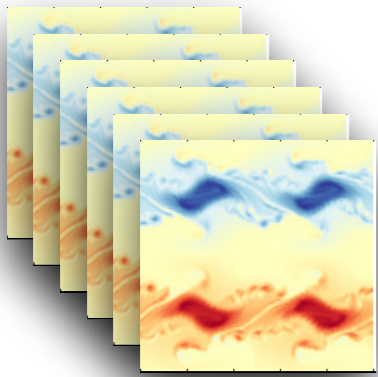
At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization

Mean



Visualization of the ensemble

At a fixed time t ,

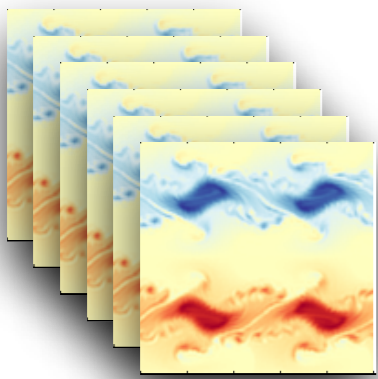
Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization

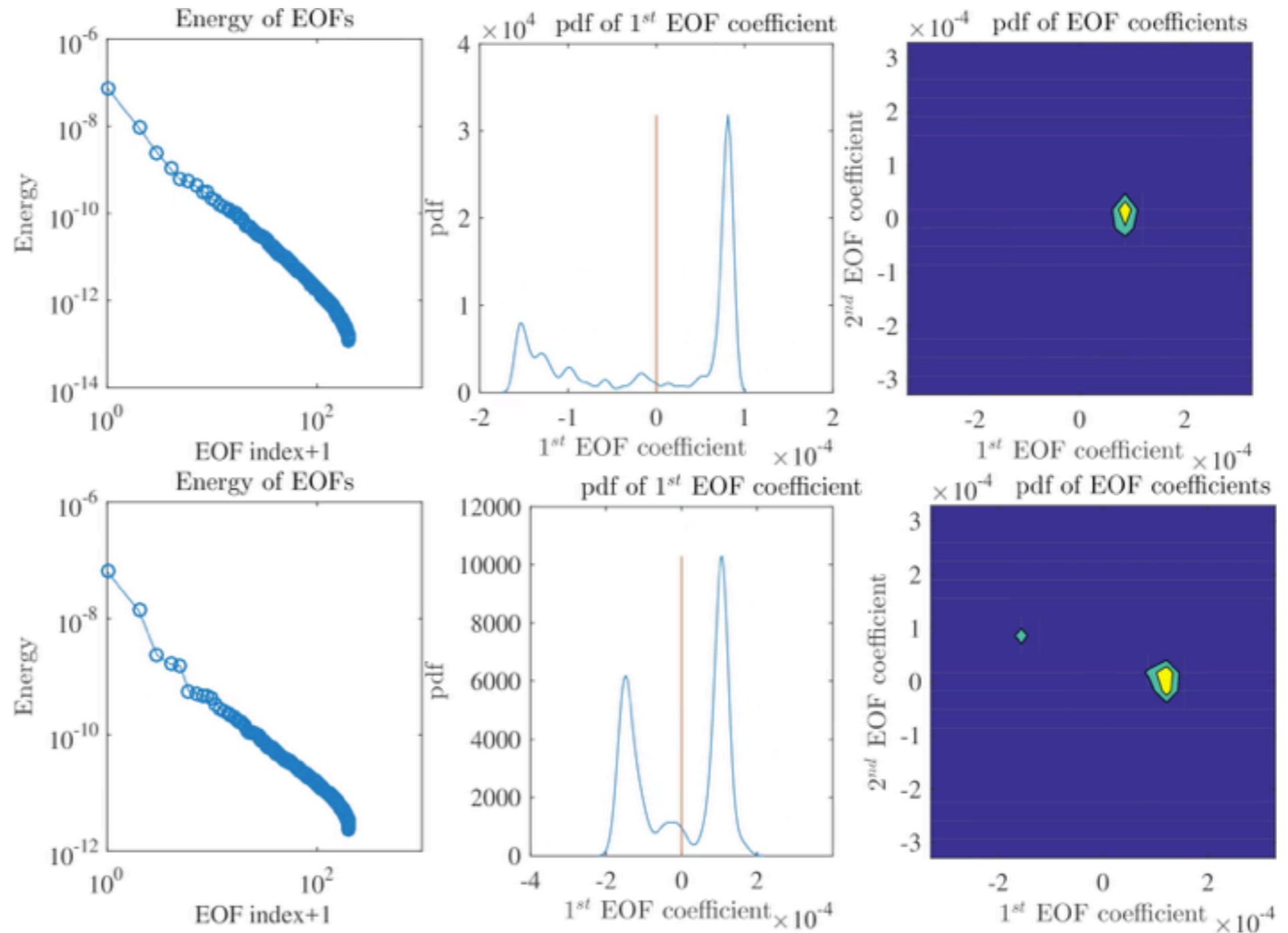
Mean

(random) EOF coefficient

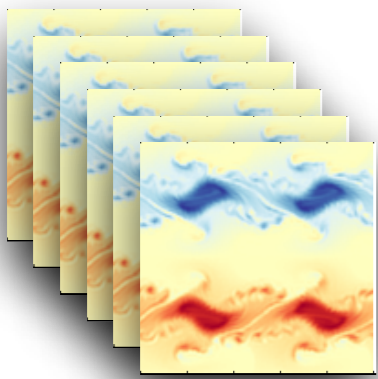


Visualization of the ensemble (200 realizations)

Random initial conditions

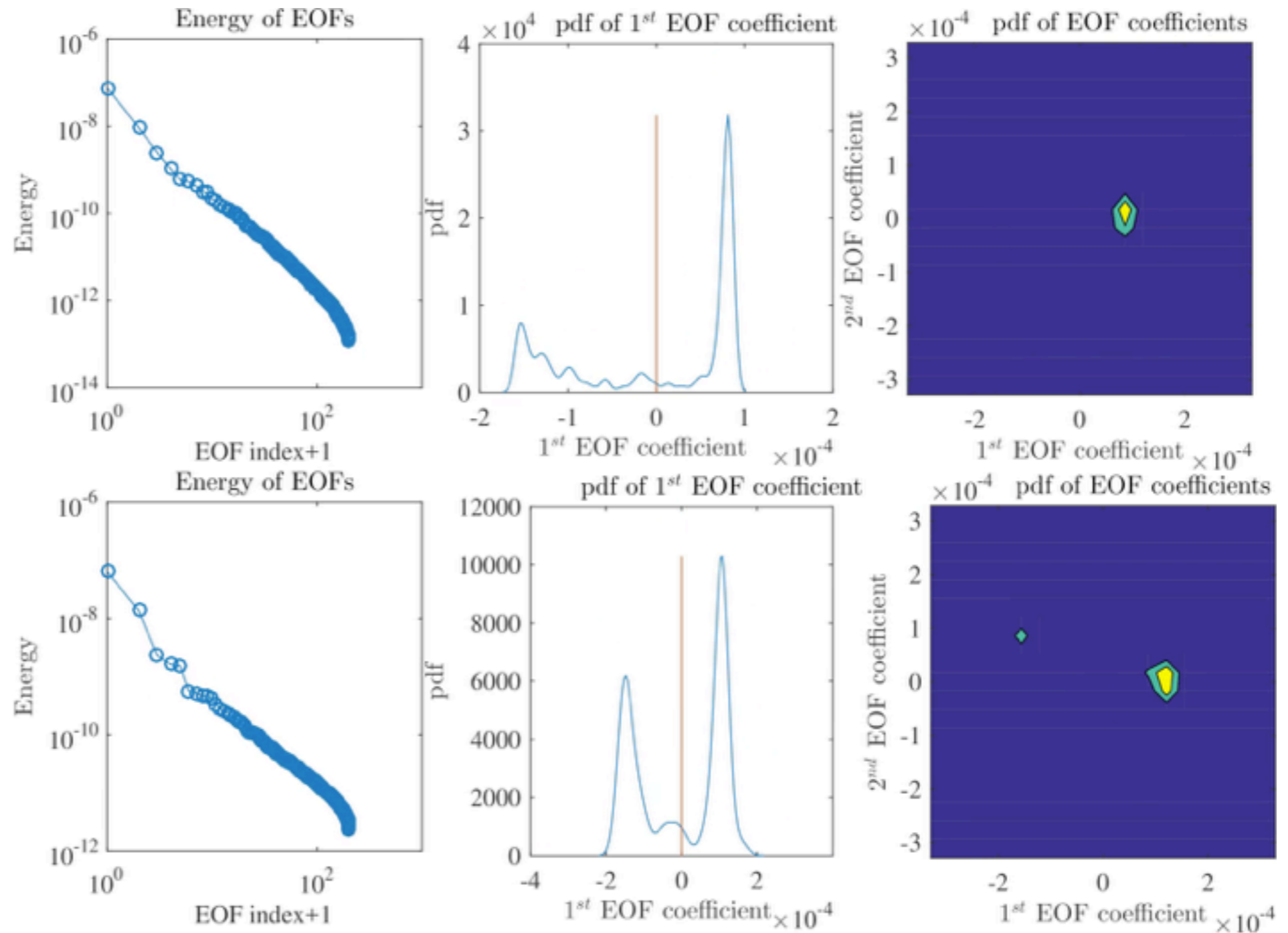


Under location uncertainty

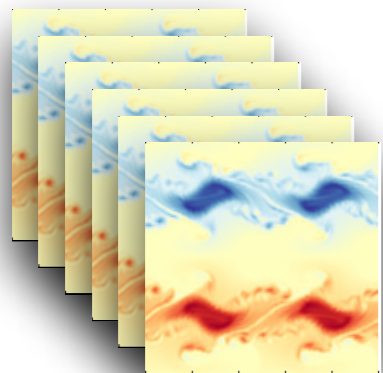


Visualization of the ensemble (200 realizations)

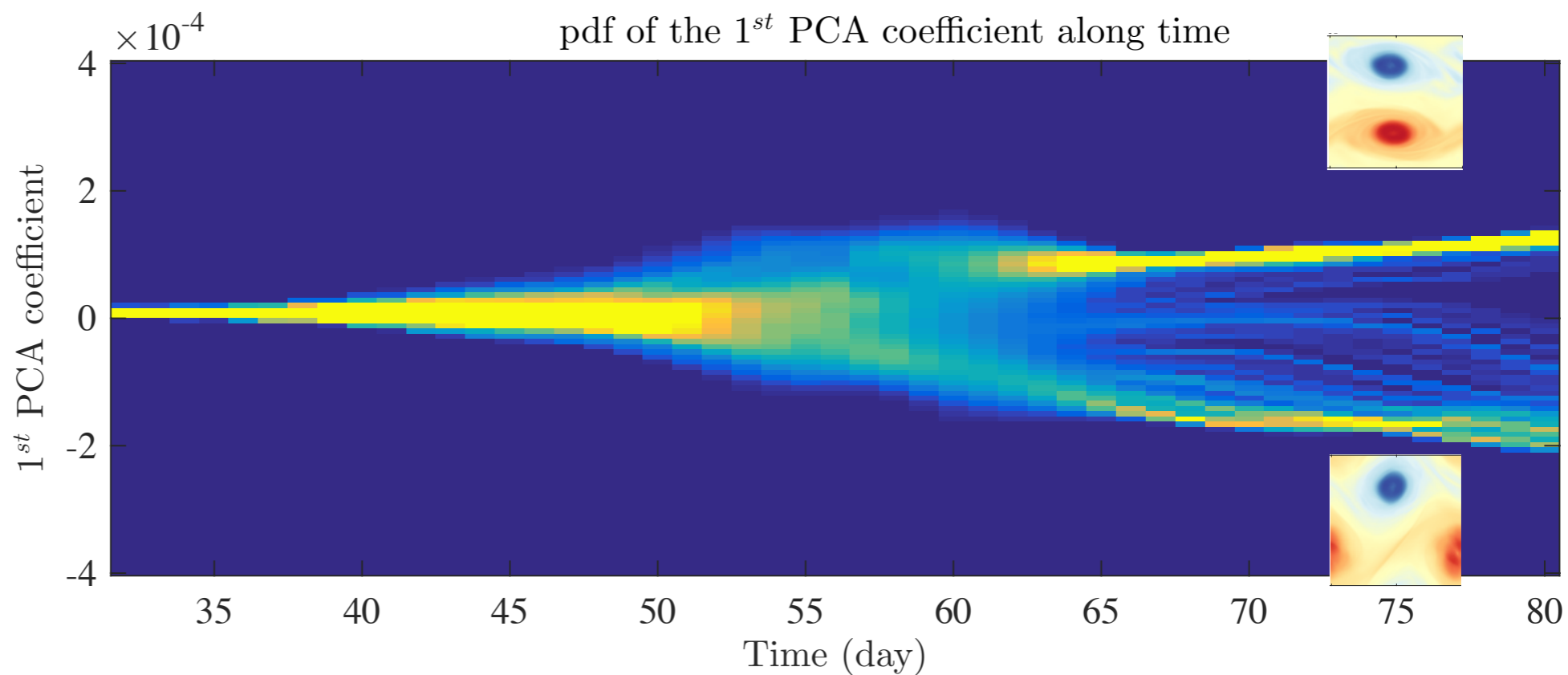
Random initial conditions



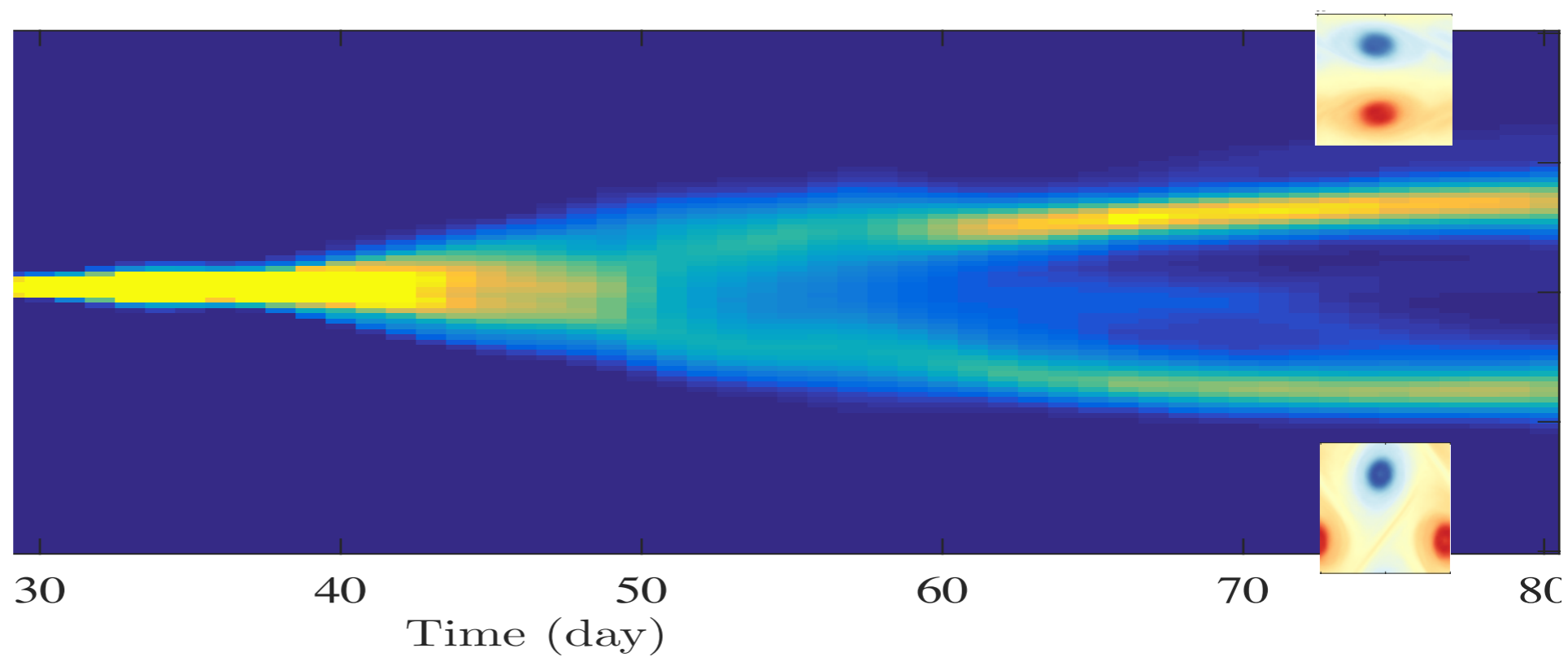
Under location uncertainty



Random
initial
conditions



Under
location
uncertainty



Conclusion

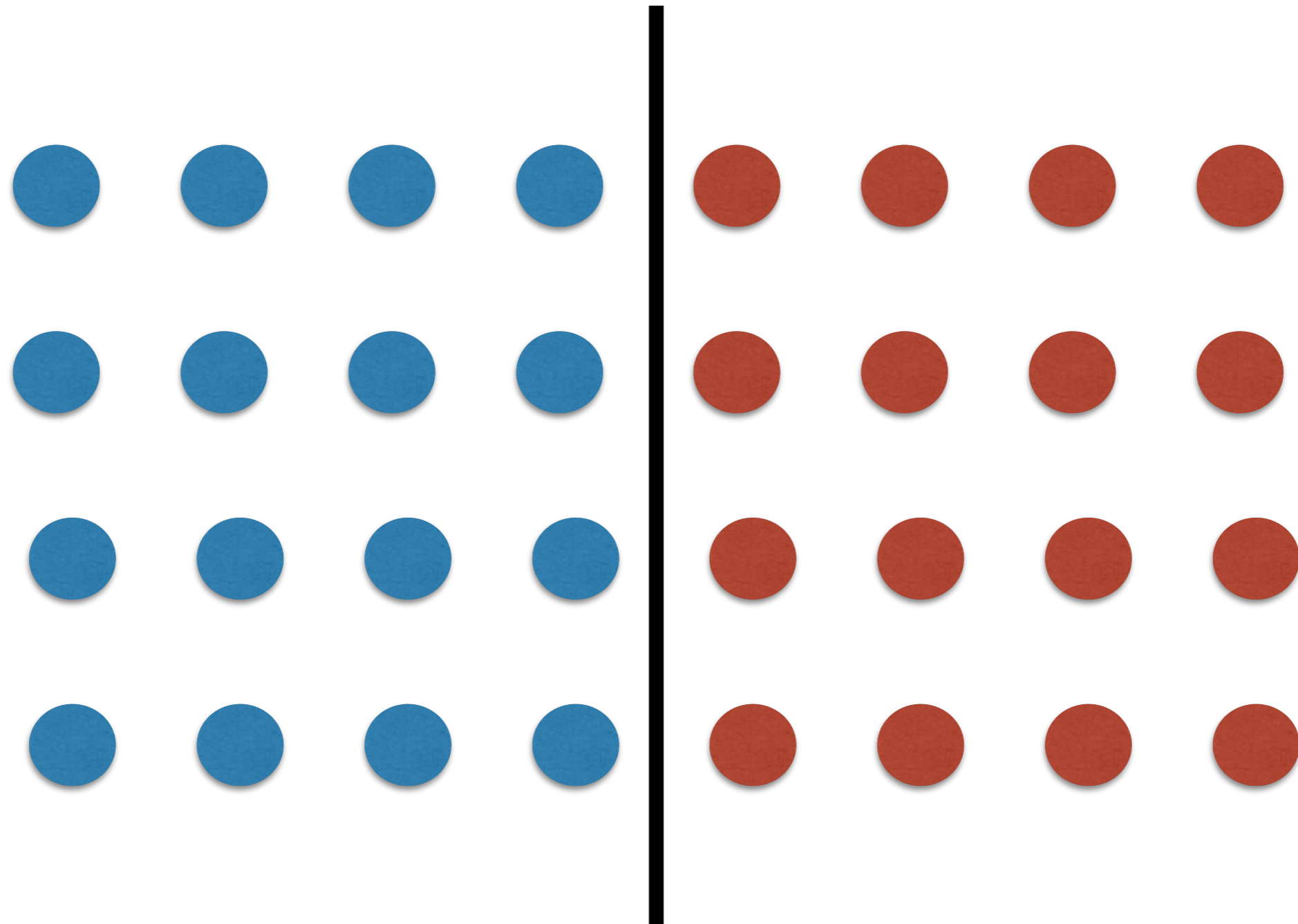
Conclusion

- Random transport applicable to any dynamics
- Link (inhomogeneous) diffusion, drift correction and multiplicative noise
- Predict likely scenarios with few realizations
- Other results:
 - Better small scales
 - Estimate positions and amplitudes of errors
 - Extreme events
 - Additional physical information

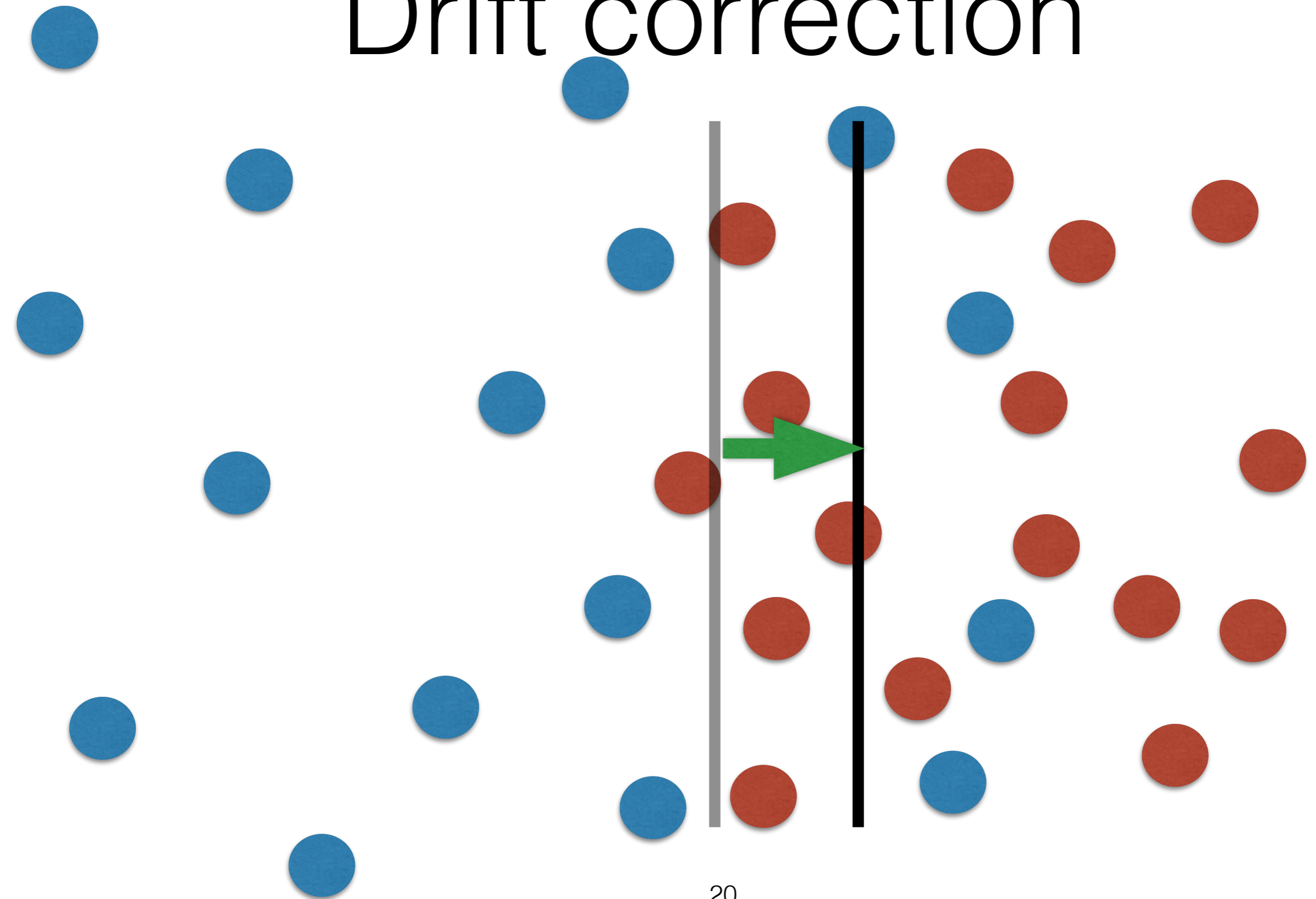
Thank you for your attention

Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

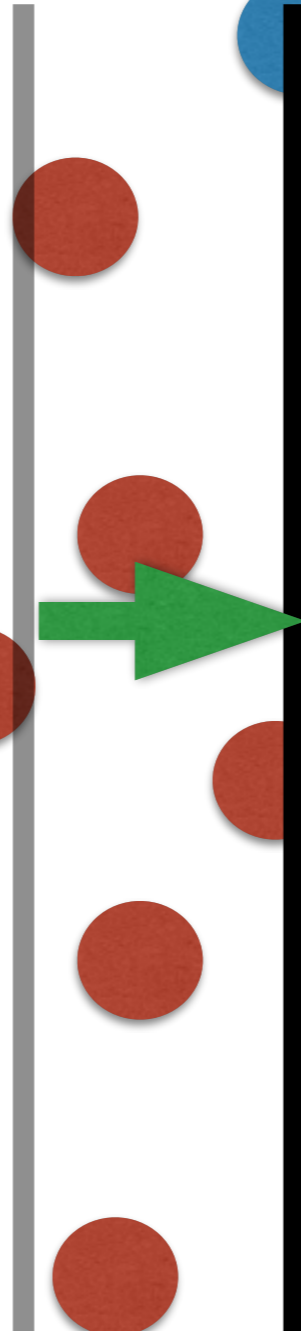


Drift correction



Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot \mathbf{a})^T$$



SQG under Moderate Uncertainty

SQG MU

Code available online

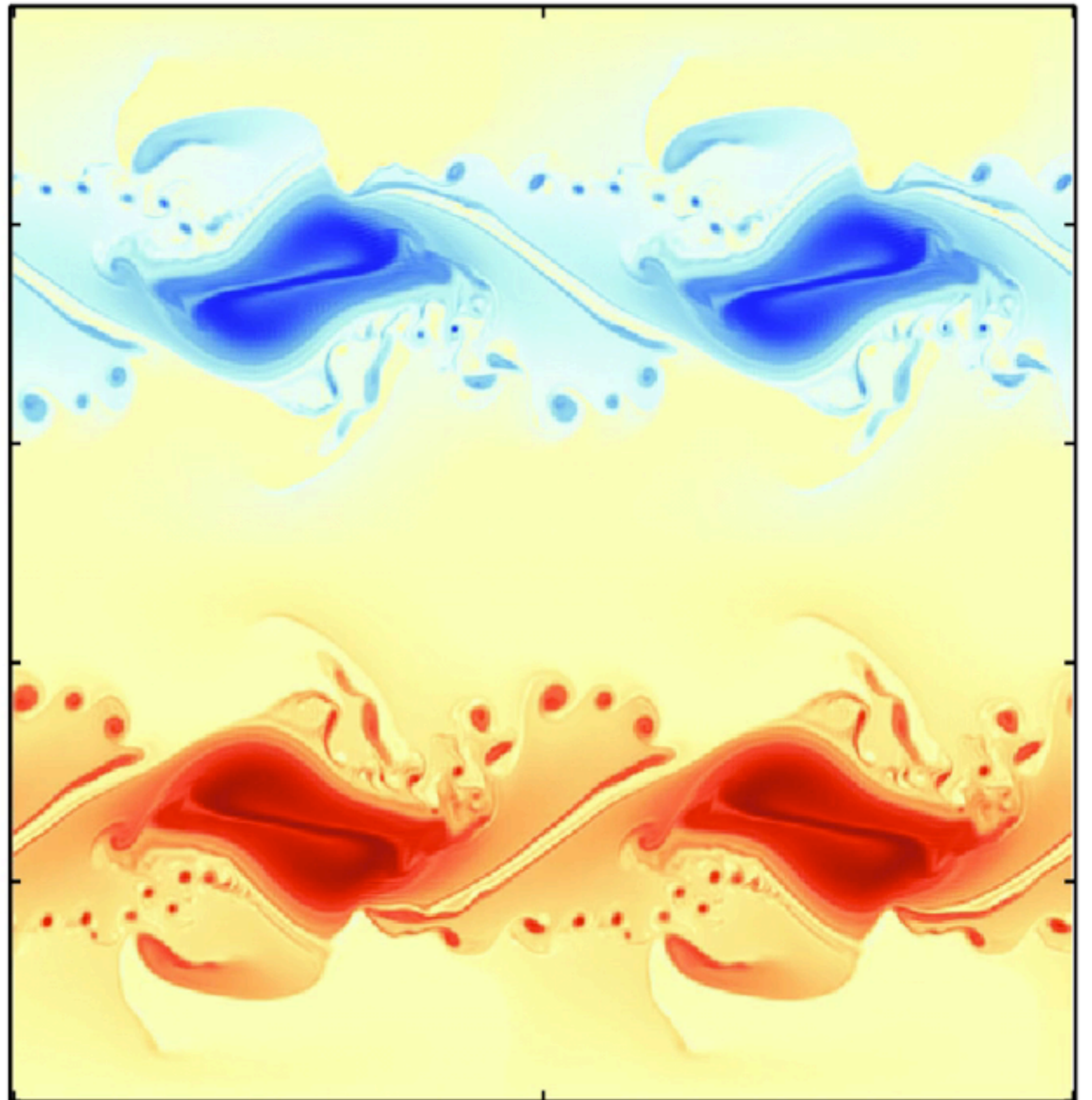
$t = 17$ days

Reference flow:

deterministic

SQG

512 x 512

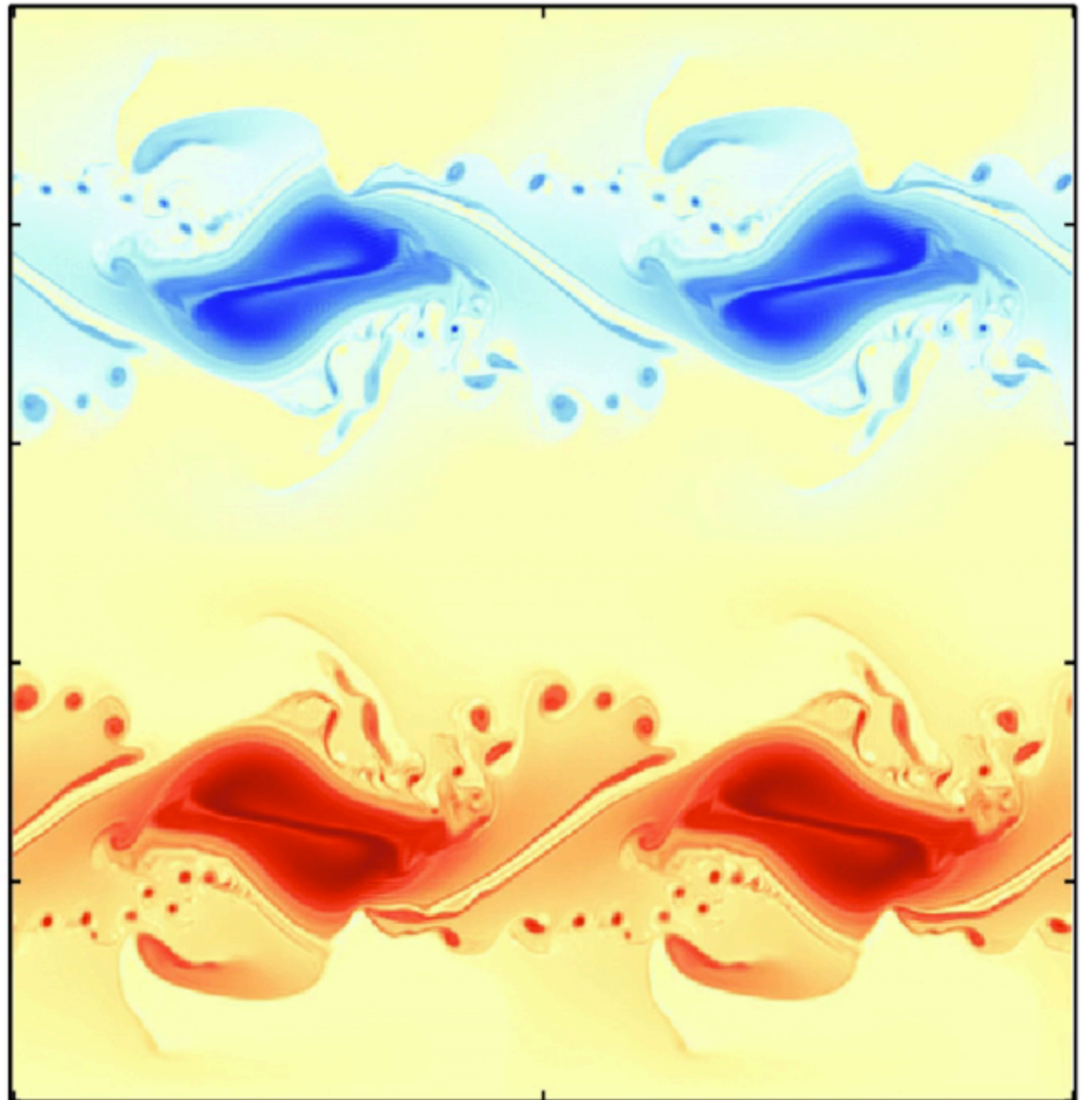


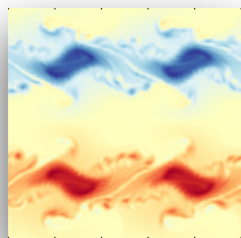
$t = 17$ days

Reference flow:
deterministic

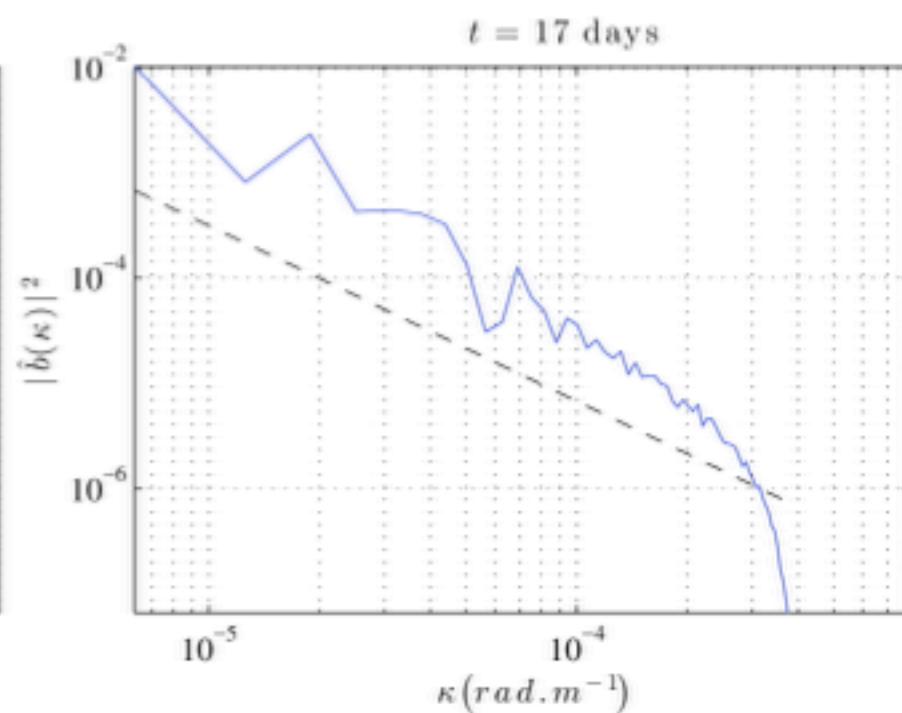
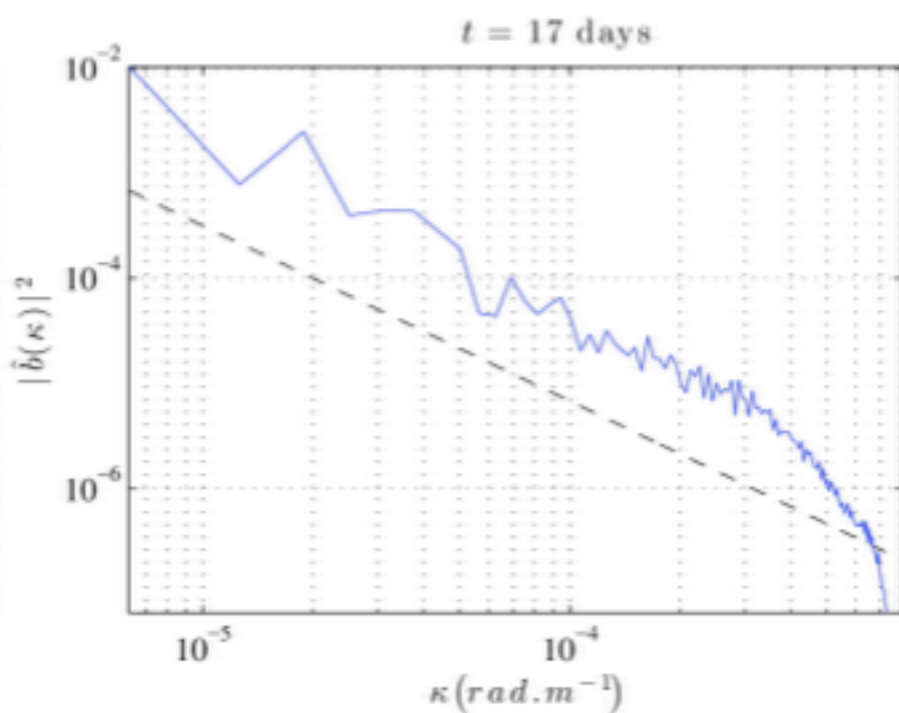
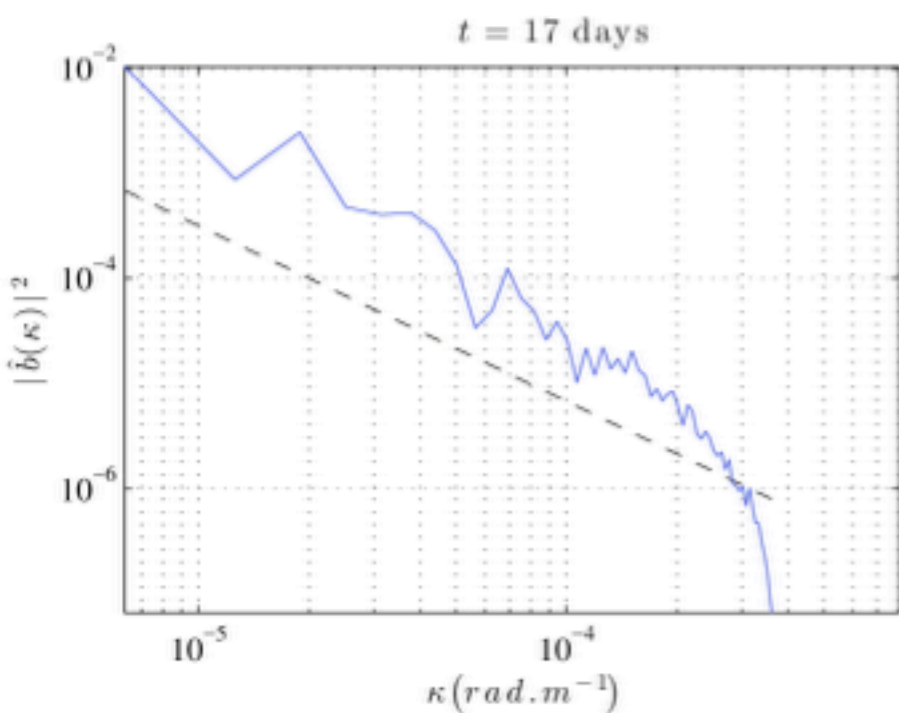
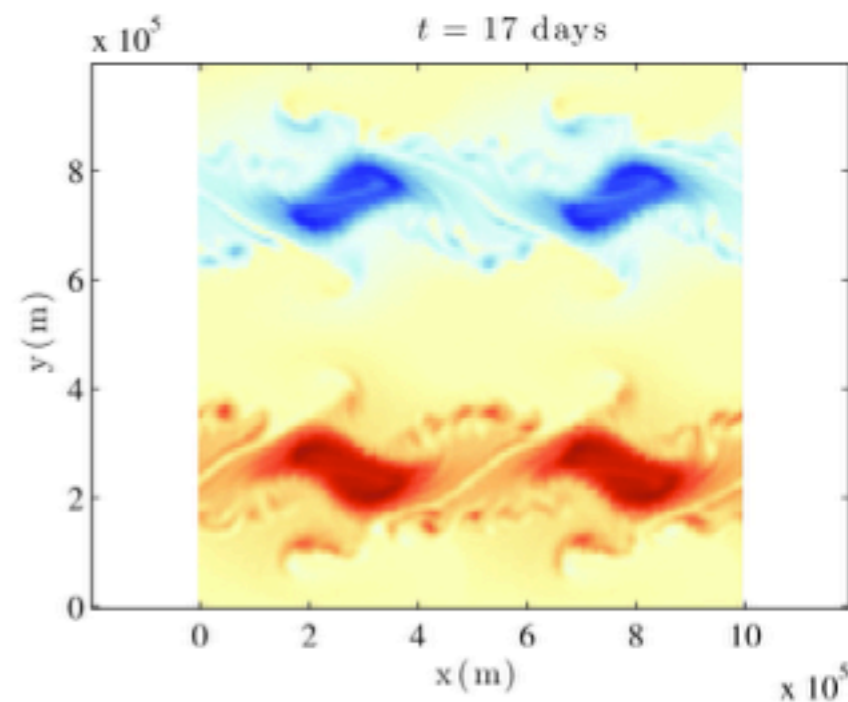
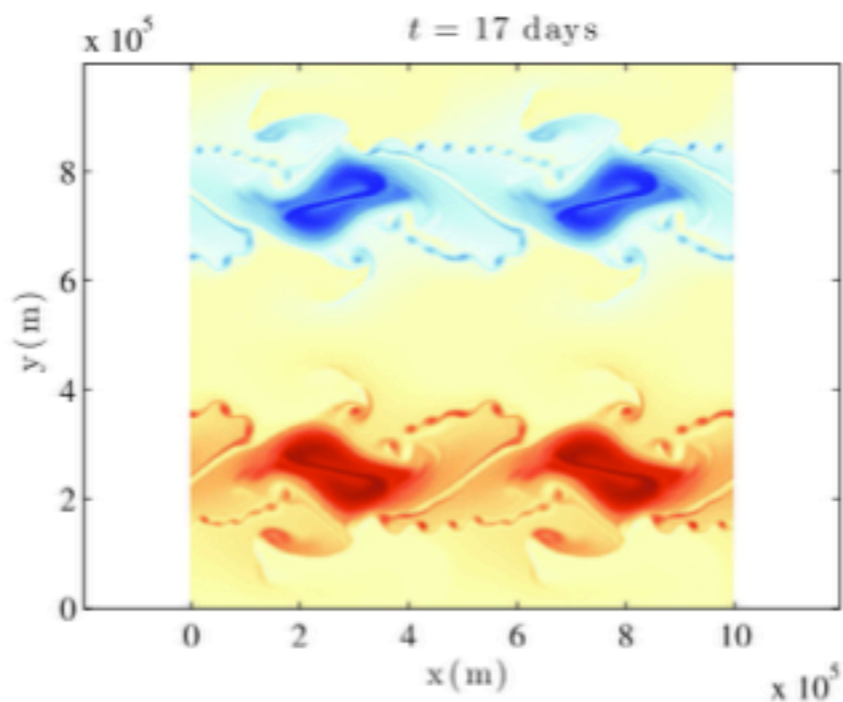
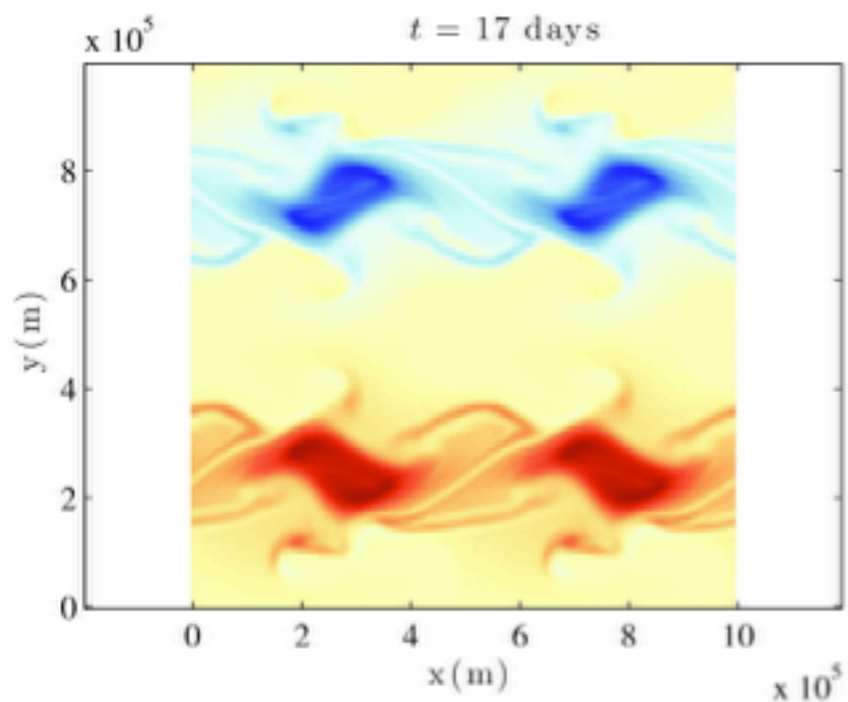
SQG

512 x 512





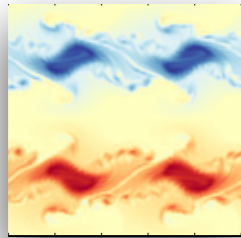
One realization



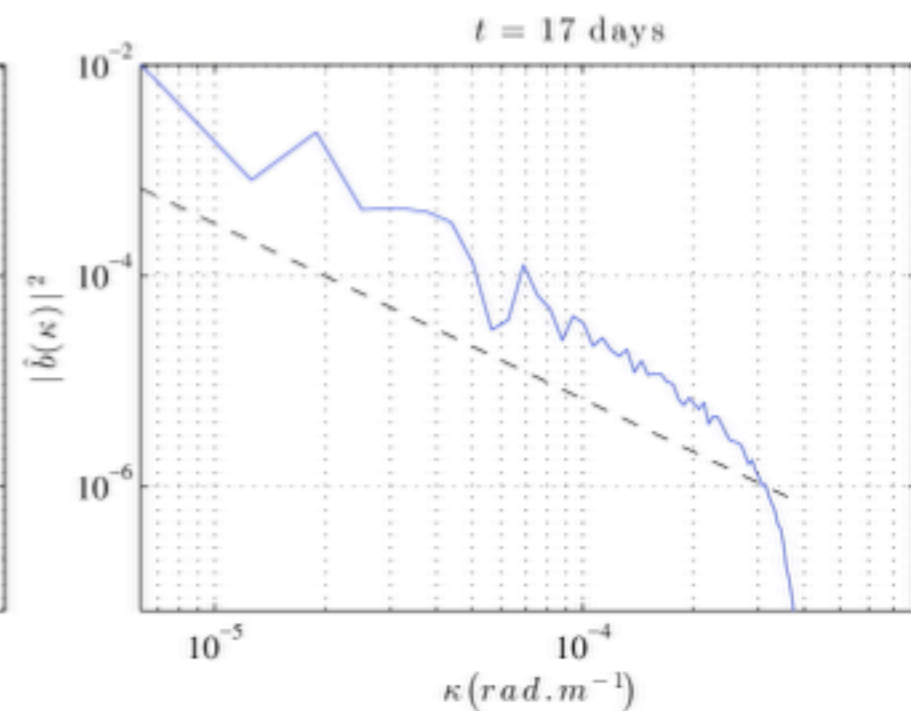
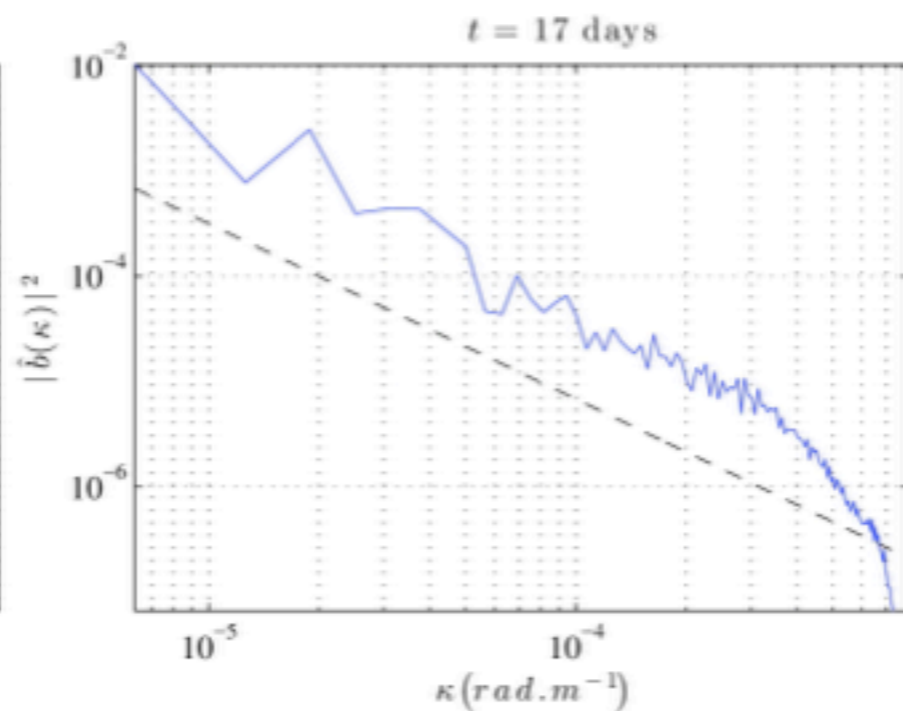
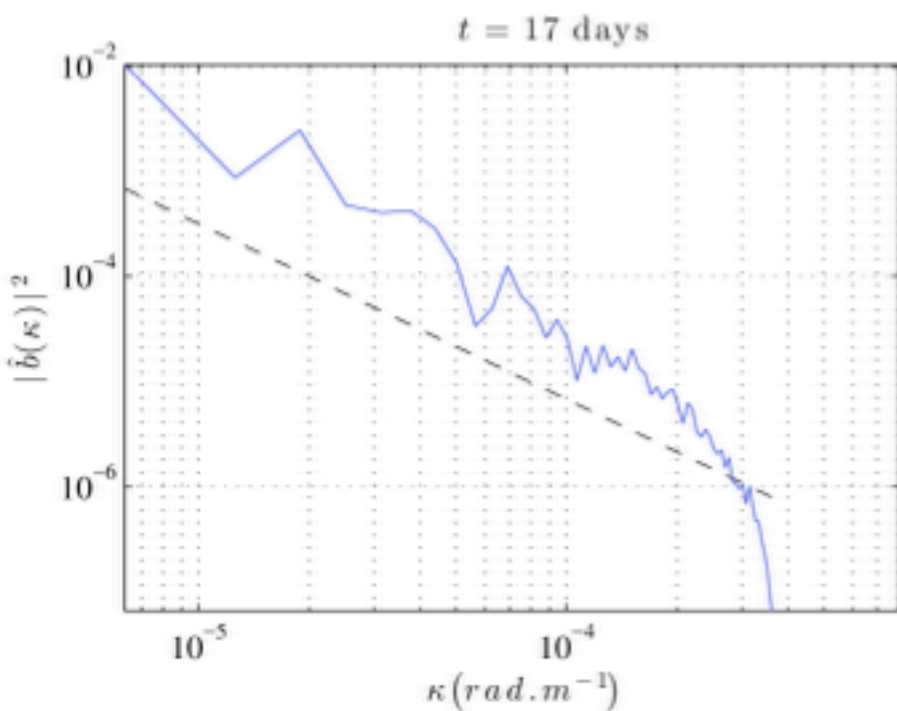
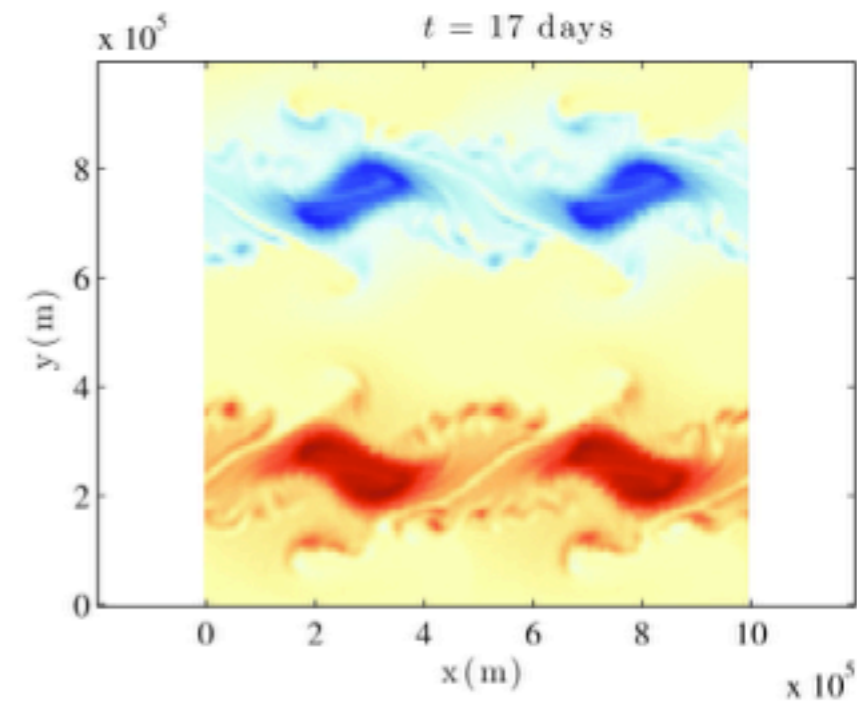
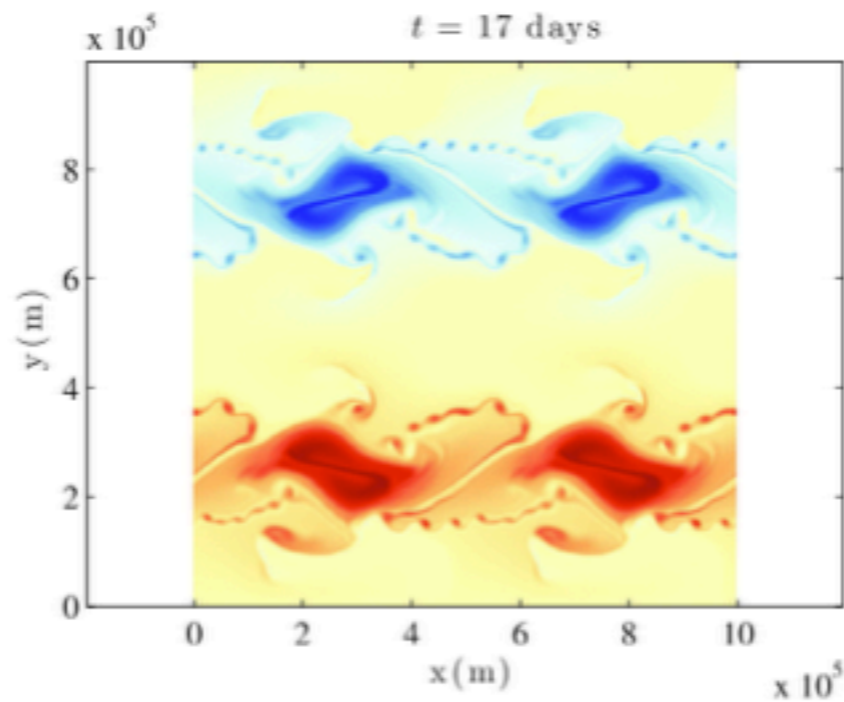
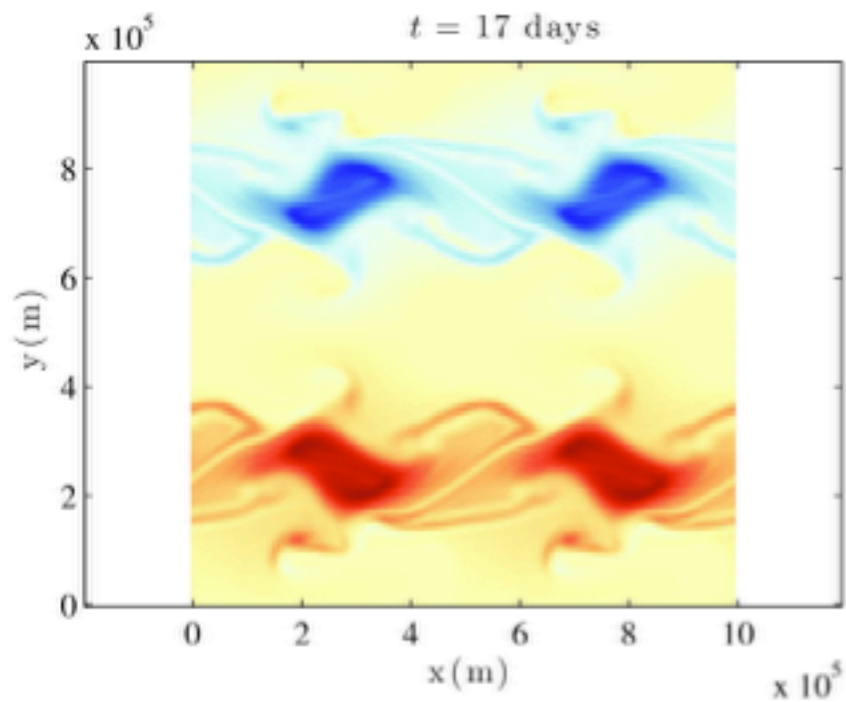
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



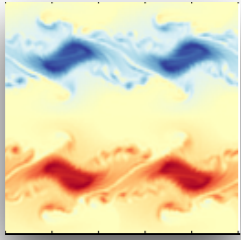
One realization



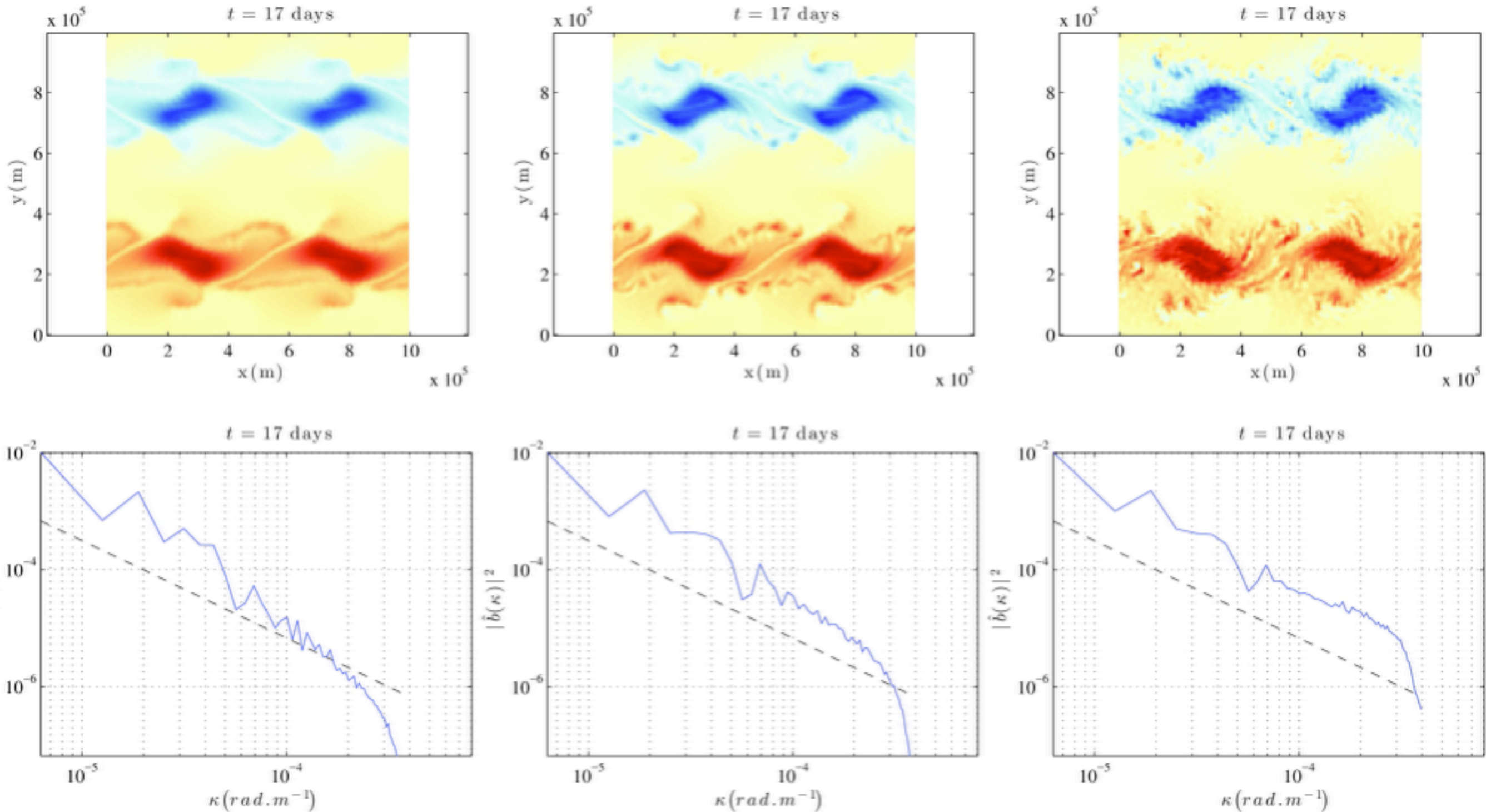
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



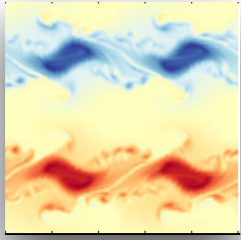
One realization



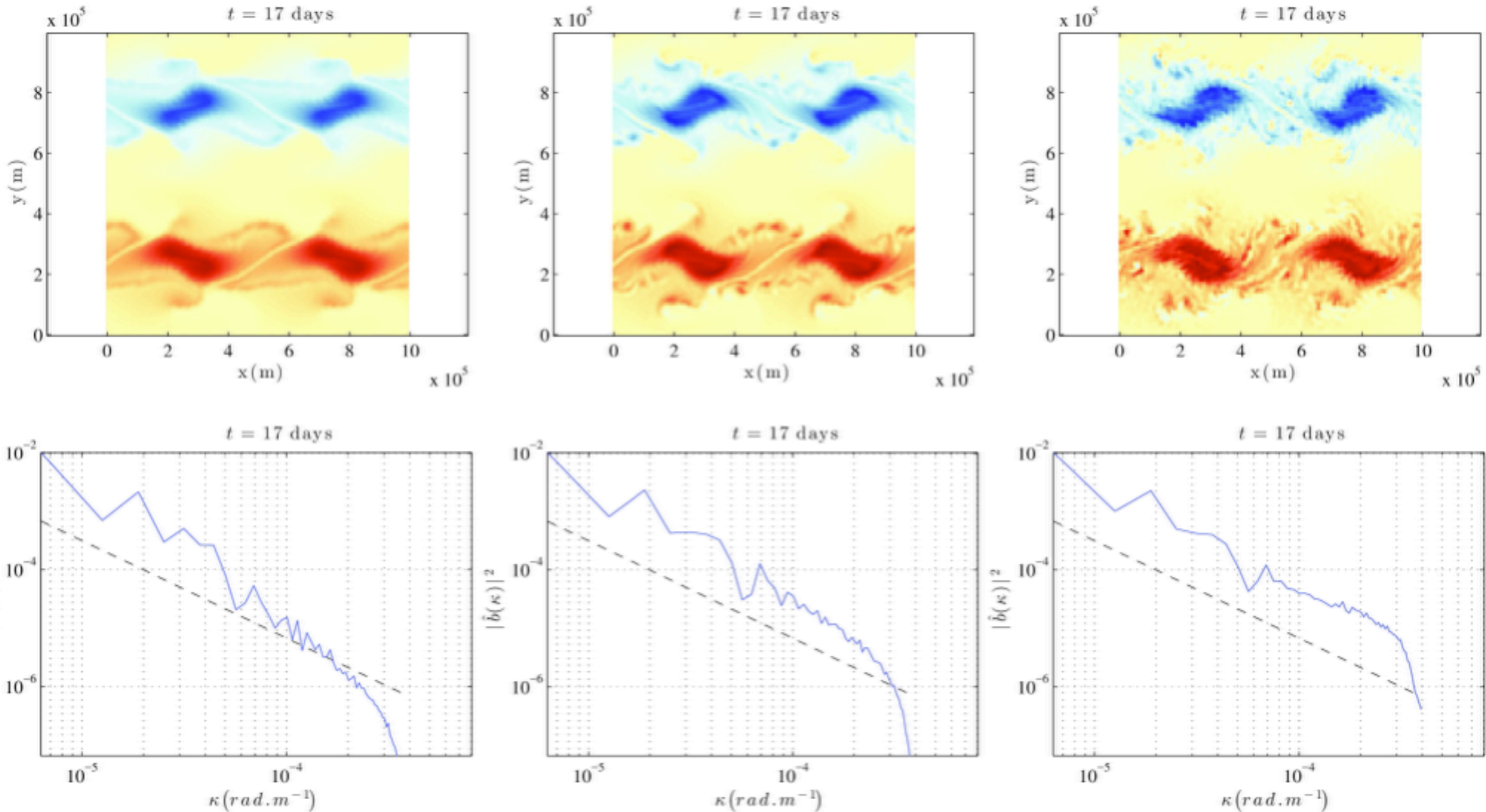
Lower noise

Our model

Larger noise



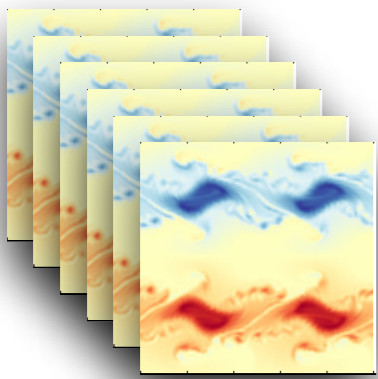
One realization



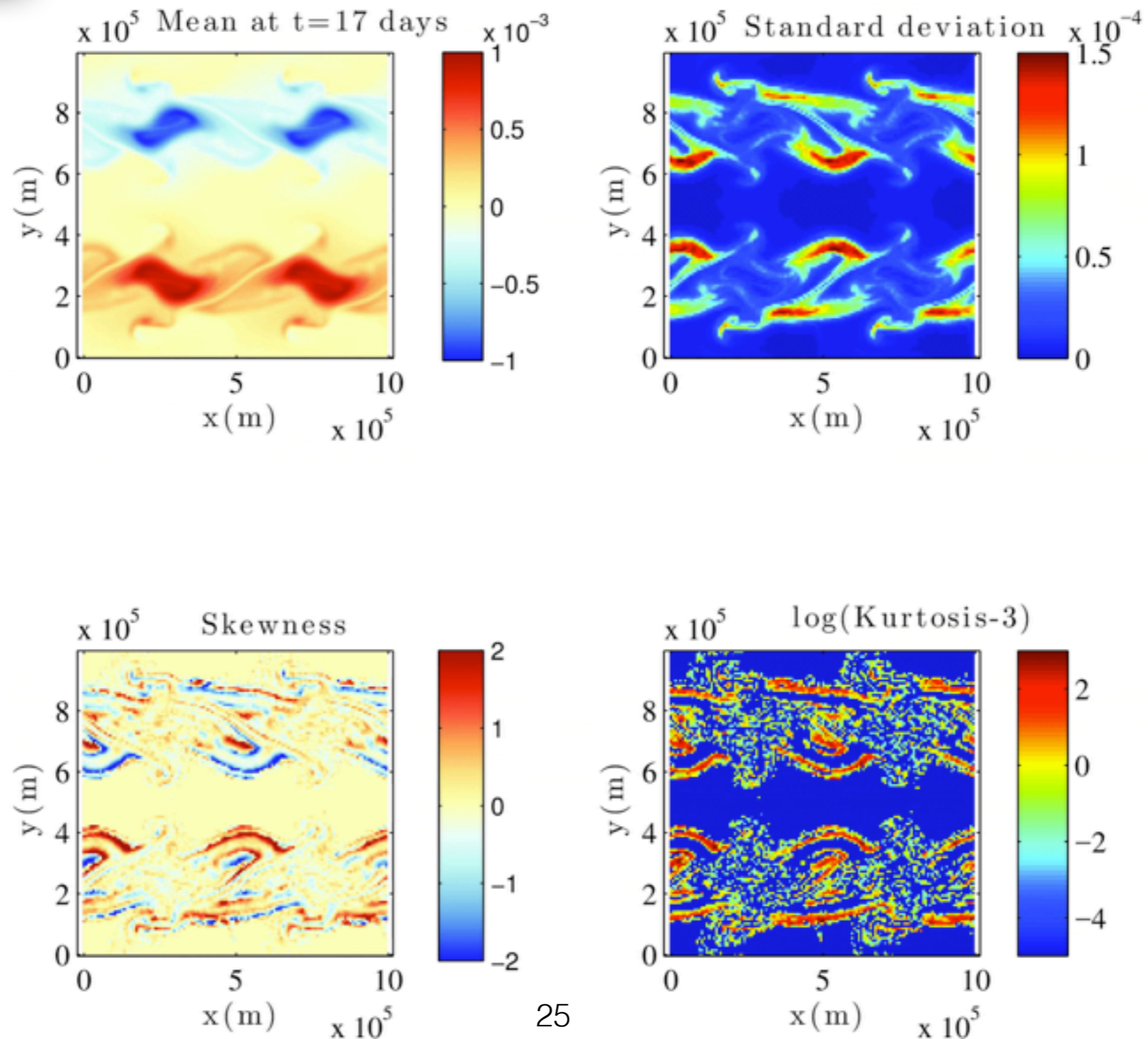
Lower noise

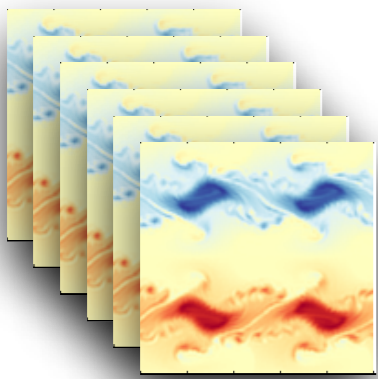
Our model

Larger noise

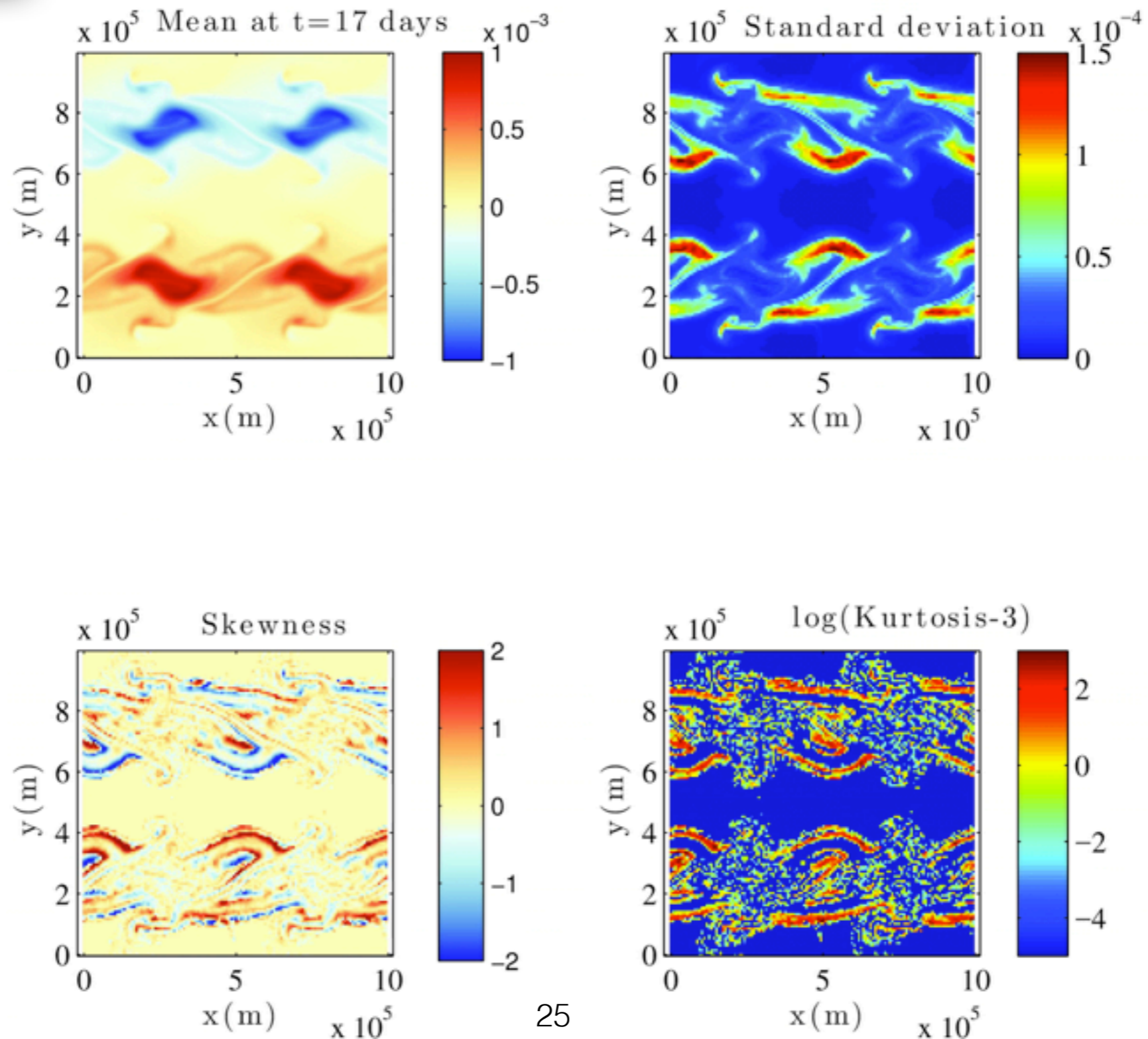


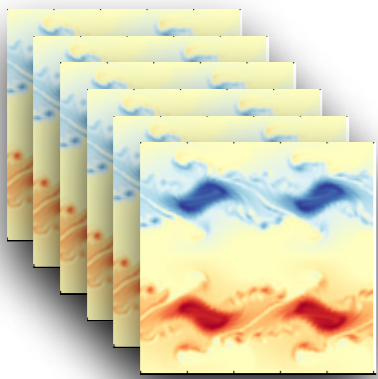
Ensemble



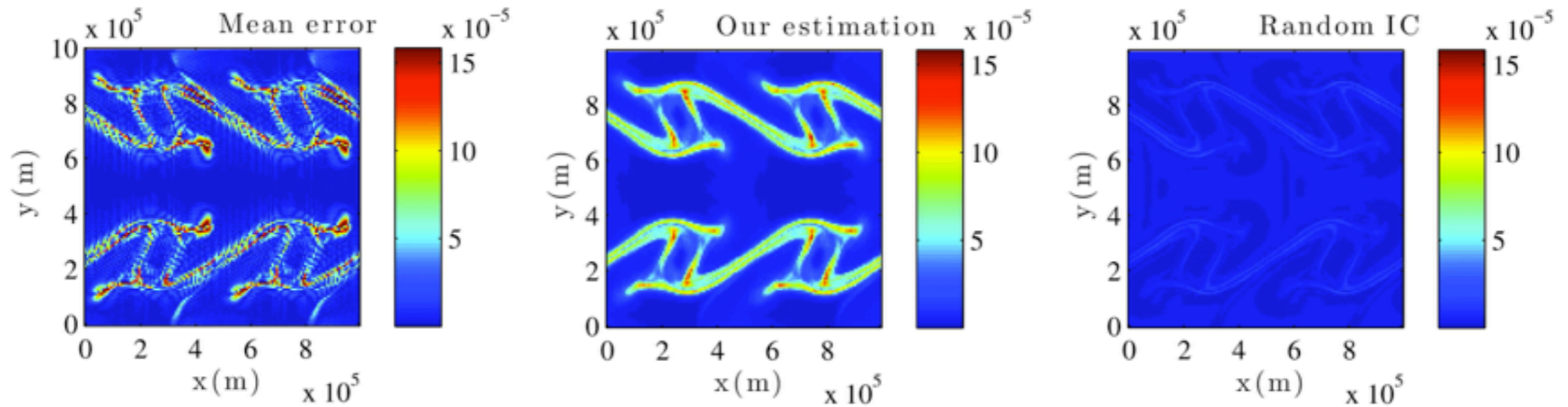


Ensemble

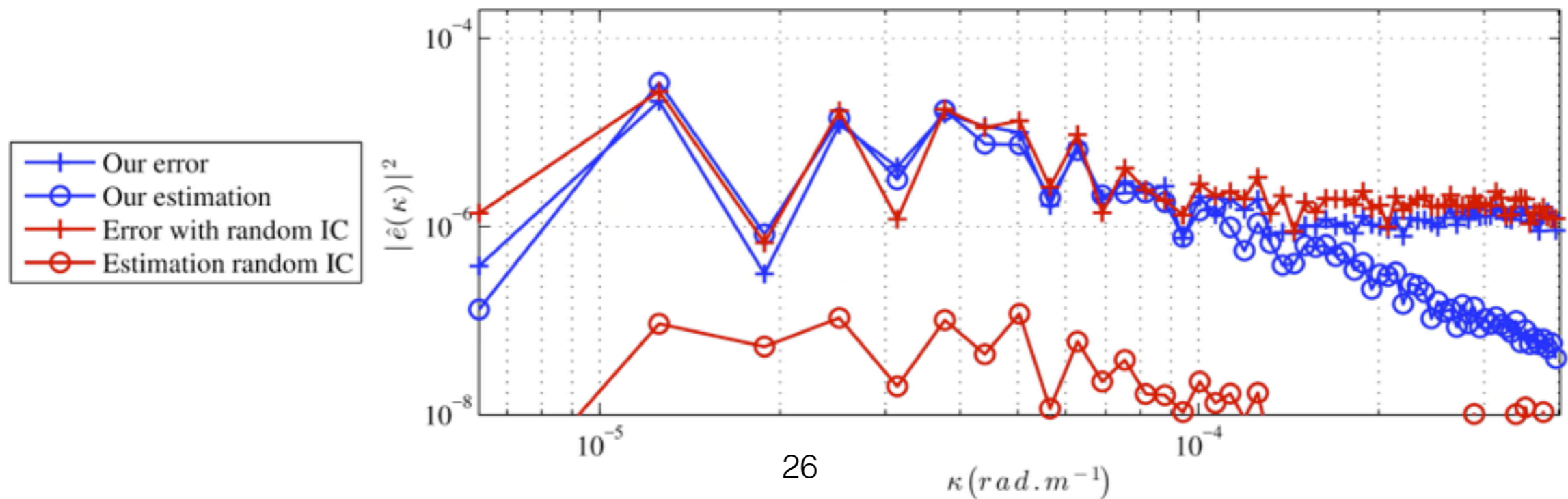


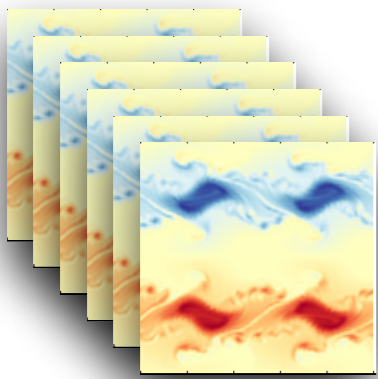


Ensemble

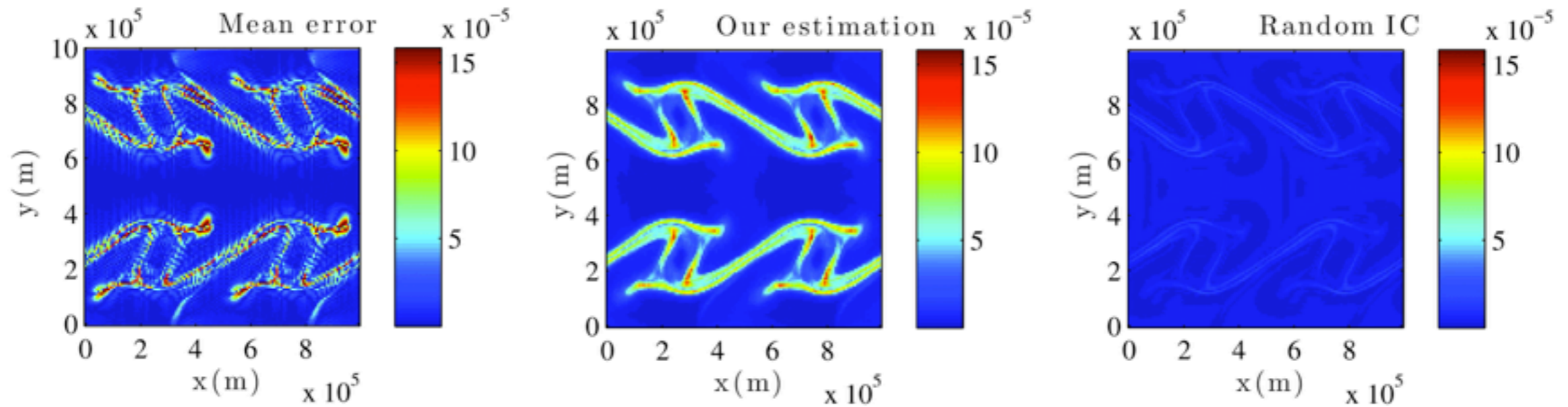


Spectrum of the errors and its estimation at $t=12$ days

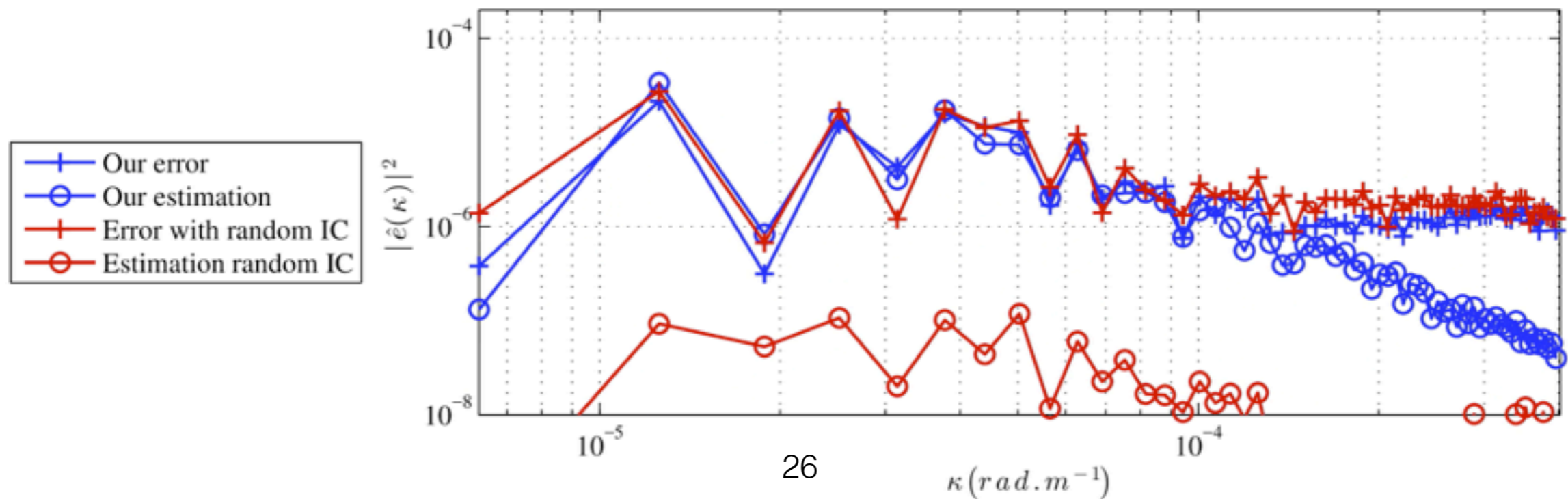




Ensemble



Spectrum of the errors and its estimation at $t=12$ days



Likely SQG scenarios

tracked by SQG MU

SQG under Strong Uncertainty

SQG SU

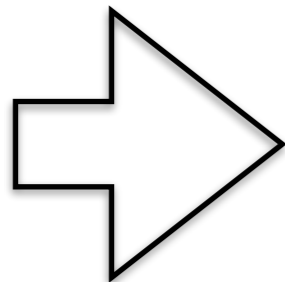
Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Horizontal
Diffusion

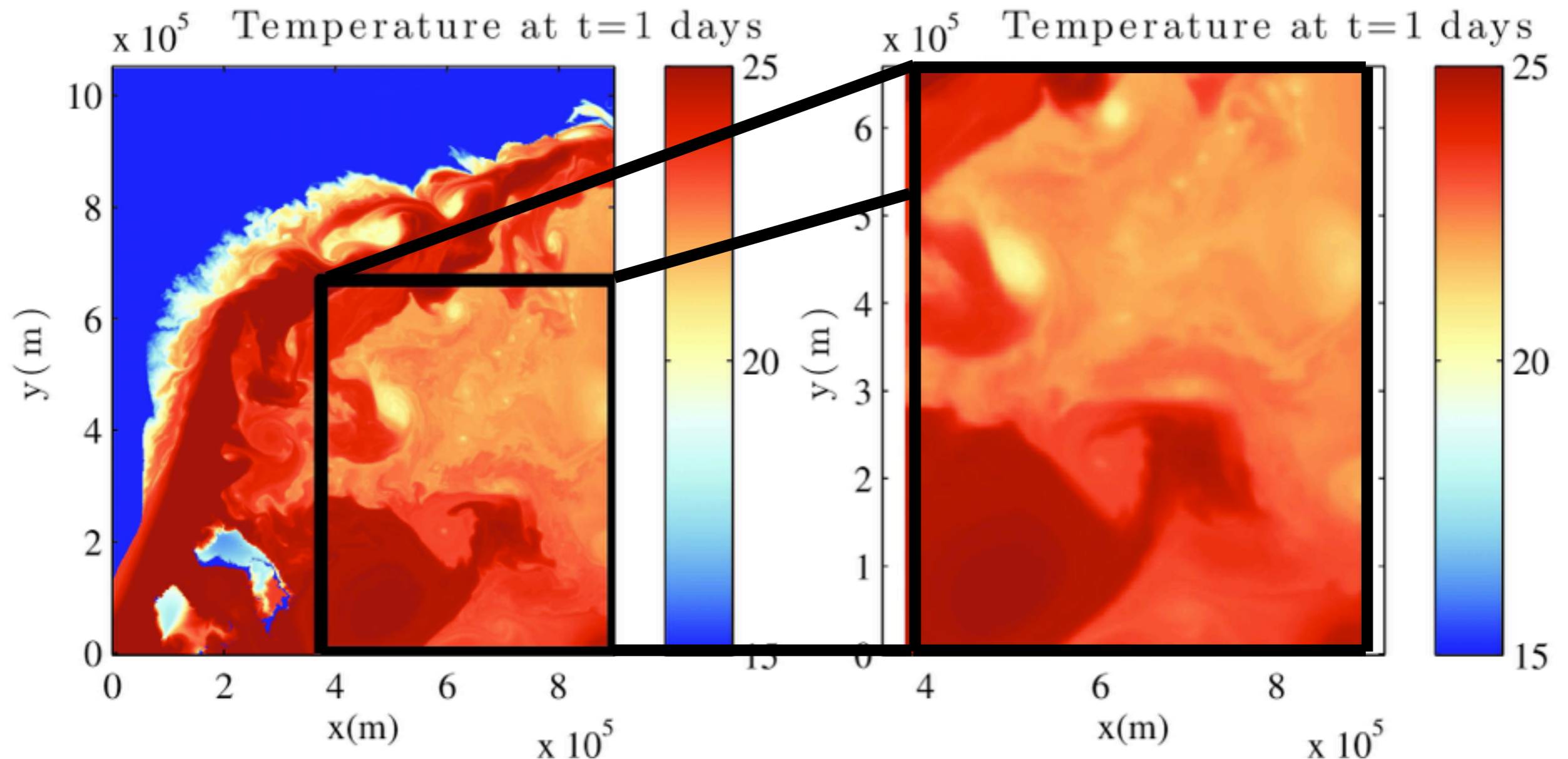
$$+ \frac{a}{2} \Delta \mathbf{u}$$



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

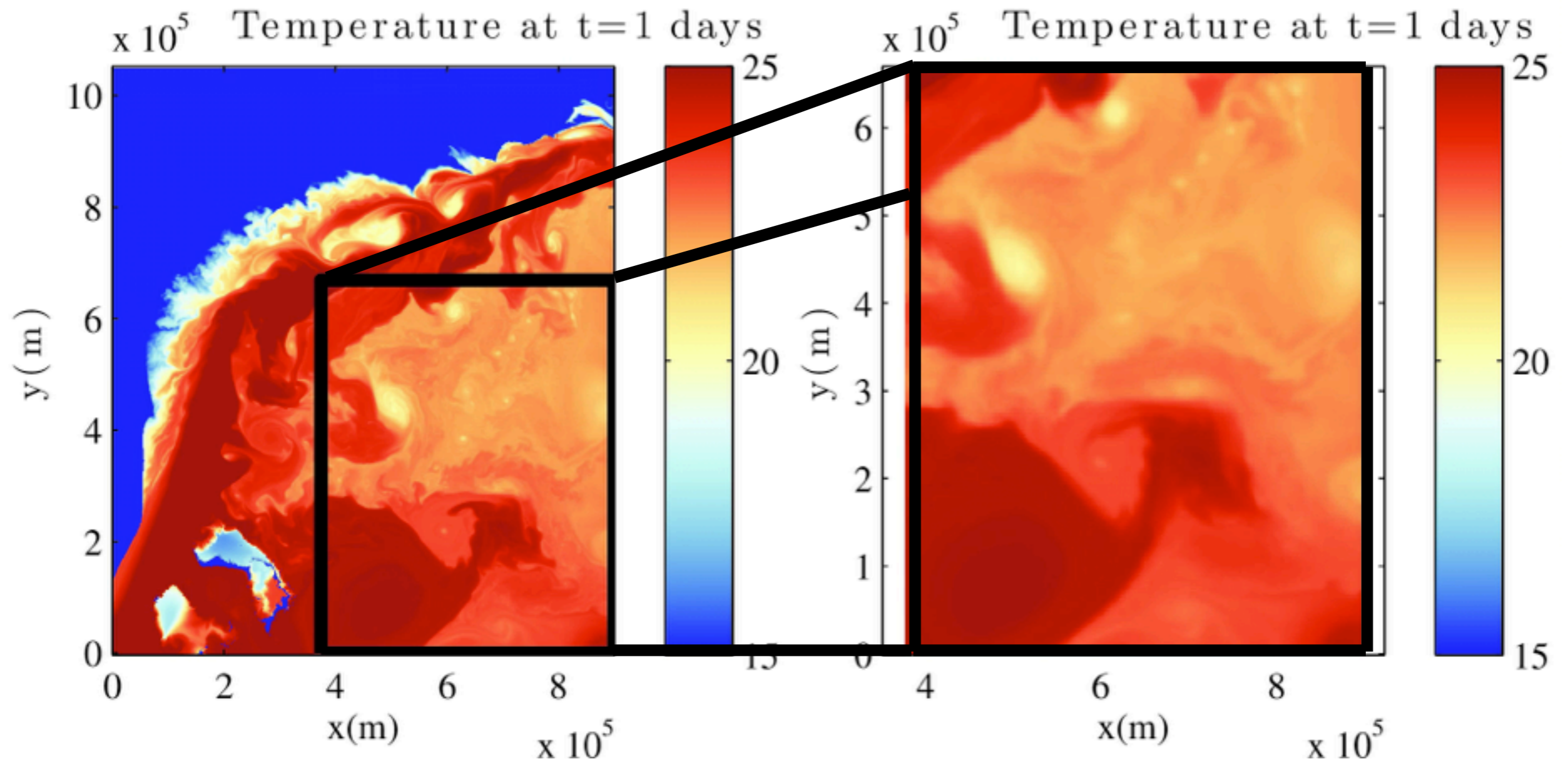
Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

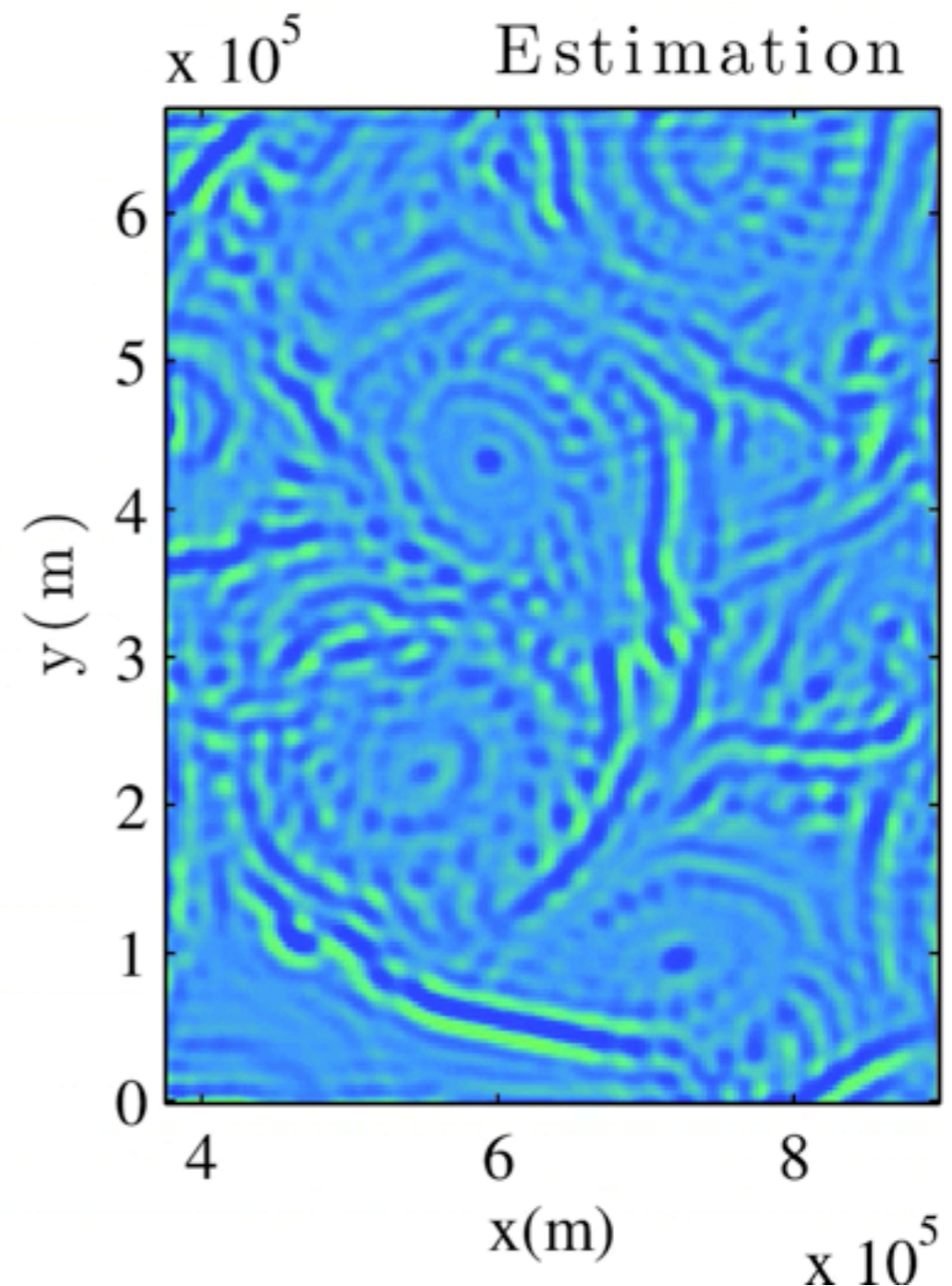
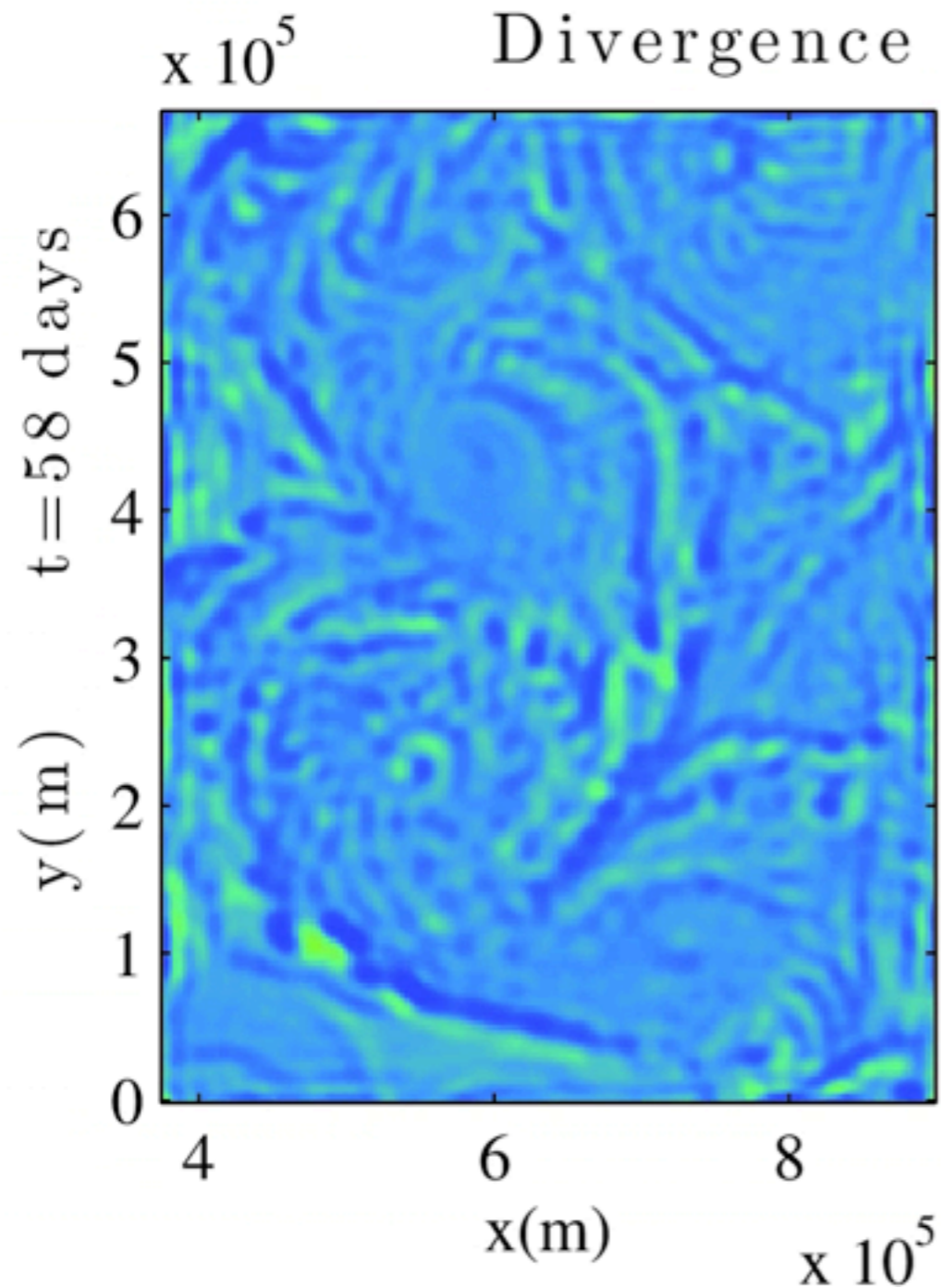


Filtering of model outputs:

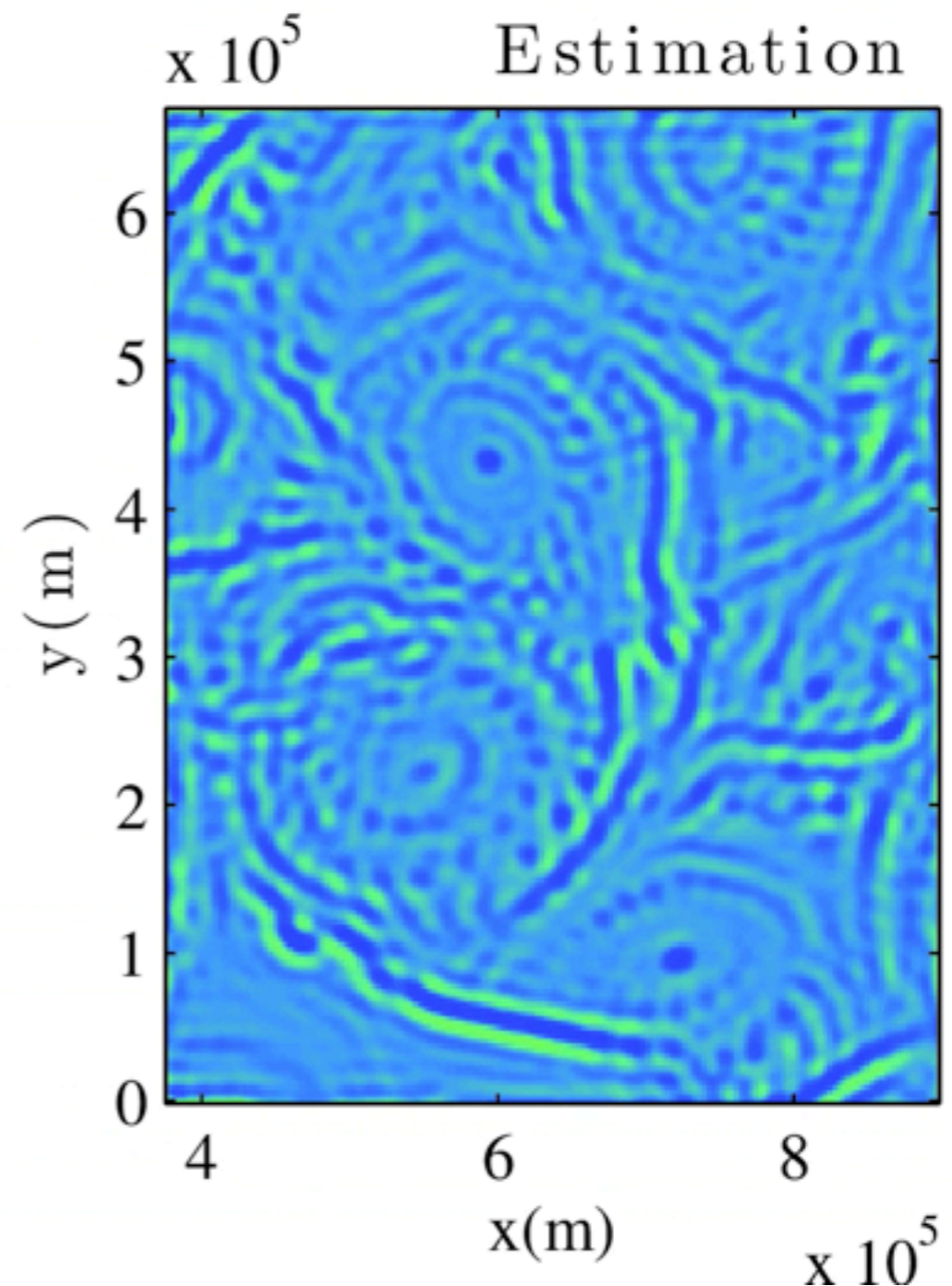
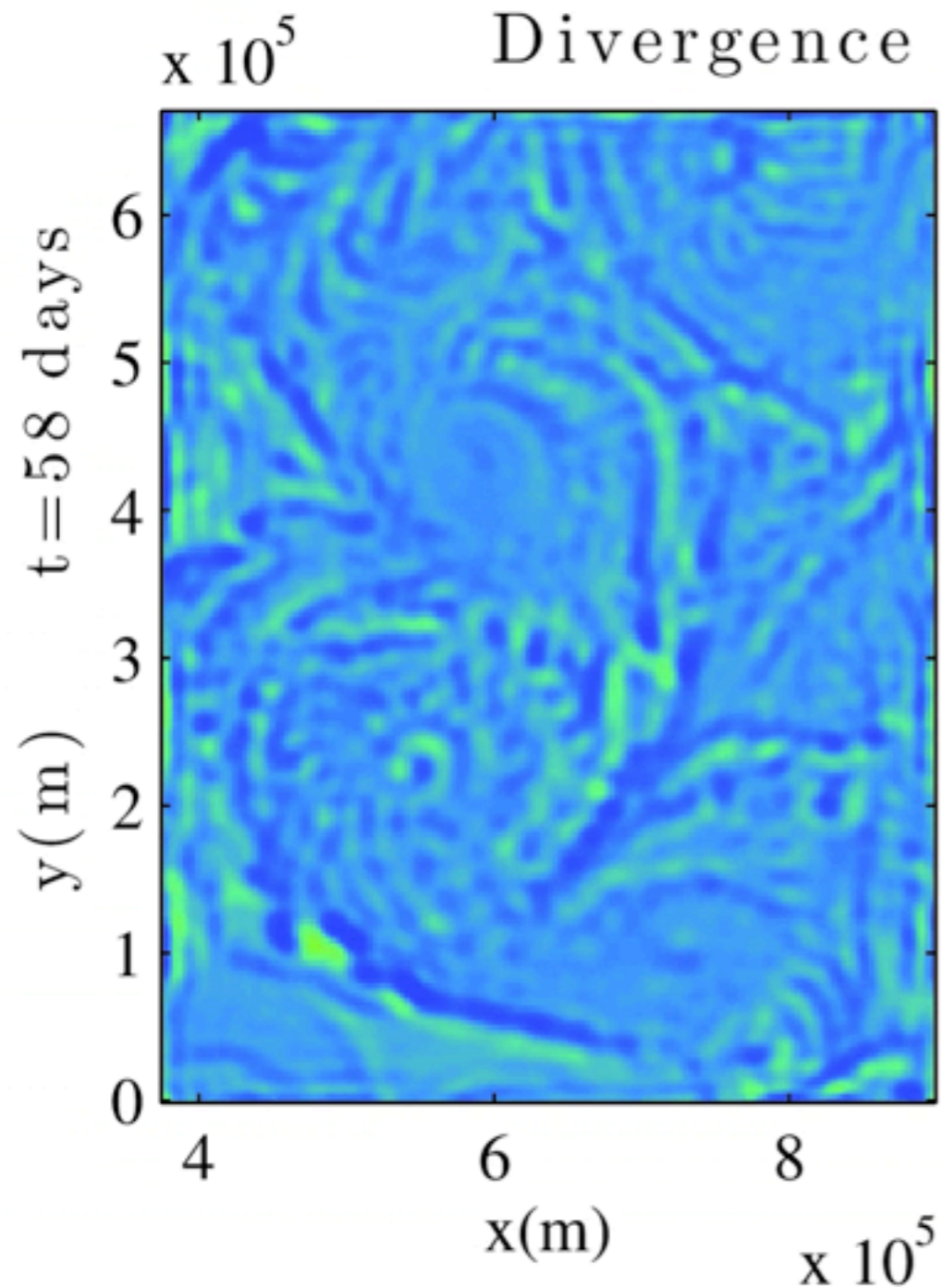
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"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.



Spatial test



Spatial test



Spectral test

