Robust Optimization-Based High Frequency Gm-C Filter Design

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Abstract. An optimization-based methodology for designing robust high frequency Gm-C filters is presented. In the first step of the design, the topology for high-frequency Gm-C filter is obtained directly from an active RC prototype, using the a transformation method proposed in [1]. where a transformation method for designing Gm-C filters from known RC filters while grounding floating capacitors is proposed. In the above mentioned paper the filter elements values are obtained by a user-defined methodology which can produce high sensitivity to component variations. In the work developed, the analytical characterization of a 2nd order Gm-C Sallen-Key LP filter is considered as well as the corresponding symbolic expressions for their sensitivity to component value variations. By using the minimization of the sensitivity as a goal function in the optimization process, parasitic capacitances are canceled and robust designs are obtained. Optimization and simulations results that validate the theoretical results are presented.

Keywords: Gm-C high-frequency filters, optimization, sensitivity minimization

1   Introduction

In recent years, the broadband communication systems, such as video applications, wireless telephony and computer networking have been a strong driving force for the IC technology. Low Pass Filters are an essential part of the analogue baseband circuit of modern communication receivers, so, they need to be low in power, small in size and they must not limit the performance of the overall system. For these applications, Gm-C has proved to be the best candidate [2].

Over the last decade several methodologies have been proposed for generating Gm-C filters topologies [3-7]. However, the necessity for designing high frequency filters poses several topological constraints, namely on what concerns the reduction of the parasitic capacitances and thus guaranteeing the non-existence of floating capacitors. In 0a transformation method for obtaining high frequency Gm-C filters from active RC prototypes is proposed. By introducing constraints on the design variables, a direct correspondence on the design equations for both the active-RC and Gm-C filter is obtained. This methodology, however, shows severe limitations on the robustness of the solutions obtained, due to the high sensitivity of the filter

characteristics to parameter variations. In this paper we propose an optimization-based methodology for the evaluation of the filter component values. High solution robustness is obtained by considering the sensitivity of the quality factor of the filter to component variations in the optimization process.

This can be obtained using the minimization of the sensitivities as a goal function in the optimization process.

2   Contribution to Value Creation

The main objective of the optimization-based methodology presented is to design high frequency Gm-C filters using deep submicron technologies. For this purpose two main concerns were considered. On one hand, the necessity for reducing the influence of parasitic capacitances, imposes topological constraints such as the non-existence of floating capacitors. On the other hand as technologies approach nanoscale, the variability of the technological parameters becomes an important issue to be taken into account during the sizing process. The topological constraint question has already been addressed in the literature, and one of the proposed methods will be described in detail in section 3.

Although there are many ways to approach the sizing problem, ranging from the manual design and knowledge-based strategies to global optimization engines, robust sizing is still a challenging task for analogue filter designers. The main contribution of the work presented in this paper consists in adopting an optimization-based sizing methodology, where the sensitivity functions of the filter characteristics to component value variations are used as goal functions. This methodology will be described in Section 4.

3   RC to Gm-C Transformation

Although several op-amp-RC to Gm-C circuit transformation methods have been presented over the years [3] , the method presented in this section allows the generation of high-performance Gm-C circuits that benefit from the advantages of well-known op-amp-RC structures. The main advantage of this method is twofold. In one hand the resulting circuits have only grounded capacitors, which makes them good candidates for high frequency applications. On the other hand the filter design equations remain valid for both cases, if some constraints on design variables are imposed 0.

In order to generate the Gm-C equivalent circuit the node equations of the op-amp-RC circuit are obtained. For a user defined output node, the corresponding equation may be written as:

$$sC_oV_o = \sum_m G_jV_j + s \sum_n C_iV_i \qquad (1)$$

where the first term on the right side of the equation represents the resistors connected to the output node and the second term appears due to the floating capacitors connected between the output node and another node. In order to ground these capacitors it is necessary to define a new capacitor, C_i, as:

$$C_i = C_0 \frac{G_{ci}}{G_0} \qquad (2)$$

Having in mind that if G_{ci} over G_o is one, then C_i must be equal to C_o. This allows to ground the floating capacitor without changing its value, since the block transfer function is not affected. Substituting (2) into (1) will generate:

$$sC_oV_o = \sum_m G_jV_j + s \sum_n \frac{C_0}{G_0} G_{ci}V_i \qquad (3)$$

which can be implemented with the Gm-C structure represented in Fig. 1(a).

   A special case from the general fist order section occurs when no floating capacitor exists in the transformation section. Thus (3) becomes the following:

$$sC_oV_o = \sum_m G_jV_j + 0 \qquad (4)$$

Which can be implemented with the Gm-C structure represented in Fig. 1(b).

(a)   (b)
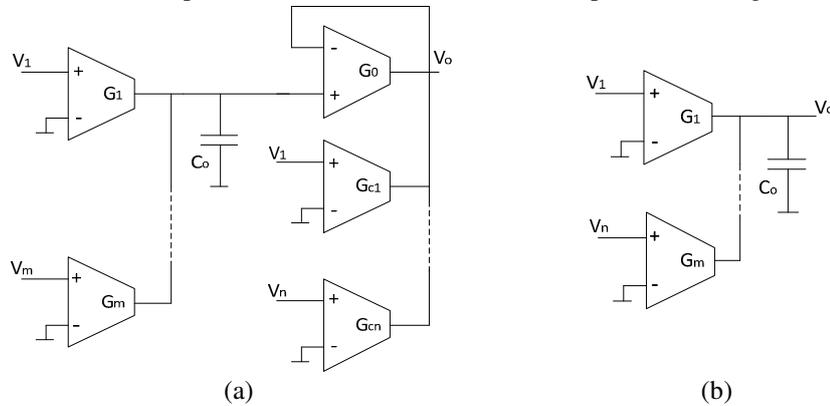
Fig. 1. Gm-C first order section obtained from RC-active prototype (a) with floating capacitor (b) without floating capacitor.

   Higher order transfer functions may be realized by interconnecting several first order Gm-C sections illustrated. Since all input voltage nodes of each section are connected to high impedance nodes then connecting several first order sections does not affect the individual transfer function of each section.

2.1 Application Example

The 2nd order Sallen-Key low-pass filter with unity gain amplifier shown in Fig. 2 allows to illustrate the transformation method presented in 0.
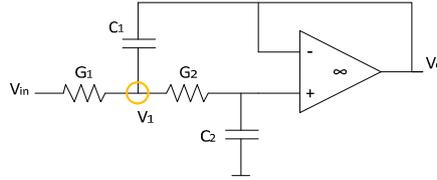
Fig. 2. Second order Sallen-Key low-pass filter topology with unity-gain amplifier 0.

Assuming an ideal opamp the filter can be described with the following nodal equations:

$$G_1(V_1 - V_{in}) + G_2(V_1 - V_o) + sC_1(V_1 - V_o) = 0 \tag{5}$$

$$sC_2V_o + G_2(V_o - V_1) = 0 \tag{6}$$

yielding the filter transfer function as:

$$H(s) = \frac{\dfrac{G_1G_2}{C_1C_2}}{s^2 + s\dfrac{G_1 + G_2}{C_1} + \dfrac{G_1G_2}{C_1C_2}} \tag{7}$$

For the generation of the corresponding Gm-C topology, (5) and (6) are manipulated according to (3). As can be seen in (5) there is a floating capacitor, C_1, connected between V_1 and V_o. In order to ground this capacitor it is necessary to define $C_1 = C_1 \cdot G_{c1}/G_0$ thus giving the following equations for both nodes:

$$sC_1V_1 = G_1(V_{in} - V_1) + G_2(V_o - V_1) + sC_1\frac{G_{c1}}{G_0}V_o \tag{8}$$

$$sC_2V_o = G_2(V_1 - V_o) \tag{9}$$

The Gm-C realization of (8) and (9) is represented in Fig. 3.

(a)   (b)
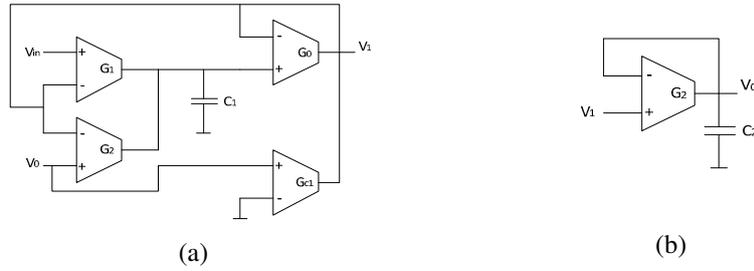
Fig. 3. (a) The Gm-C first order section (a) implementation of (8) with $G_{ci} = G_o$[1] (b) implementation of (9) [1].

By interconnecting the two first order sections represented in Fig. 3, the Gm-C biquadratic topology illustrated in Fig. 4 is obtained.
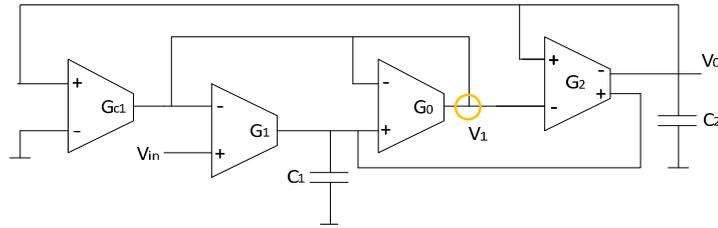
Fig. 4. The transformed Gm-C filter proposed in 0

From (8) and (9) it is possible to calculate the Gm-C filter transfer function

$$H(s) = \frac{\dfrac{G_1 G_2}{C_1 C_2}}{s^2 + s\left(\dfrac{G_1 + G_2}{C_1} + \dfrac{G_2}{C_2}\left(1 - \dfrac{G_{c1}}{G_0}\right)\right) + \dfrac{G_1 G_2}{C_1 C_2}} \tag{10}$$

Having in mind the general 2nd order low pass filter transfer function with unity-gain given by

$$H(s) = \frac{\omega_0^2}{s^2 + \dfrac{\omega_0}{Q}s + \omega_0^2} \tag{11}$$

the analytical expressions for both the filter cutoff frequency, ω_0, and quality factor, Q, are obtained. These expressions are represented in Table 1.

Table 1. Equations for the cutoff frequency and quality factor

	op-amp-RC filter	Gm-C equivalent
ω_0	$\sqrt{\dfrac{G_1 G_2}{C_1 C_2}}$	$\sqrt{\dfrac{G_1 G_2}{C_1 C_2}}$
Q	$\dfrac{\sqrt{\dfrac{G_1 G_2}{C_1 C_2}}}{\dfrac{G_1 + G_2}{C_1}}$	$\dfrac{\sqrt{\dfrac{G_1 G_2}{C_1 C_2}}}{\dfrac{G_1 + G_2}{C_1} + \dfrac{G_2}{C_2}\left(1 - \dfrac{G_{c1}}{G_0}\right)}$

The inspection of the design equations for the Gm-C structure leads to the conclusion that assuming $G_{c1} = G_0$ will make the design equations equal to those of the RC-equivalent circuit. This will enable the Gm-C filter designer to take advantage of the design methodology usually adopted for the active RC-prototype.

4   Robust Optimization-Based Filter Sizing

4.1   Introduction

The most common sizing methodology for analogue filters consists in applying constraints to/between component values as a way of getting rid of the several freedom degrees and then immediately obtain the remaining component values. Although this way will work for most academic cases, it does not exploit all scenario cases and sometimes produces poor quality filters with high sensitivities to component and temperature variation.

In this section an optimization-based design methodology where sensitivities minimization is used as a goal function will be presented.

4.2   Optimization-Based Biquad Sizing

For the description of the robust optimization methodology the 2nd order Gm-C Sallen-Key low-pass filter presented in section 2 will be considered.

The optimization constrains used refer to the error in the cut-off frequency and quality factor. Considering the expressions in Table 1, and assuming $\alpha = G_{c_1}/G_0$, constraints and may be written as

$$\varepsilon_{\omega 0} = \left| \frac{G_1 G_2}{C_1 C_2} - \omega_0^2 \right| < 10^{-6} \tag{12}$$

$$\varepsilon_Q = \left| \frac{\sqrt{\frac{G_1 G_2}{C_1 C_2}}}{\frac{G_1 + G_2}{C_1} + \frac{G_2}{C_2}(1 - \alpha)} - Q \right| < 10^{-6} \tag{13}$$

The cutoff frequency and the quality factor sensitivities to component variation are good parameters to evaluate the transfer function stability. Since the cutoff frequency, ω_0, is independent from α then the same sensitivity expressions are valid for both cases and are presented in [9] .

The following expressions represent the quality factor sensitivity to component variation:

$$S_{G_1}^Q = -S_{G_2}^Q = \frac{1}{2} \frac{G_2\left(1 - \frac{C_1}{C_2}(\alpha - 1)\right) - G_1}{G_2\left(1 - \frac{C_1}{C_2}(\alpha - 1)\right) + G_1} \tag{14}$$

$$S_{C_1}^Q = -S_{C_2}^Q = \frac{1}{2} \frac{C_2(G_1 + G_2) + G_2 C_1(\alpha - 1)}{C_2(G_1 + G_2) - G_2 C_1(\alpha - 1)} \tag{15}$$

$$S_\alpha^Q = \frac{\alpha G_2 C_1}{C_2(G_1 + G_2) - G_2 C_1(\alpha - 1)} \tag{16}$$

The optimization goal will be the minimization of the filter quality factor sensitivity to component variation. For the optimization we used the MATLAB® optimization toolbox.

4   Working Example

As a working example the previous 2nd order Sallen-Key low-pass Gm-C topology will be considered. A filter with a cutoff frequency at 100 kHz and a quality factor of 20 is desired.

Since the cutoff frequency sensitivity to component variation is a constant value, only optimizations to the quality factor sensivity minimization were performed.

Several scenarios were considered. In the first optimization stage, only the S_G^Q, S_C^Q and $\left|S_G^Q\right| + \left|S_C^Q\right|$ were computed. As can be seen from the Table 2, α tends to be unity. This indicates that $\alpha = 1$ is an optimum value for minimizing the above mentioned sensibilities and it is also important to remember that it is a key factor in the transformation method presented in 0. However, a deeper sensitivity analysis shows that using this value will produce high sensibilities values for S_α^Q. As an example, the

filter design method in [1] using f_c = 100 kHz, Q = 20 and G_1 = G_2 = G will produce a S_α^Q = 800 while having very low S_G^Q and S_C^Q (Table 3). It is then obvious that while performing the transformation the designer has to take into account the filter sensitivity to α variations when designing the filter.

The second optimization stage consisted in the minimization of the quality factor sensitivity to α variations. The minimal sensitivity was aprox. 39.5, obtained while considering α different from one. When considering α equal to one the minimum sensitivity reached was aprox. 400.

At last, in order to avoid he need for a multi-objective optimization and for the sake of simplicity, several optimizations of S_α^Q were computed and the minimal sum of $|S_\alpha^Q| + |S_G^Q| + |S_C^Q|$ was chosen in order to create a robust filter. The resulting data is available in Table 2 and Table 3.

As we may easily conclude from results in Table 3, with the methodology proposed we manage to obtain a more robust filter against transistor mismatch as the α term represented a ratio between trans-conductances in the circuit. We may also conclude, that in spite of having performed an optimization envisaging minimizing the sensivity of the filter quality factor, we obtained a solution where the sensivity of the filter cut-off frequency to transconductance variations is also lower.

**Table 2.** Quality factor sensitivities and component values

	S_G^Q	S_C^Q	$|S_G^Q| + |S_C^Q|$	S_α^Q	$|S_\alpha^Q| + |S_G^Q| + |S_C^Q|$
Minimal value for	-0.0006	0.0507	0.0197	39.5041	67.4920
G_1 (μF)	7496.9192	6437.9246	5672.7815	198.9700	329.3413
G_2 (μF)	9.5264	1.2176	0.0550	0.02456	0.7449
C_3 (pF)	117.8985	53.0331	38.4772	270.5219	970.7345
C_4 (pF)	239.1468	58.3561	3.2037	0.04576	0.6402
α	1.0006	0.9997	0.9995	2.3119	1.2652

Table 3. Quality factor sensitivities values for the proposed design method and the method presented in [1].

Design approach	$S_{G_1}^{\omega_0} = S_{G_2}^{\omega_0}$	$S_{C_1}^{\omega_0} = S_{C_1}^{\omega_0}$	$S_\alpha^{\omega_0}$	$S_{G_1}^Q = -S_{G_2}^Q$	$S_{C_1}^Q = -S_{c_2}^Q$	S_α^Q
In [1]	1	-1/2	0	0	-1/2	800
Proposed Method (total sensitivity sum)	1/2	-1/2	0	-10.3246	10.3246	46.8671

The validation of the results obtained was performed through simulations with LTspice. A Monte Carlo analysis with 1% tolerance for the transconductances values and 10% tolerance for the capacitances values was considered. The transconductance amplifiers were considered ideal.

The results of the Monte Carlo analysis toward alpha variation is presented in Fig. 5(a) and Fig. 5(b) where it is possible to compare the design approach made in [1] and the proposed optimization.
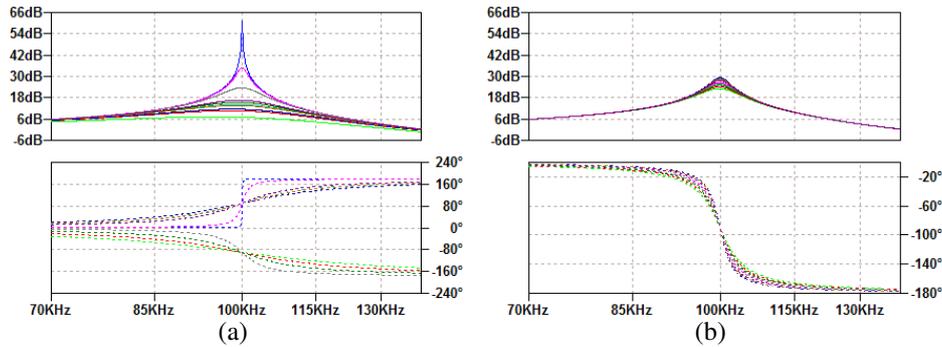
Fig. 5. Bode Diagram of the Monte Carlo analysis performed (a) with the design approach made in [1] which has high quality sensitivity to α variations (b) with the proposed design approach. The quality factor variation in this approach is much lower.

5 Conclusions

In this paper an optimization-based method for designing high frequency Gm-C filters was presented. The methodology presented in [1] allows the transformation from op-amp-RC circuits to Gm-C circuits while grounding floating capacitors. In order to have a fully equivalent transformation it is necessary to have α equal to unity, causing the circuit to become very sensitive to α variations. In an attempt to overcome the previously mentioned, the sensitivity to α was taken into account in the optimization procedure and a sensivity S_α^Q of 39.5041 was reached but with α different from unity. Although this does not make a fully equivalent Gm-C filter, the transformation method in [1] allows starting from a well-known op-amp-RC filter and then optimizing it in order to minimize the circuit sensivities, while cancelling the parasitic capacitances. Without using multi-objective optimization a global sensitivity optimization was performed achieving an optimum point with S_α^Q of 46.8671, S_G^Q of -10.3246 and S_C^Q of 10.3246, with a total sum of 67.4920.

In future work in order to improve these results it is necessary to use either another more powerful optimization tool or to create another architectural transformation, without creating such circuit sensitivity to the parameter α. The next simulation step would be to introduce nonlinearities in the transconductance amplifiers and capacitors with the aim to implement the circuit in IC technology. In order to do this it would be necessary to use a more realistic simulation environment, such as CADENCE.

Further evaluation of the methodology proposed to other biquadratic sections envisaging the implementation of filters for wireless applications should also be considered [10] .

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