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On Integral Sum Numbers of Cycles

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Abstract. In this paper we determine that integral sum number of graph C_n , namely for any integer $n \geq 5$, then $\xi(C_n) = 0$, therefore we prove that the graph $C_n (n \geq 5)$ is an integral sum graph.

Key Words: Integral sum number, Integral sum graph, Graph C_n

1 Introduction

The graph in this paper discussed are undirected、no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this paper referred to reference [1],[2].

F.Harary^[3] introduce the concept of integral sum graphs. The integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (V, E) , where $V = S$ and $uv \in E$ if and only if $u + v \in S$. A graph G is an integral sum graph if it is isomorphic to the integral sum graph number of $G^+(S)$ of some $S \subset Z$. The integral sum number of a given graph G , denoted by $\xi(G)$, is defined as the smallest nonnegative integer S such that $G \cup sk_1$ is an integral sum graph. For

convincing, an integral sum graph is written as an integral sum graph in references [3, 4]. Obviously, graph G is an integral sum graph iff $\xi(G) = 0$.

It is very difficult to determine $\xi(G)$ for a given graph G in general. All paths and matchings are verified to be integral sum graph in references [3], and we see from references [4] that $\xi(C_n) \leq 1$ for all $n \neq 4$. And further, an open conjecture was posed in references [4] as followss:

Conjecture^[4]: Is it ture that any old cycle is an integral sum graph?

Definition 1.1. If a graph is isomorphism graph $G^+(S)$, then we call graph G is Integral sum graph, denoted by $G \cong G^+(S)$.

Definition 1.2. For graph G , if it exists nonnegative integer S such that $G \cup sk_1$ is an integral sum graph, then we call number s is integral sum number of G , denoted by $\xi(G) = s$.

2 Main Results and certification

Theorem 2.1. For any integer $n \geq 3$, then

$$\xi(C_n) = \begin{cases} 3, & \text{when } n = 4; \\ 0, & \text{when } n \neq 4. \end{cases}$$

Proof. It is immediate from references [3, 4] that $\xi(C_3) = 0$ and

$\xi(C_4) = 3$. And it is clear that $C_5 \cong G^+\{2, 1, -2, 3, -1\}$ and

$C_7 \cong G^+\{1, 2, -5, 7, -3, 4, 3\}$.

Next we consider two cases: For all $C_{2j}(j \geq 3)$ and $C_{2j+1}(j \geq 4)$, we will show that the two classes of cycle are integral sum graph.

Let the vertices of $C_{2j}(j \geq 3)$ and $C_{2j+1}(j \geq 4)$ be marked as the methods in Fig.1 and Fig.2.

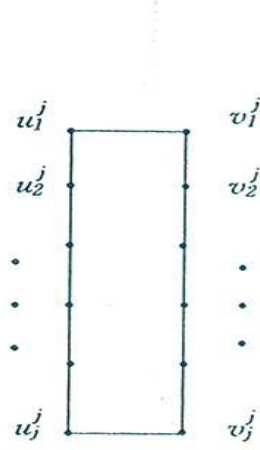


Fig.1. marking of C_{2j}

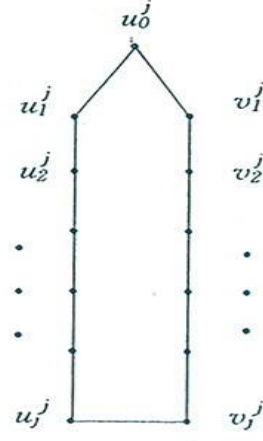


Fig.2. marking of C_{2j+1}

Case 1 When $n = 2j$ ($j \geq 3$)

We first give the labels of C_6 and C_8 as follows:

$$\text{Let } u_1^3 = 4, u_2^3 = -1, u_3^3 = 5; \quad v_1^3 = 1, v_2^3 = 3, v_3^3 = -2.$$

and

$$u_1^4 = 7, u_2^4 = -2, u_3^4 = 9, u_4^4 = -11; \quad v_1^4 = 2, v_2^4 = 5, v_3^4 = -3, v_4^4 = 8.$$

Then $C_6 \cong G^+ \{4, -1, 5, -2, 3, 1\}$ and

$C_8 \cong G^+ \{7, -2, 9, -11, 8, -3, 5, 2\}$, therefore $\xi(C_6) = 0$ and $\xi(C_8) = 0$.

When $j \geq 5$, we give the labels of C_{2j} as follows:

$$\text{Let } u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \geq 5); \quad u_2^j = u_2^{j-1} + u_2^{j-2} \quad (j \geq 5);$$

$$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \geq 5); \quad v_2^j = v_2^{j-1} + v_2^{j-2} \quad (j \geq 5).$$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 3, 4, \dots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 3, 4, \dots, j).$$

By labeling of above, we know $\xi(C_{2j}) = 0$ ($j \geq 5$).

The labeling of above is illustrated in Fig.3.

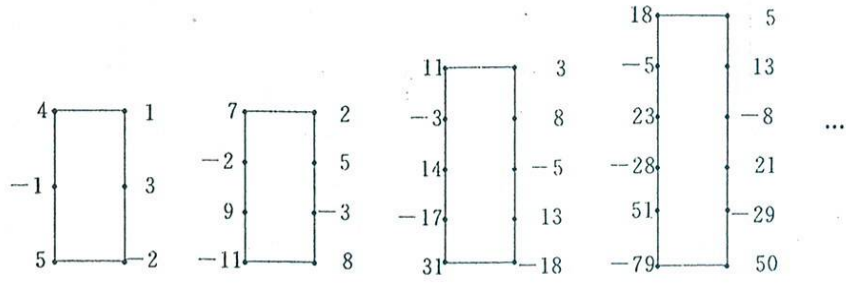


Fig.3. labeling of C_{2j} ($j \geq 3$)

Case 2 When $n = 2j + 1$ ($j \geq 4$)

We first give the labels of C_9 and C_{11} as follows:

$$\text{Let } u_0^4 = 2, u_1^4 = 5, u_k^4 = u_{k-2}^4 - u_{k-1}^4 \quad (k = 2, 3, 4);$$

$$v_1^4 = -5, v_2^4 = 7, v_k^4 = v_{k-2}^4 - v_{k-1}^4 \quad (k = 3, 4).$$

and

$$u_0^5 = 3, u_1^5 = 8, u_k^5 = u_{k-2}^5 - u_{k-1}^5 \quad (k = 2, 3, 4, 5);$$

$$v_1^5 = -8, v_2^5 = 11, v_k^5 = v_{k-2}^5 - v_{k-1}^5 \quad (k = 3, 4, 5).$$

Then $C_9 \cong G^+ \{2, 5, -3, 8, -11, 19, -12, 7, -5\}$ and

$C_{11} \cong G^+ \{3, 8, -5, 13, -18, 31, -49, 30, -19, 11, -8\}$, therefore $\xi(C_9) = 0$

and $\xi(C_{11}) = 0$.

When $j \geq 6$, we give the labels of C_{2j+1} as follows:

$$\text{Let } u_0^j = u_0^{j-1} + u_0^{j-2} \quad (j \geq 6); \quad u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \geq 6);$$

$$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \geq 6);$$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 2, 3, \dots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 2, 3, \dots, j).$$

Where $v_0^j = u_0^j$ for $j \geq 6$.

By labeling of above, we know $\xi(C_{2j+1}) = 0$ ($j \geq 6$).

The labeling of above is illustrated in Fig.4.

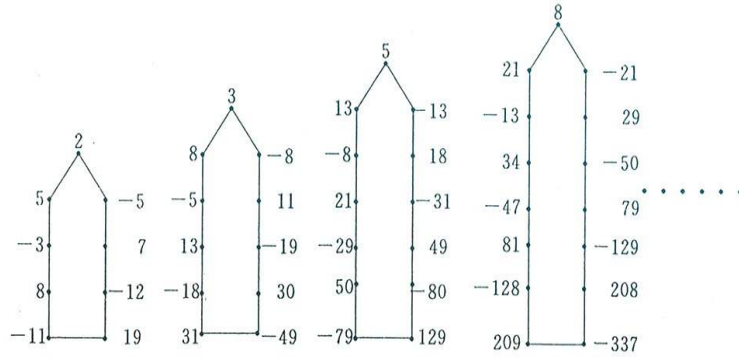


Fig.4. labeling of C_{2j+1} ($j \geq 6$)

Theorem 2.2. For any integer $n \geq 5$, the graph C_n is integral sum graph.

Proof. From theorem 2.1, we know $\xi(C_n) = 0$ ($n \geq 5$), therefore for any integer $n \geq 5$, the graph C_n is integral sum graph.

References

1. F. Harary. Graph Theory[M]. Addison Wesley. Reading MA, 1969.
2. Bondy J A, Murty U S R. Graph Theory with Applications[M]. Elsevier North-Holland, 1976.
3. F. Harary. Sum graphs over all the integers[J]. Discrete Math. 1994(124):99~105.
4. Baogen Xu. On integral sum graphs[J]. Discrete Math. 1999(194):285~294.
5. Achaya B D and Hegde S M. Arithmetic graphs[J]. Journal of Graph Theory, 1990, 18(3):275-299.
6. Hui Yu. The Windmill W_n^* is the integral sum Graph and Mod integral sum Graph[J]. Journal of Heze University. 2006, 28(3):18-19.
7. Gao Xiu-lian. Several kinds Integral Sum Tree, Journal of Dezhou University. 2011, 17(2):18-22.