

## FOREWORD

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MEGA stands for Effective Methods in Algebraic Geometry, and its translations in various languages. It is a biennial series of conferences, which celebrated its 25th anniversary during the 2015 edition in Trento. Following the MEGA tradition, the conference was succeeded by a call for submissions to a standard issue of the Journal of Symbolic Computation on the occasion of MEGA 2015. Accepted contributors to the conference were encouraged to submit, but the call was open to everyone. The result of this call lies before you.

*Algebraic Geometry* in the title of MEGA is interpreted broadly—including effective methods and theoretical and practical complexity issues in commutative algebra, geometry, real geometry, algebraic number theory, algebraic geometry, arithmetic geometry, and related fields such as algebraic analysis of differential equations, differential geometry, associative algebras, group theory, algebraic groups and Lie algebras, algebraic and differential topology; and also including applications of these fields, e.g. to statistics, coding theory, robotics, *etc.*

The papers accepted for this issue reflect this diversity: there is motivic and classical algebraic geometry (papers by Stout and Sturmfels), commutative algebra (papers by Assi–García-Sánchez–Micale, Boffi–Logar, and Markwig–Ren–Wienand), real-algebraic aspects (papers by Blekherman–Sinn and Fernando–Ueno), arithmetic aspects (the paper by Kelley–Owen), and applications to liaison linkages as well as algebraic statistics (papers by Gallet–Nawratil–Shicho, Horobet–Rodriguez, Mohammadi–Sáenz-de-Cabezón–Wynn). The stimulating interaction between these various areas, with their common emphasis on explicit computations, is at least partly responsible for the success of MEGA conferences in general and MEGA 2015 in particular.

We conclude this foreword with a very brief discussion of each of the papers, which we have ordered alphabetically by first author.

- (1) *Bases of subalgebras of  $K[[x]]$  and  $K[x]$*  by A. Assi, P.A. García-Sánchez, and V. Micale deals with the effective computation of the semigroup of orders (respectively, degrees) of elements of the subalgebra generated by a number of formal power series (respectively, polynomials)  $f_1, \dots, f_s$  in  $K[[x]]$  and  $K[x]$ . This is highly relevant in the study of degenerations of the space curve parameterised by  $(f_1, \dots, f_s)$ .
- (2) *Extreme rays of Hankel spectrahedra for ternary forms* by Grigoriy Blekherman and Rainer Sinn concerns the set of extreme rays of the cone dual to the cone  $\Sigma_{2d}$  of sums of squares of homogeneous degree- $d$  polynomials in three variables. Since  $\Sigma_{2d}$  is strictly contained in the cone  $P_{2d}$  of nonnegative polynomials for  $d > 2$ ,  $\Sigma_{2d}^\vee$  must have extreme rays that are not extreme rays of  $P_{2d}^\vee$ . The paper proves a much stronger statement by showing that

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the Zariski closure of the set of extreme rays has much higher dimension; and it studies this set of rays in more detail for small values of  $d$ .

- (3) *Border bases for lattice ideals* by Giandomenico Boffi and Alessandro Logar concerns the construction of all possible division-closed sets of monomials forming a basis of a quotient of a polynomial ring by a lattice ideal. In particular, it is shown that the number of such sets is finite. Each of them gives rise to a different border basis of the ideal. As border bases can differ wildly in terms of numerical stability, having access to all of them rather than just to one, is very useful in applications.
- (4) *A short proof for the open quadrant problem* by José F. Fernando and Carlos Ueno is motivated by the real-algebraic question which subsets of real vector spaces are the images of vector spaces under polynomial maps. Using computer calculations, it had been proved in 2003 that the strictly positive quadrant in  $\mathbb{R}^2$  is such an image. The current paper gives a simpler, computer-free proof of this fact, via a map that factors as the composition of three low-degree maps from  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ . This technique may well prove applicable to other instances of the problem, as well.
- (5) *Liaison linkages* by Matteo Gallet, Georg Nawratil, and Josef Schicho concerns hexapods: mechanical devices consisting of a base and a platform connected via six rigid legs that can rotate at their endpoints. The set of positions of a hexapod is embedded as a variety in a suitable projective space, and the maximal possible degree of that variety is determined. Of particular interest are hexapods whose configuration variety is a curve. Such hexapods have not yet been classified, and the paper represents an important step towards such a classification.
- (6) *The maximum likelihood data singular locus* by Emil Horobeț and Jose Israel Rodriguez is motivated by the computation of the maximum likelihood estimate for a data point in some algebro-statistical model. This estimate is one of the solutions to a system of polynomial equations that, for generic data, has a fixed number of solutions in the smooth locus of the model. The locus studied in the paper is the set of data points where one of these solutions wanders off into the singular locus of the model. The paper offers a lower approximation and an upper approximation of this locus, and gives examples where these containments are equalities.
- (7) *Estimating the number of roots of trinomials over finite fields* by Zander Kelley and Sean W. Owen is about counting the roots of a univariate polynomial of the form  $x^n + ax^s + b$  over a finite field  $\mathbb{F}_q$ . An upper bound on the number of roots is proved in terms of  $q, n, s$ , and this upper bound is shown to be tight for an infinite class of finite fields and trinomials. For prime  $q$ , however, it seems that the maximum number of roots is actually much lower than the bound. This leads to a conjectured stronger bound, which is supported by extensive computational evidence.
- (8) *Standard bases in mixed power series and polynomial rings over rings* by Thomas Markwig, Yue Ren, and Oliver Wienand deals with standard bases for submodules of free modules over rings of the form  $R[[t]][x]$ , i.e., polynomials in  $x$  with formal power series in  $t$  as coefficients. Here both  $t$  and

$x$  are tuples of variables, and where moreover  $R$  comes from a rather general class of base rings. The paper unifies and generalises much existing literature.

- (9) *Types of signature analysis in reliability based on Hilbert series* by Fatemeh Mohammadi, Eduardo Sáenz-de-Cabezón, and Henry Wynn is motivated by the statistical analysis of certain systems. Each state of such a system is represented by a monomial in  $n$  variables, and the failure states are captured by a monomial ideal  $I$ . It was observed earlier that the Hilbert series of this ideal facilitates the computation of the system's unreliability—but the Hilbert series is not sufficient for quantifying simultaneous failures or for signature analysis. The finer algebraic invariants required for this arise from free resolutions of the ideals in certain filtrations of  $I$ .
- (10) *On the auto Igusa-zeta function of an algebraic curve* by Andrew Stout introduces a subtle invariant associated to pair  $(X, p)$  where  $X$  is a scheme over a field  $k$  and  $p$  is a point of  $X$ : for each natural number  $n$ , the set of endomorphisms of the  $n$ -jet of  $X$  at  $p$  is a scheme  $A_n$  over the residue field of  $X$  at  $p$  dubbed the  $n$ -th *auto-arc space* of  $X$  at  $p$ . The auto Igusa-zeta series organises the entire sequence  $(A_n)_n$  (or their reductions) into a generating series with coefficients in a suitable Grothendieck ring. The paper contains several explicit computations of this series for algebraic curves  $X$  at a singularity  $p$ , which gives rise to natural conjectures.
- (11) *The Hurwitz Form of a Projective Variety* by Bernd Sturmfels studies, given a codimension- $d$  projective variety  $X$  in projective  $n$ -space, the locus of dimension- $d$  subspaces that do not meet  $X$  in the right number of (reduced) points. Typically, this locus is a hypersurface in the Grassmannian of  $d$ -spaces in  $n$ -space, and, along with several examples, the paper presents explicit techniques for computing the equation for this hypersurface.

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