

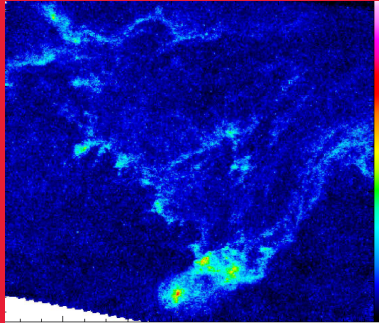
# Surprising “quantum statistical” treatment of Landau damping

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thanks to: H. Yahia, K. Daoudy, N. Brodu, N. Schneider, S. Bontemps

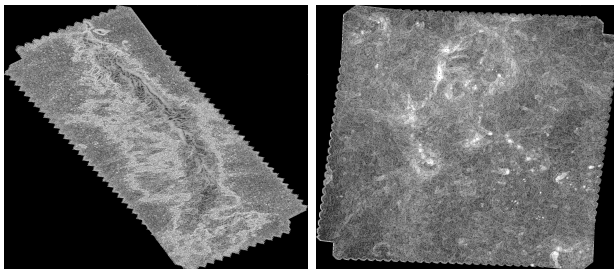
Statistical mechanics of systems with long range interactions  
“Quantum statistical” treatment of Landau damping  
Path integrals in toy models of desynchronisation

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# Statistical mechanics of systems with long range interactions

# Stationary states for long range interactions



Herschel, Musca, Cygnus. N. Schneider, S. Bontemps, H. Yahia

- ▶ Relaxation towards stationary states?
- ▶ Homogeneity?
- ▶ Discriminate information content of spatio-temporal gravitational structures from the rest?
  - ▶ Globular clusters, galaxies, halos, solar wind, interstellar dust.

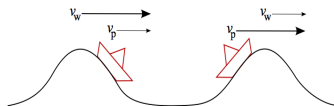
# Statistical mechanics of systems with long range interactions

**Plasma parameter:** pair wise interaction vs mean field

$$\Gamma = \frac{Gm^2R}{NmT} \sim N^{-\gamma}$$

- ▶ Outstanding challenges deterministically **and** statistically:
- ▶ Large  $N$  collisionless.
- ▶ True equilibrium relaxation is slow  $\tau \sim N^\gamma$ .
- ▶  $U = \sum_{i,j}^N V_{i,j} \sim N^2$  and  $S = K \ln \Omega \sim N$ . Kac scaling for extensivity  $m \rightarrow 0$ , while  $mN = M$ .
- ▶ Non additivity.

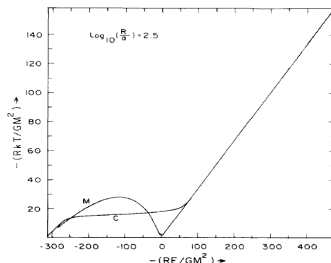
# Wave-particle interaction scheme



Almost right picture

- ▶ Mean field generates waves.
- ▶ Spatial simplification: great reduction of DoF.
- ▶ Synchronisation transition (Kuramoto75/HMF Firpo00).
- ▶ Wave particle resonant interaction and **phase mixing** (Landau46, Van Kampen55, Lynden-Bell62).
- ▶ Collisionless **fast** relaxation to Quasi Stationary States.

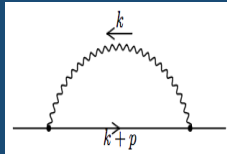
# Ergodicity breaking



$\mu$ -canonical temperature in binary system, T. Padmanabhan 1989

- ▶ Non-additivity. Negative specific heat for  $\Gamma = \mathcal{O}(1)$ .
- ▶ **Inequivalence of ensembles**: Similar to a phase transition.
- ▶ Ergodicity breaking near transitions to inhomogeneous QSSs (non-Gaussian velocity distribution, anomalous diffusion).

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# “Quantum statistical” treatment of Landau damping

# Wave modulations

$$\partial_t \rho = i\omega \rho.$$

- ▶ Greater reduction of DoF: modulations.
- ▶ Plasma frequency is  $\omega = \omega_p + \delta\omega$ . Modulations from trapped particles with density  $\rho$ .
- ▶ Density oscillates with  $\delta\omega \sim \rho\omega_p^{-1}$ .
- ▶ Resonances mode locking (Arnold's tongues) and chaos.
- ▶ Not sufficient to understand violent relaxation.



# Wave scattering

$$\partial_t \rho = i\omega \rho.$$

- ▶ Ansatz of wave-(particles-)wave scattering.
- ▶ From PDE to SPDE with  $\delta\omega$  a random variable.
- ▶ Trapped particle diffusion time (O'Neil65)  $\tau^{-1} \sim \sqrt{\rho}$
- ▶ Underlying high DoF: High frequencies are “forgotten” then injected back into lower frequencies (effectively driven-damped).
- ▶ Formally from symplectic to Hilbertian.

# Self scattering

Green's function  $G_0 = \lim_{\eta \rightarrow 0} \frac{1}{\nu - \omega_p + i\eta}$  and

$$G = \lim_{\eta \rightarrow 0} \frac{1}{\nu - \omega + i\eta}$$

Dyson's equation:  $G = G_0 + G_0 \delta\omega G_0 + G_0 \delta\omega G_0 \delta\omega G$

$$\omega_L + i\gamma = \lim_{\eta \rightarrow 0} \sum_f \frac{1}{\nu_i - \nu_f + i\eta} \delta\omega^2$$

- ▶ Phase mixing and intrinsic noise related by equipartition.
- ▶ Dyson's renormalisation scheme: **Gap opening** for excitations.
- ▶ Phase mixing is mass + **Lamb shift** of ground state.

# Fluctuation-Dissipation

Linear susceptibility  $\chi(\omega) = \int d\nu \frac{\delta\omega^2(\nu)}{\nu - \omega}$

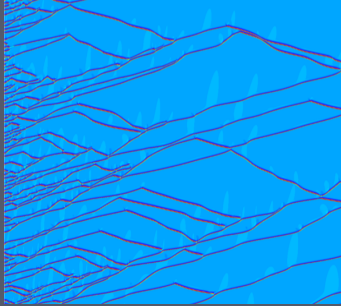
Equipartition in moving frame  $\epsilon(\nu) \sim \nu - \omega$ :  $\delta\omega^2(\nu) = \frac{T\rho(\nu)}{\nu - \omega}$

Resonant condition  $\nu = kv$  returns Landau's damping term:

$$\chi(\omega) = \frac{T}{k} \int d\nu \frac{1}{kv - \omega} \partial_\nu \rho$$

- ▶ Assume  $\mu$ -canonical phase space mixing and equipartition (estimation of the relaxed state after the instability).
- ▶ This is good **away** from phase transitions.
- ▶ Trapping particles **is** a phase transition (O'Neil65, Firpo00).

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## Path integrals in toy models of desynchronisation

# Transport and desynchronisation

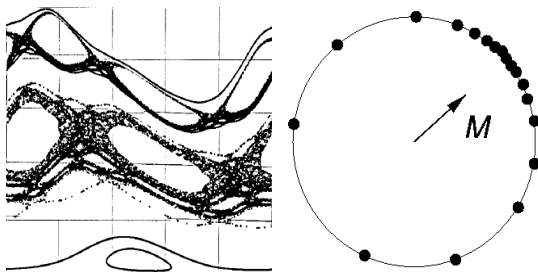


Illustration of one particle transport. Coupling and synchronisation.

- ▶ Stochasticity over island chains scaling (Escande81).
- ▶ Assume transport effectively **locally** couples density troughs.
- ▶ Canonical partition function description is relevant.

# Glassy Behavior

$$Z = \int ds_i \exp \left( -\frac{1}{2} \sum_{\langle i,j \rangle} K_{i,j} s_i \cdot s_j \right)$$

Follow the transport of a locally synchronised cluster

$s(x, t) \cdot s(x + dx, t) = 1$ : Random paths

Probability density  $\rho = \int dx(t) e^{\int_0^t dt' \mathcal{L}(x(t'))}$

$$\partial_t \rho = D \partial_{x,x}^2 \rho + i \chi \rho$$

- ▶ In or out of phase transport creates **dynamical frustration**  $K_{i,j} \in \{-1; 1\}$  (Pluchino04).
- ▶ Chaotic renormalisation orbits possible (MacKay82).
- ▶ **Cascade** in phase space: Landscape and large deviations.
- ▶ Effectively random susceptibility  $\chi(\omega)$  (low frequency) and effectively diffusing high frequencies  $D$ .

# Conclusions

Large deviations:  $\delta\rho = \Pi_j e^{i \int_{t_j} dt' \chi(t')}$

$$\delta\rho \sim \int dh \, p(h) t^h$$

Relation to explore with  $t \sim \epsilon$

$$\left\langle \int_{\epsilon} dt' \chi(t') \right\rangle = \inf_h h - (d - D_{\epsilon}(h))$$

- ▶  $\epsilon$ -entropy scales as  $S_{\epsilon} \sim \epsilon^{-\alpha}$ .
- ▶ Random susceptibility  $\chi(\omega)$  yields singularities.
- ▶ Information transport processes monitored by singularities.
- ▶ Classes of non equilibrium processes (DP, KPZ etc).

**Thank you!**