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A tentative reconstruction of Bürgi's sine table at 2" intervals (ca. 1600)

Denis Roegel

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A tentative reconstruction of Bürgi's sine table at $2''$ intervals (ca. 1600)

Denis Roegel

2016

(last version: 22 march 2016)

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This document gives a tentative reconstruction of a $2''$ sine table which may have been computed by Jost Bürgi. The original table does not seem to have survived.

We have based our reconstruction on Bürgi's smaller table giving the sines for every minute,[1] which is part of Bürgi's *Fundamentum Astronomiæ* recently discovered by Menso Folkerts [3, 4] and edited by Dieter Launert [5].

Even if Bürgi's table was organized differently, our reconstruction should give a good idea of the amount of data contained in such a table, and of the amount of work needed to compute it. It is still not certain that the original table really was complete or as accurate as a number of people claim it. It is always easy to speak of the accuracy of something that nobody has seen for 400 years, and where the only accounts may themselve not have focused that much on the accuracy.

In any case, our reconstruction gives the sines for every two seconds of the quadrant and to 8 sexagesimal places. The sexagesimal values are given such that the *sinus totus* is equal to 60. The last value of the table is therefore 60,0,0,0,0,0,0,0 corresponding to the sine of $90°$. The sine of $2''$ is 0,0,2,5,39,49,20,32 corresponding to the modern value

$$(((((32/60 + 20)/60 + 49)/60 + 39)/60 + 5)/60 + 2)/60/60/60 = 0.00000969627362\ldots$$

Given a modern value of the sine, we can easily obtain its sexagesimal digits by multiplying repeatedly by 60 and taking the fractional parts. The integer parts are the sexagesimal digits. For instance, if we start with $\sin 1'$, we have

$$\sin 1' = 0.00029088820456\ldots \quad (1)$$

$$60 \sin 1' = 0.01745329227380\ldots \quad (2)$$

$$\rightarrow \quad d_0 = 0$$

$$(60 \sin 1' - d_0) \times 60 = 1.04719753642832\ldots \quad (3)$$

$$\rightarrow \quad d_1 = 1$$

$$((60 \sin 1' - d_0) \times 60 - d_1) \times 60 = 2.83185218569971\ldots \quad (4)$$

$$\rightarrow \quad d_2 = 2$$

$$(((60 \sin 1' - d_0) \times 60 - d_1) \times 60 - d_2) \times 60 = 49.91113114198276\ldots \quad (5)$$

$$\rightarrow \quad d_3 = 49$$

$$((((60 \sin 1' - d_0) \times 60 - d_1) \times 60 - d_2) \times 60 - d_3) \times 60 = 54.66786851896614\ldots \quad (6)$$

$$\rightarrow \quad d_4 = 54$$

$$(7)$$

[1] This table is available on line at `http://www.bibliotekacyfrowa.pl/dlibra/doccontent?id=45624`, as well as at `http://locomat.loria.fr`. We have also reconstructed this table, see [11].

and we see that the sexagesimal expression of $\sin 1'$ is in fact 0,1,2,49,54,40,4,. . . .

Each page of the table covers five minutes. The values of the sines are rounded. The differences between consecutive values are shown in red. These are the differences between the rounded values, not the rounded exact differences, of course.

It is likely that the original computations were done to an even greater number of places, but that only eight places were kept. This table was computed using pivots, that is values which were computed beforehand, for instance using Bürgi's iterative algorithm, and then by applying an interpolation scheme based on the accumulation of higher differences. These techniques are described by Launert [5], as well as in several of our notes [8, 9, 10].

References

The following list covers the most important references[2] related to Bürgi's table. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.

[1] Jost Bürgi. *Arithmetische und Geometrische Progress Tabulen, sambt gründlichem Unterricht, wie solche nützlich in allerley Rechnungen zugebrauchen, und verstanden werden sol.* Prague, 1620. [These tables were recomputed in 2010 by D. Roegel [7]]

[2] Kathleen Clark. *Jost Bürgi's Aritmetische und Geometrische Progreß Tabulen (1620).* New York: Springer, 2015.

[3] Menso Folkerts. Eine bisher unbekannte Schrift von Jost Bürgi zur Trigonometrie. In Rainer Gebhardt, editor, *Arithmetik, Geometrie und Algebra in der frühen Neuzeit*, pages 107–114. Annaberg-Buchholz: Adam-Ries-Bund, 2014. [not seen]

[4] Menso Folkerts, Dieter Launert, and Andreas Thom. Jost Bürgi's method for calculating sines, 2015. [uploaded on arXiv on 12 October 2015, id 1510.03180v1; a preprint dated 19 September 2015 is also available online; a second version was put on arXiv on 2 February 2016]

[5] Dieter Launert. *Nova Kepleriana : Bürgis Kunstweg im Fundamentum Astronomiae — Entschlüsselung seines Rätsels.* München: Bayerische Akademie der Wissenschaften, 2015.

[6] Christian Riedweg. Bürgi's "Kunstweg" — geometric approach, 2016. [uploaded on arXiv on 25 February 2016]

[7] Denis Roegel. Bürgi's *Progress Tabulen* (1620): logarithmic tables without logarithms. Technical report, LORIA, Nancy, 2010. [This is a recalculation of the tables of [1].]

[8] Denis Roegel. Jost Bürgi's skillful computation of sines. Technical report, LORIA, Nancy, 2015.

[9] Denis Roegel. A note on the complexity of Bürgi's algorithm for the computation of sines. Technical report, LORIA, Nancy, 2016.

[10] Denis Roegel. A preliminary note on Bürgi's computation of the sine of the first minute. Technical report, LORIA, Nancy, 2016.

[2]**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

[11] Denis Roegel. A reconstruction of Bürgi's sine table at $1'$ intervals (ca. 1587). Technical report, LORIA, Nancy, 2016.

Bürgi's 2″ sine table (tentative reconstruction, D. Roegel, 2016)

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Bürgi's 2″ sine table (tentative reconstruction, D. Roegel, 2016)

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Bürgi's 2″ sine table (tentative reconstruction, D. Roegel, 2016)

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