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# Simplified theory of an active lift turbine with controlled displacement

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## Abstract

It is presented in this article, a simplified theory of the active lift turbine which has been the subject of several patent[4, 5, 11].

A simplified theory is proposed to extend the Betz limit of the yield on vertical axis wind turbine. This work can be extended either on wind driven or marine current turbine. Based on kinetic energy calculation, that theory demonstrates that the radial force acting on the blade can be used to extend the maximum recoverable power, mainly by transforming a linear motion into a rotating motion. The geometry of this new type of vertical axis turbine is described. Finally the driving power is calculated through the tangential power coefficients. Consequences on the velocity variation generation and induced vibrations are exposed.

Two parts is treated in this article : The calculation of power coefficient and the calculation natural modes of vibration

## Keywords

Darrieus, betz limit, VAWT, Vertical axis turbine, Alternative linear motion

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\*Inventor of the active lift turbine concept

## 1 prelude

Vertical axis turbines, driven by wind or water, have been intensively developed after the 70's (South and Rangi [12] from the National Research Council of Canada). The Darrieus wind turbine type was initially patented in the USA in 1931 (Darrieus [7]) in which both curved blade shape (Troposkien) and straight bladed rotor were designed. Continuous technical developments have been pursued thanks to experimental works in wind tunnels (Sheldahl et al. [9], Blackwell et al. [8]). H-type rotor including straight blades and speed control have been studied by Musgrove [10] in the mid 80's.

## 2 The calculation of power coefficient

### 2.1 Introduction :

German physicist Albert Betz[1] has proven that the recoverable maximum power for a wind turbine is closed to 60% ( $16/27$ ) of the power of the wind that runs through the area swept by the sensor. That theory is based on the calculation of the kinetic energy. Modern wind turbines are reaching aeronautical yields close to the Betz limit. These wind turbines are subjected to very substantial structural stresses. When the wind speed becomes too high, in the order of 40 m/s, it is necessary to force these large wind turbines to stall and then to stop. The main purpose presented in this paper is to transform these constraints into additional energy recovery. The wind turbines with horizontal axis are facing constant structural stresses without a complementary system, this type of turbine can not transform directly these constant stresses into energy.

The (vertical or horizontal axis) Darrieus [7] turbines are facing stresses alternatively in each arm during each rotation. These structural stresses in the arms are of compression or extension type. the forces causing alternative constraints can be transformed to recover the extra energy.

The blades of the Darrieus-type turbines have an incidence angle which varies during the rotation. The induced force of the fluid velocity on the profile of the blade depends essentially on the fluid velocity and the angle of incidence.

This induced force can be decomposed into a tangential force and a radial force. The tangential force associated to the arm radius creates a torque. The radial force causes alternative constraints on the structure of turbine arm. Much research is performed on the angle of incidence in order to maximize the value of the tangential force or whether to try to keep that angle

at a constant value for each half turn of revolution.

With aircraft type profiles (NACA, etc.), the radial force is substantially greater than the tangential force and thus creates alternative structural stresses on the arms.

The first version of the active lift turbine [4, 3] was to put linear stiff generators directly inside the arms. The rotation frequency of the turbines is low thus the yields of these generators are very low. The second version of the active lift turbine is to transform the alternative linear motion into a rotating motion. This solution allows the energy recovery directly on the same rotation axis due to the tangential force, associated to the arm radius, but also extra energy due to alternative constraints. Induced force created by the fluid on the blade profile is due to the relative speed which is the combination of the fluid velocity and of the rotation speed of the profile. With the active lift turbine, the relative velocity is the combination of the fluid velocity, the rotational velocity of the profile and the linear radial velocity.

## 2.2 Geometry of the Active Lift Turbine :

The point O is the main axis of the turbine. The gear F with its center A is not coincident with the main axis O. The gear F is fixed relative to the direction of flow. The satellite gear S (center B) is turning without sliding around the fixed gear F (center A). The slide support is one of the turbine's arm and turns around the axis O. The profile (blade) is fixed with the slide bar in E. The slide bar has a reciprocating translatory motion with a low speed of movement. The rod is connected to the slide bar by a pivot connection in D and is connected to the satellite gear S by a pivot connection in C. The slide bar is driven in rotation by the slide support.

$$\bar{OA} = e \quad \bar{AB} = 2r \quad \bar{BC} = r_b \quad \bar{CD} = L \quad \bar{DE} = d \quad \vec{OE} = R(\beta)\vec{i}$$

$$R : \text{radius} \quad \bar{R} : \text{mean radius} \quad \dot{R} = \frac{\partial R}{\partial t}$$

**Standard Darrieus Turbine :**

$$R = 2r + d + L \quad \bar{R} = R \quad \dot{R} = 0$$

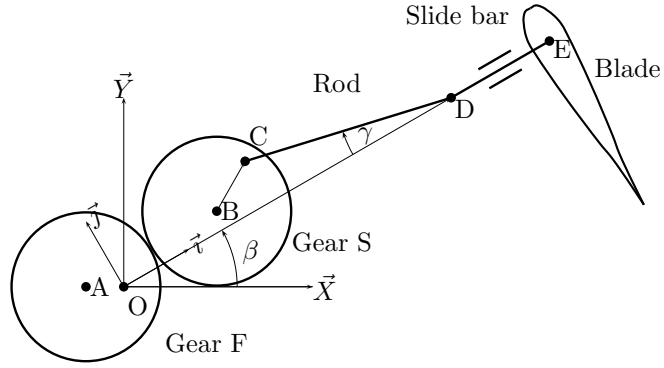


Figure 1: geometry

**Active lift turbine :**

$$\gamma = \widehat{ODC} \quad \cos \gamma \approx 1 \quad \text{avec} \quad L > 2r$$

$$R \approx 2r + d + L + (r_b - e) \cos \beta \quad \bar{R} \approx 2r + d + L$$

$$R \approx \bar{R} + (r_b - e) \cos \beta \quad (1)$$

$$\dot{R} = (-r_b + e) \dot{\beta} \sin \beta \quad (2)$$

## 2.3 Theory on Power Coefficient :

### 2.3.1 Transformation of the slide force into a torque :

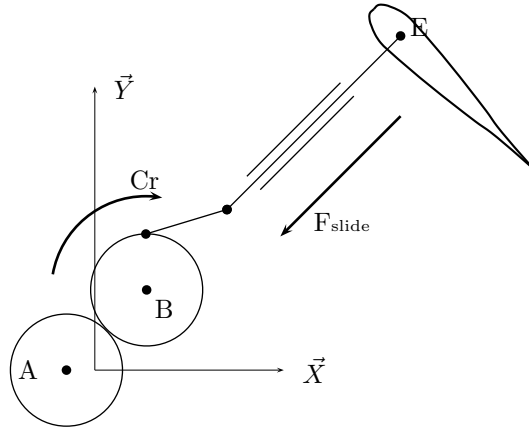


Figure 2: Actions on the rod

$$\bar{AE} \approx 2r + d + L + (r_b - e) \cos \beta \quad \bar{AB} = 2r - e \cos \beta$$

The variation of the length  $AE$  is negligible compared to the variation of  $AB$

$$r_b \sim e \quad \Delta AE_{\max} = |r_b - e| \quad \Delta AB_{\max} = e \quad \Delta AE \ll \Delta AB \quad \Delta AE \approx 0$$

The system is equivalent to a crank rod.

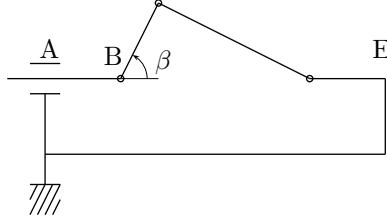


Figure 3: crank rod

Equality of powers provides the couple

$$C_r \dot{\beta} = F_{slide-bar} \frac{d(\bar{AB})}{dt} = F_{slide-bar} \dot{R}$$

By using (2), the torque is equal to:

$$C_r = \{e \sin \beta\} F_{slide-bar} \quad (3)$$

### 2.3.2 Power coefficient calculation

#### 2.3.2.1 The Bernoulli equations

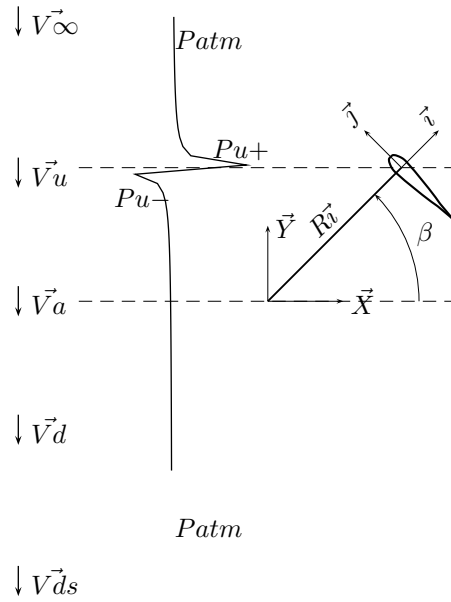


Figure 4: Pressure variations

$$\vec{V}_\infty = V_\infty \vec{Y} \quad \text{with} \quad V_\infty \leq 0$$

$$\begin{aligned} P_{atm} + \frac{1}{2}\rho V_\infty^2 &= P_{u+} + \frac{1}{2}\rho V_u^2 \\ P_{u-} + \frac{1}{2}\rho V_u^2 &= P_{atm} + \frac{1}{2}\rho V_a^2 = P_{d+} + \frac{1}{2}\rho V_d^2 \\ P_{d-} + \frac{1}{2}\rho V_d^2 &= P_{atm} + \frac{1}{2}\rho V_{ds}^2 \end{aligned}$$

From these equations, we can defined these pressure differences

$$P_{u+} - P_{u-} = \frac{1}{2}\rho(V_\infty^2 - V_a^2) \quad P_{d+} - P_{d-} = \frac{1}{2}\rho(V_a^2 - V_{ds}^2) \quad (4)$$

### 2.3.2.2 The continuity equation along a streamtube

$$dAx_u V_u = dAx_d V_d \quad dAx_u = R_u d\beta_u |\sin \beta_u| dH \quad dAx_d = R_d d\beta_d |\sin \beta_d| dH$$

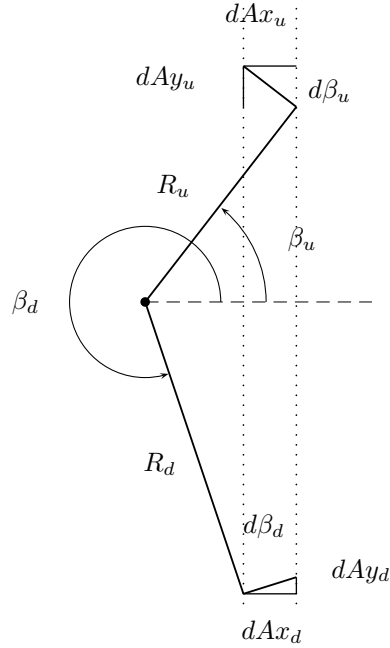




Figure 5: streamtube

$$dAy_u = R_u d\beta_u |\cos \beta_u| dH \quad dAy_d = R_d d\beta_d |\cos \beta_d| dH$$

$dH : \text{elementary height of the blade}$

We can define the relation between these two elementary surfaces

$$dAx_d = dAx_u \frac{V_u}{V_d} \quad (5)$$

### 2.3.2.3 Forces on a blade crossing a streamtube

$$0 \leq \beta \leq \pi \quad dFy_u = (P_{u+} - P_{u-}) dAx_u \quad dFx_u = (P_{u+} - P_{u-}) dAx_u \frac{1}{\tan \beta}$$

$$\pi \leq \beta \leq 2\pi \quad dFy_d = (P_{d+} - P_{d-}) dAx_u \frac{V_u}{V_d} \quad dFx_d = (P_{d+} - P_{d-}) dAx_u \frac{V_u}{V_d} \frac{1}{\tan \beta}$$

By using(5), these equations become

$$dFy_u = \frac{1}{2} \rho (V_\infty^2 - V_a^2) dAx_u \quad dFy_d = \frac{1}{2} \rho (V_a^2 - V_{ds}^2) dAx_u \frac{V_u}{V_d} \quad (6)$$

$$dFx_u = \frac{1}{2} \rho (V_\infty^2 - V_a^2) dAx_u \frac{1}{\tan \beta} \quad dFx_d = \frac{1}{2} \rho (V_a^2 - V_{ds}^2) dAx_u \frac{V_u}{V_d} \frac{1}{\tan \beta} \quad (7)$$

### 2.3.2.4 Euler Equations

$$dFy_u = (V_\infty - V_a) dm_u \quad dm_u = \rho V_u dAx_u$$

$$dFy_d = (V_a - V_{ds}) dm_d \quad dm_d = \rho V_u dAx_u \quad (8)$$

By using(6), the previous equations, we determine the following relations between the velocities

$$V_a = 2V_u - V_\infty \quad V_{ds} = 2(V_d - V_u) + V_\infty$$

Finally, the Euler equations give

$$dFy_u = 2(V_\infty - V_u) \rho V_u dAx_u \quad dFy_d = 2(2V_u - V_d - V_\infty) \rho V_u dAx_u \quad (9)$$

**2.3.2.5 Projection of the aerodynamic force** Induced velocity creates an aerodynamic force on the profile which can be decomposed into a lift and a drag force.

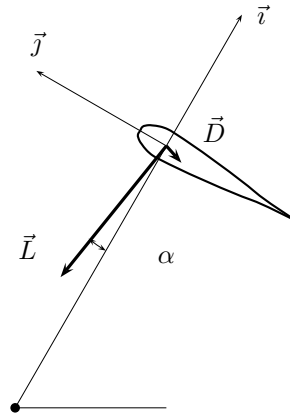
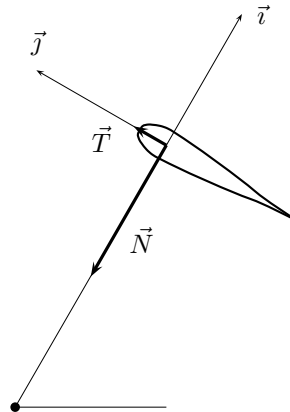


Figure 6: Lift and drag force on a blade

At the turbine level, it is important to know the decomposition of this force into a normal force  $N$ , and a tangential force  $T$ .



Forces are set according to X and Y axis from the normal force  $N$  and the tangential force  $T$

Figure 7: Axial and normal force on a blade

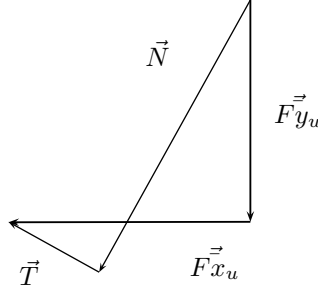


Figure 8: Resolution of forces

$$\begin{aligned}
 Fy_u &= N \sin \beta - T \cos \beta & Fx_u &= N \cos \beta + T \sin \beta \\
 T &= L \sin \alpha - D \cos \alpha & N &= L \cos \alpha + D \sin \alpha \\
 L &= C_L \frac{1}{2} \rho W_u^2 c d H & D &= C_D \frac{1}{2} \rho W_u^2 c d H & c : \text{blade chord} \\
 T &= C_T \frac{1}{2} \rho W_u^2 c d H & N &= C_N \frac{1}{2} \rho W_u^2 c d H
 \end{aligned}$$

Thanks to mathematical approximation, drag and lift coefficients can be defined for a symmetrical profile of a Naca0012 type and for low incidence values.

$$C_L = 2\pi \sin \alpha \quad C_D = \text{constant}$$

$C_D$  has a value far below  $C_L$  in the order of  $C_D < 10 C_L$   $C_D \approx 0$

$$C_T = 2\pi \sin^2 \alpha - C_D \quad C_N = 2\pi \sin \alpha \cos \alpha \quad (10)$$

$$\begin{aligned}
 Fy_u &= \frac{1}{2} \rho W_u^2 c d H \sin \beta (C_N - C_T \frac{\cos \beta}{\sin \beta}) \\
 \text{and } Fy_d &= \frac{1}{2} \rho W_d^2 c d H \sin \beta (C_N - C_T \frac{\cos \beta}{\sin \beta}) \quad (11)
 \end{aligned}$$

**2.3.2.6 Calculation of the value of  $Fy_u$**  For a two blades turbine, only one blade follows a half-turn.[2]

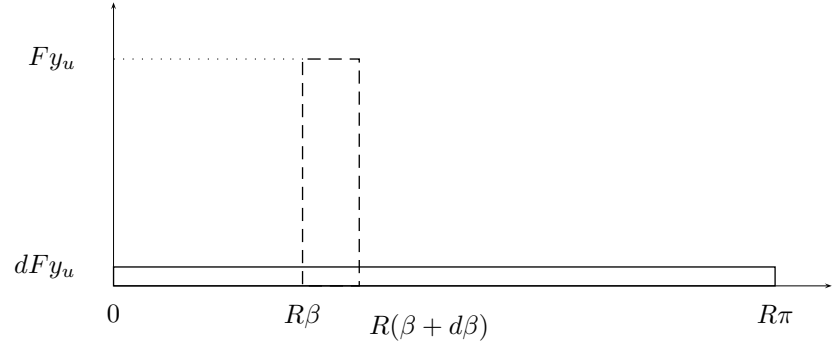


Figure 9: Equality of surfaces

The following equation is given for a turbine with  $N_p$  blades and by integration :

$$dFy_u R\pi = Fy_u R d\beta \frac{N_p}{2} \quad Fy_u = dFy_u \frac{2\pi}{N_p d\beta} \quad Fy_d = dFy_d \frac{2\pi}{N_p d\beta}$$

by using the equations (9) and (11) and previous equations

$$\begin{aligned} \frac{1}{8\pi} \frac{N_p c}{R} W_u^2 (C_N - C_T \frac{\cos \beta}{\sin \beta}) &= (V_\infty - V_u) V_u \\ \frac{1}{8\pi} \frac{N_p c}{R} W_d^2 (C_N - C_T \frac{\cos \beta}{\sin \beta}) &= (2V_u - V_d - V_\infty) V_u \quad (12) \end{aligned}$$

**2.3.2.7 velocities triangle**

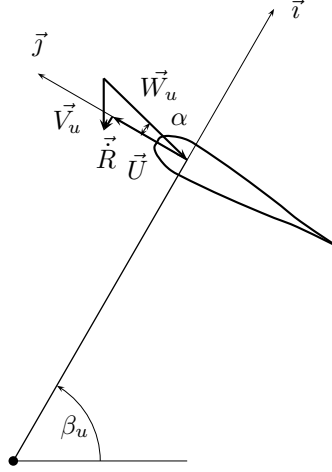


Figure 10: velocities triangle

$$\vec{V}_u = \vec{W}_u + \vec{U} + \dot{R}\vec{i} \quad \vec{U} = R\dot{\beta}\vec{j} \quad W_u : \text{vitesse relative}$$

$$\vec{V}_u = -V_u\vec{Y} \quad \vec{U} = U\vec{j} \quad U \geq 0$$

$$U = R\dot{\beta} \quad \dot{R} = (-r_b + e)\dot{\beta} \sin \beta \quad R \gg (-r_b + e) \quad \vec{V}_u \approx \vec{W}_u + \vec{U}$$

The Naca profile is following pretty much a circle trajectory during one rotation turn. ( $\dot{R} \approx 0$ )

$$V_u \sin \beta = -\dot{R} + W_u \sin \alpha \quad V_u \cos \beta = W_u \cos \alpha - R\dot{\beta}$$

$$\bar{\sigma} : \text{stiffness coefficient} \quad \bar{\sigma} = \frac{N_p c}{R}$$

$$\bar{\lambda} : \text{velocity coefficient} \quad \bar{\lambda} = \frac{\bar{R}\dot{\beta}}{V_\infty}$$

$$b = \bar{\sigma} \bar{\lambda}$$

$$\sin \alpha = \frac{V_\infty}{W_u} \frac{V_u}{V_\infty} \sin \beta \quad \cos \alpha = \frac{V_\infty}{W_u} \left\{ \frac{V_u}{V_\infty} + \frac{\bar{\lambda}}{\cos \beta} + \bar{\lambda} \frac{r_b - e}{\bar{R}} \right\} \cos \beta \quad (13)$$

by using equations (2),(10),(12) and also the above equations.

$$\begin{aligned}
C_{Tu} &= 2\pi \frac{V_\infty^2}{W_u^2} \sin^2 \beta \left\{ \frac{V_u}{V_\infty} \right\}^2 - C_D \\
C_{Nu} &= 2\pi \frac{V_\infty^2}{W_u^2} \sin \beta \cos \beta \frac{V_u}{V_\infty} \left\{ \frac{V_u}{V_\infty} + \frac{\bar{\lambda}}{\cos \beta} + \bar{\lambda} \frac{r_b - e}{\bar{R}} \right\} \\
C_{Nu} - C_{Tu} \frac{\cos \beta}{\sin \beta} &= 2\pi \frac{V_\infty^2}{W_u^2} \sin \beta \bar{\lambda} \frac{V_u}{V_\infty} \left\{ 1 + \frac{r_b - e}{\bar{R}} \cos \beta \right\} + C_D \frac{\cos \beta}{\sin \beta} \quad \text{with } C_D \approx 0 \\
A_u &= \frac{1}{V_\infty} \{V_\infty - V_u\} \quad A_d = \frac{1}{V_\infty} \{2V_u - V_d - V_\infty\} \\
A(V, \beta, W) &= \frac{1}{V} \frac{1}{V_\infty} \left\{ \frac{1}{8\pi} \bar{\sigma} W_u^2 (C_N - C_T \frac{\cos \beta}{\sin \beta}) \right\} \\
A(V, \beta, W) &= \frac{b}{4} \sin \beta (1 + \dots) \tag{14}
\end{aligned}$$

$$A_u = A(V_u, \beta, W_u) \quad A_d = A(V_d, \beta, W_d)$$

$$A_m = \frac{b}{4} \sin \beta \tag{15}$$

*In order to simplify the calculations, we will use  $A_m$  instead of  $A_u$  or  $A_d$*

*in the calculation of  $V_u$  or  $V_d$*

$$\begin{aligned}
A_m &= \frac{1}{V_\infty} (V_\infty - V_u) \quad A_m = \frac{1}{V_\infty} (2V_u - V_d - V_\infty) \\
\frac{V_u}{V_\infty} &= 1 - A_m \quad \frac{V_d}{V_\infty} = 1 - 3 A_m \tag{16}
\end{aligned}$$

### 2.3.2.8 Calculation of the mean dimensional values of Vu et Vd

$$\bar{\sigma} : \text{stiffness coefficient} \quad \bar{\sigma} = \frac{N_p c}{R}$$

$$\bar{\lambda} : \text{velocity coefficient} \quad \bar{\lambda} = \left| \frac{\bar{R} \dot{\beta}}{V_\infty} \right|$$

$$b = |\bar{\sigma} \bar{\lambda}|$$

$$A_{max} = -\frac{1}{4} b |\sin \beta|$$

$$\bar{v}_u = \left( \frac{V_u}{V_\infty} \right)_{moyen} = \frac{1}{\pi} \int_0^\pi \left( 1 - \frac{1}{4} b \sin \beta \right) d\beta = 1 - \frac{1}{2\pi} b$$

$$\text{with } 1 - \frac{1}{4} b \leq 1$$

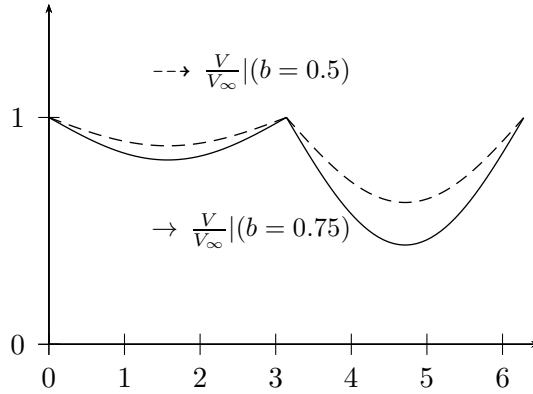
$$\bar{v}_d = \left( \frac{V_d}{V_\infty} \right)_{moyen} = \frac{1}{\pi} \int_\pi^{2\pi} \left( 1 - 3 \frac{1}{4} b \sin \beta \right) d\beta = 1 - \frac{3}{2\pi} b$$

$$\text{with } 1 - \frac{3}{4} b \leq 1 \rightarrow b \leq \frac{4}{3}$$

Defining

$$a = 1 \text{ for } 0 \leq \beta \leq \pi \quad a = -3 \text{ for } \pi \leq \beta \leq 2\pi$$

$$\frac{V}{V_\infty} = 1 - a \frac{b}{4} \sin \beta$$



One can define the mean value of  $\frac{V}{V_\infty}$

$$\left( \frac{\bar{V}_x}{V_\infty} \right) = \frac{1}{\pi} \left( \pi - a \frac{b}{4} \right) \quad x = u \text{ or } v$$

Figure 11: curve of Vu and Vd

### 2.3.2.9 Tangential velocity, angular velocity and rotational velocity

In order to follow the maximum angle  $\alpha_{max}$  of the profil, with  $\dot{R} = 0$   $\beta = \frac{\pi}{2}$

$\alpha_{max}$  maximun incidence angle of the profil  $(\beta = \frac{\pi}{2})$

$$U_{min} = \frac{(1 - \frac{b}{4} \sin \frac{\pi}{2}) V_{\infty}}{\tan \alpha_{max}}$$

The angular velocity is following :  $\omega = \dot{\beta} = \frac{U_{min}}{\bar{R}}$

Rotational velocity is  $N_{rpm} = \frac{\omega}{2\pi} 60$

$$U = (\bar{R} - (r_b - e) \cos \beta) \omega \quad \lambda = \frac{\omega \bar{R}}{V_{\infty}} \quad \sigma = \frac{b}{\lambda} \quad c = \frac{\sigma \bar{R}}{N_p} \quad b \leq \frac{4}{3}$$

### 2.3.2.10 Incidence angle By using (1)(2) and (13)

$$\tan \alpha = \frac{\sin \beta}{\cos \beta} \frac{[(1 - a \frac{b}{4}) + \frac{(-r_b + e)}{\bar{R}} \lambda]}{[(1 - a \frac{b}{4}) + \frac{\lambda}{\bar{R}} + \frac{(r_b - e)}{\bar{R}} \lambda]} \quad (17)$$

$$\tan \alpha = \tan \beta \left( \frac{1}{1 + \frac{\lambda}{(1 - a \frac{b}{4} \sin \beta)} (\frac{1}{\cos \beta} + \frac{r_b - e}{\bar{R}})} \right) \quad (18)$$

Incidence angle having a low value,  $\alpha \approx \tan \alpha \approx \sin \alpha \quad \cos \alpha \approx 1 - \frac{\alpha^2}{2} \approx 1$

The incidence angle must be below the maximum incidence angle of the profile.



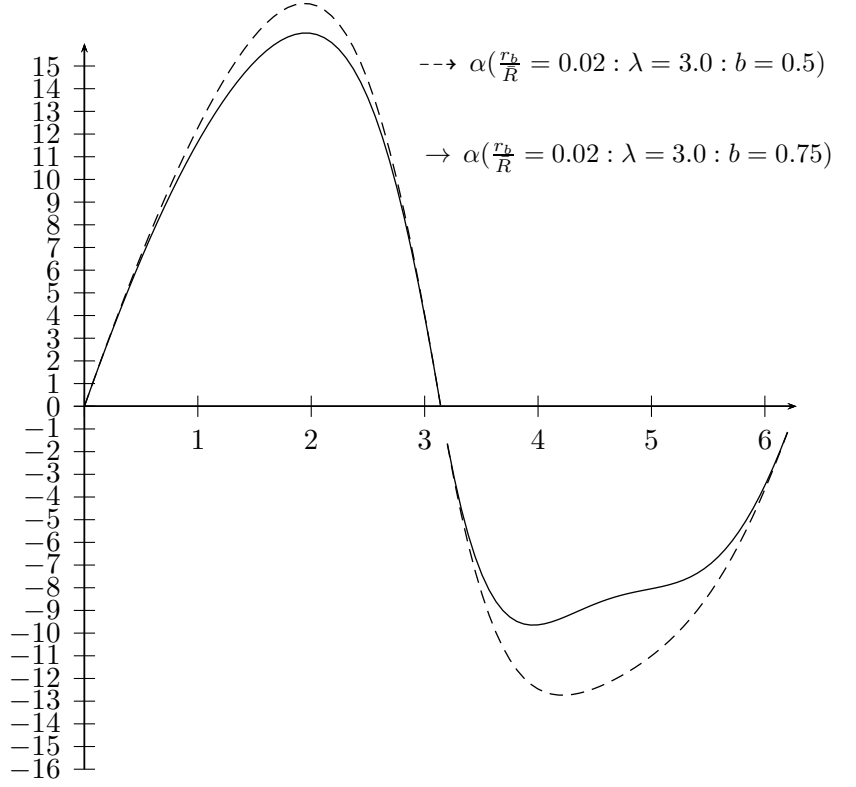


Figure 12: incidence angle variations

#### 2.3.2.11 Calculations of the tangential and normal coefficients

By using (10), the following incidence angle definition below

$$avec C_D = 0 \quad C_T = 2\pi \sin^2 \alpha \quad C_N = 2\pi \sin \alpha \cos \alpha$$

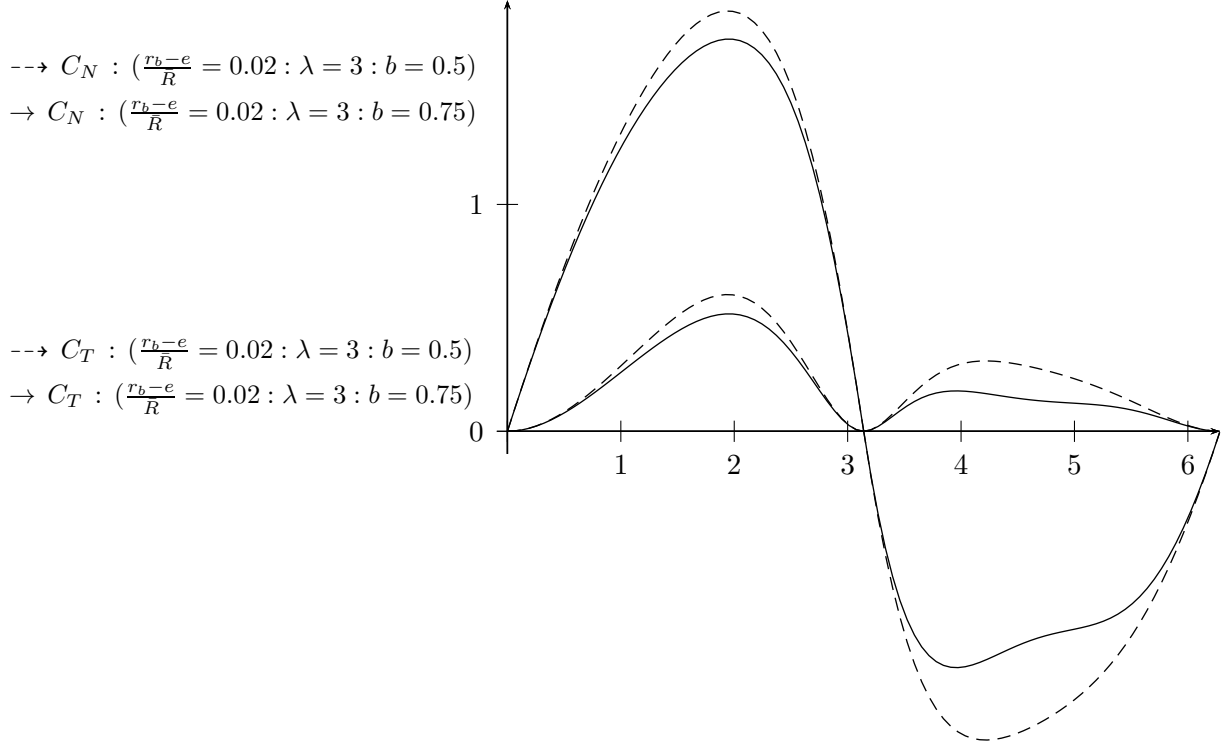


Figure 13: tangential and normal coefficient

**2.3.2.12 Calculation of the relative velocity :** by using (13) dimensional relative velocity is

$$\frac{W^2}{V_\infty^2} = \frac{V^2}{V_\infty^2} (1 + \lambda^2 + 2\lambda \cos \beta)$$

$$\frac{V}{V_\infty} = 1 - a \frac{b}{4} \sin \beta$$

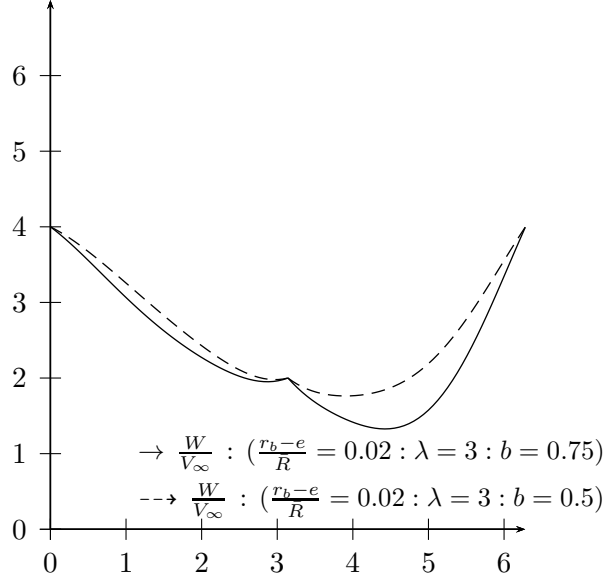


Figure 14: relative velocity

The dimensional relative velocity  $\frac{W}{V_\infty}$  can be approximated by

$$\left(\frac{\bar{W}_x}{V_\infty}\right) = \lambda \left(\frac{\bar{V}_x}{V_\infty}\right) = \lambda \frac{1}{\pi} \left(\pi - a \frac{b}{4}\right) \quad x = u \text{ or } v$$

**2.3.2.13 Torque and power calculations** Following the document [6], a mean deviation angle can be defined  $\phi$  like being the mean value of  $\phi = \frac{\beta_u + \beta_d}{2}$  and the section of the flow area  $dAx_m$  of a streamtube ruled by the velocity  $V_m$ . The mean flow area is defined by  $dAx_m = \bar{R} d\phi \sin \phi dH$ .

$$dAx_u V_u = dAx_d V_d = dAx_m V_m \quad dAx_m = \frac{dAx_u + dAx_d}{2}$$

$$dAx_u = R_u d\beta_u \sin \beta_u dH = R_u \sin \beta_u \frac{d\beta_u}{d\phi} d\phi dH$$

$$dAx_d = R_d d\beta_d \sin \beta_d dH = R_d \sin \beta_d \frac{d\beta_d}{d\phi} d\phi dH$$

We infer

$$\begin{aligned}\frac{dAx_u}{dAx_m} &= \frac{V_m}{V_u} = \frac{2V_d}{V_u + V_d} & \frac{dAx_d}{dAx_m} &= \frac{V_m}{V_d} = \frac{2V_u}{V_u + V_d} \\ \frac{dAx_u}{dAx_m} &= \frac{R_u d\beta_u \sin \beta_u}{\bar{R} d\phi \sin \phi} & R_u &\approx \bar{R} & \frac{d\beta}{d\phi} &= \frac{\sin \phi}{\sin \beta_u} \frac{dAx_u}{dAx_m} = \frac{\sin \phi}{\sin \beta_u} \frac{2V_d}{V_u + V_d} \\ dAx_u &= R_u \sin \beta_u \frac{d\beta_u}{d\phi} d\phi dH & R_u &\approx \bar{R} \\ \frac{dAx_u}{\sin \beta_u dH} &= \bar{R} \frac{d\beta_u}{d\phi} d\phi = \bar{R} \frac{\sin \phi}{\sin \beta_u} \frac{2V_d}{V_u + V_d} d\phi\end{aligned}$$

The angle  $\phi$  is of the same order of magnitude than  $\beta$ .

$$\frac{dAx_u}{\sin \beta_u dH} = \bar{R} \frac{2V_d}{V_u + V_d} d\beta_u$$

likewise

$$\frac{dAx_d}{\sin \beta_d dH} = \bar{R} \frac{2V_u}{V_u + V_d} d\beta_d$$

The torque due to the tangential force  $dT_u$  is defined by :

$$\begin{aligned}T_u &= C_T \frac{1}{2} \rho W_u^2 c dH & T_u &= dT_u \frac{2\pi}{N_p d\beta} \\ dCT_u &= R_u dT_u & dAx_u &= R_u d\beta_u \sin \beta_u dH \\ dCT_u &= \frac{1}{4\pi} \sigma \bar{R} C_T \rho W_u^2 \frac{dAx_u}{\sin \beta_u} = \frac{1}{4\pi} \sigma C_T \rho W_u^2 \bar{R}^2 \frac{2V_d}{V_u + V_d} d\beta_u\end{aligned}$$

likewise

$$dCT_d = \frac{1}{4\pi} \sigma \bar{R} C_T \rho W_d^2 \frac{dAx_d}{\sin \beta_d} = \frac{1}{4\pi} \sigma C_T \rho W_d^2 \bar{R}^2 \frac{2V_u}{V_u + V_d} d\beta_d$$

**The wind power is**

$$P_{wind} = \frac{1}{2} \rho S_{wept-area} V_\infty^3 = \frac{1}{2} \rho 2\bar{R} dH V_\infty^3$$

**The power coefficient is**

$$\begin{aligned}C_p &= \frac{Power_{turbine}}{P_{wind}} = \frac{Torque_{turbine} \dot{\beta}}{P_{wind}} & \dot{\beta} &= \frac{\lambda V_\infty}{\bar{R}} \\ C_p &= \lambda \frac{Torque_{turbine}}{\rho \bar{R}^2 H V_\infty^2} = \lambda C_{p_{dimless}} & C_{p_{dimless}} &= \frac{Torque_{turbine}}{\rho \bar{R}^2 H V_\infty^2}\end{aligned}$$

the torque due to the tangential forces is equal to

$$Torque_{ForcesT} = \left( \int_0^\pi dCT_u d\beta + \int_\pi^{2\pi} dCT_b d\beta \right) H$$

$$CpT_{dimless} = \frac{\sigma}{2\pi} \left\{ \int_0^\pi \frac{V_d}{V_u + V_d} \left( \frac{W_u}{V_\infty} \right)^2 d\beta_u + \int_\pi^{2\pi} \frac{V_u}{V_u + V_d} \left( \frac{W_d}{V_\infty} \right)^2 d\beta_d \right\}$$

By using (3)  $Cr = \{e \sin \beta\} F_{slide-bar}$

$$F_{slide-bar} = -C_N \frac{1}{2} \rho W_u^2 c dH = -N \quad C_T = 2\pi \sin^2 \alpha - C_D \quad C_N = 2\pi \sin \alpha \cos \alpha$$

$$C_D \approx 0 \quad \cos \alpha \approx 1 \quad \sin \beta = \frac{W}{V} \sin \alpha \quad \left( \frac{\bar{W}}{V_\infty} \right) \approx \lambda \left( \frac{\bar{V}}{V_\infty} \right)$$

$$CpN_{dimless} \approx \frac{e}{R} \lambda CpT_{dimless}$$

The dimensionnal power coefficient has a value of

$$Cp_{dimless} = \left( 1 + \frac{e}{R} \lambda \right) CpN_{dimless}$$

Thanks to an analytical calculation, the analytical power coefficient can be defined by

$$Cp_{analytical} = \left( 1 + \frac{e}{R} \lambda \right) \left( \frac{9\pi}{27} b^3 - \frac{2^2}{3} b^2 + \frac{\pi}{2} b \right) \quad b = \sigma \lambda \quad \text{with} \quad Cp_{analytical} < 1.$$

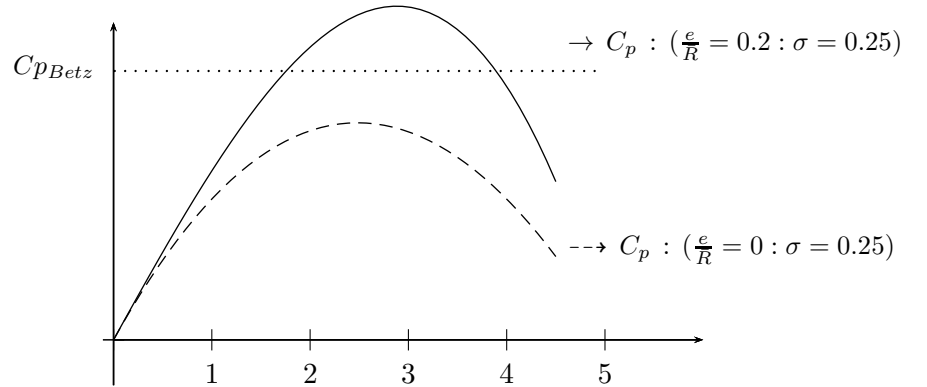


Figure 15: power coefficient

## 2.4 Conclusion about Power Coefficient

Mathematician Betz determined, thanks to the kinetic energy (with appropriate parameters), that it is possible to collect 60% of the wind energy. The remaining energy (40%) is not lost and generates forces on the turbine.

For a standard Darrieus turbine, the normal force creates compressive stress and extension of the arms. The active lift turbine is partially transferring these normal forces thanks to the crank rod system into kinetic energy.

Concerning standard Darrieus turbine, the driving power is calculated through the tangential power coefficients  $C_{pT}$ .

Concerning active lift turbine, the driving power is calculated through the tangential and normal power coefficients  $C_{pT}$  and  $C_{pN}$ .

### 3 Theory on vibrations :

#### 3.1 Resonant modes

The relative velocity is not uniform during each turn. This generates velocity variations of the rotational velocity and induced vibrations. By using Lagrange equations (virtual powers), we will estimate these vibrations

The velocity variations are

$\dot{\beta}$  has been considered as constant  $\dot{\beta} = \omega = \text{constant}$

The rotational angular velocity  $\omega$  follows the variations.

In order to take into account these variations, the angular velocity can be defined as

$$\dot{\beta} = \dot{\bar{\beta}} + \dot{\beta} \quad \dot{\bar{\beta}} = \omega = \text{Mean constant velocity}$$

The velocity variations compared to the mean velocity are corresponding to the terms  $\dot{\bar{\beta}}$

First approximation is giving  $\dot{\bar{\beta}}$  by  $a\ddot{\beta}$  a being a constant

$$\dot{\beta} = \dot{\bar{\beta}} + a\ddot{\beta} = \omega + a\ddot{\beta}$$

The kinetic energy is

$$T_e = \frac{1}{2}m\{(R\dot{\beta})^2 + \dot{R}^2\} = \frac{1}{2}m\dot{\beta}^2\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\}$$

Considering the profil mass being in one point  $m$  for simplify calculations

The Lagrange equations terms at the kinetic energy level are

$$\frac{\partial T_e}{\partial \dot{\beta}} = m\dot{\beta}\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\}$$

$$\frac{d}{dt}\left\{\frac{\partial T_e}{\partial \dot{\beta}}\right\} = m\ddot{\beta}\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\} - 2(r_b - e)\bar{R}m\dot{\beta}^2\sin\beta$$

$$\frac{\partial T_e}{\partial \beta} = -m(r_b - e)\bar{R}\dot{\beta}^2\sin\beta$$

$$\frac{d}{dt}\left\{\frac{\partial T_e}{\partial \dot{\beta}}\right\} - \frac{\partial T_e}{\partial \beta} = m\ddot{\beta}\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\} + m(r_b - e)\bar{R}\dot{\beta}^2\sin\beta$$

## 4 Powers calculation

$$Power_{Driving} = \{N\vec{i} + T\vec{j}\} \wedge \{\dot{R}\vec{i} + TR\dot{\beta}\vec{j}\} = N\dot{R} + TR\dot{\beta}$$

$$N\dot{R} + TR\dot{\beta} = -(r_b - e)N\dot{\beta} \sin \beta + T(\bar{R} + (r_b - e) \cos \beta)\dot{\beta}$$

$$Power_{Resisting} = -C_{Resisting} \dot{\beta}$$

$C_{Resisting}$  : resisting torque due electrical generator , to the friction, etc

The lagrange equation terms of the power level are

$$A_\beta = T\bar{R} + (r_b - e)T \cos \beta - (r_b - e)N \sin \beta - C_{Resisting}$$

The Lagrange equation is

$$\frac{d}{dt} \left\{ \frac{\partial T_e}{\partial \dot{\beta}} \right\} - \frac{\partial T_e}{\partial \beta} = A_\beta$$

$$m\ddot{\beta} \{ \bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R} \cos \beta \} =$$

$$T\bar{R} + (r_b - e)T \cos \beta - (r_b - e)N \sin \beta - m(r_b - e)\bar{R}\dot{\beta}^2 \sin \beta - C_{Resisting}$$

The Lagrange equation for a Darrieus turbine type r=0

$$m\ddot{\beta} \bar{R}^2 = T\bar{R} - C_{rsistant} \quad T = C_T \frac{1}{2} \rho c H W^2$$

$$\ddot{\beta} = \frac{1}{m\bar{R}^2} \{ T\bar{R} - C_{Resisting} \}$$

T depends on  $C_T$  and on  $W^2$ , for a blade, simplified curve  $|\ddot{\beta}|$  is

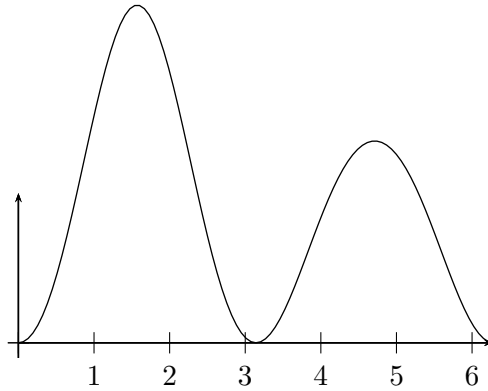




Figure 16: approximately curve of  $|\ddot{\beta}|$

For one blade, there will be two pulsations per turn. The blade pace will be equivalent (translation of 120) for a three blades case. The association of the three paces is lowering the vibrations and is increasing the vibration frequency. Six pulsations will be obtained per turn.

The Lagrange equation for an active lift turbine with a controlled displacement is

$$\ddot{\beta} = \frac{1}{m\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\}} \times \{T\bar{R} + (r_b - e)T\cos\beta - (r_b - e)N\sin\beta - m(r_b - e)\bar{R}\dot{\beta}^2\sin\beta - C_{Resisting}\} \quad (19)$$

The same kind of equations is obtained for standard Darrieus turbine type.

The attenuation of the amplitudes has almost the value of  $\frac{(r_b - e)}{\bar{R}}$ .

#### 4.1 Conclusion on vibrations

The same range of vibrations is generated for a Darrieus turbine type or for an active lift turbine with a controlled displacement. Vibrations are reduced slightly.

These frequencies can be a source of fatigue for the structure or of noise generation. For Darrieus type turbines, manufacturers are building helicoidal blades or stepped blades. The yield of the active lift turbine is higher than any other turbine types but the vibrations are not cancelled. These are slightly dampened.

## 5 Conclusion

cf. conclusions : 2.4 and 4.1

It has been theoretically demonstrated that active lift turbine is delivering a power coefficient greater than defined by Betz [1] for classical vertical axis Darrieus [7] type turbine. The active lift turbine will not solve the generated vibrations problem. It will be necessary to add devices (such as twisted blades for example) in order to limit the vibrations. The first version of this turbine was the subject of a Phd work (numerical and experimental)[3] which validated the concept. The latest version of the active lift turbine provides a real gain on the theoretical power and requires a research plan to validate and optimize this concept in order to achieve an industrial version.

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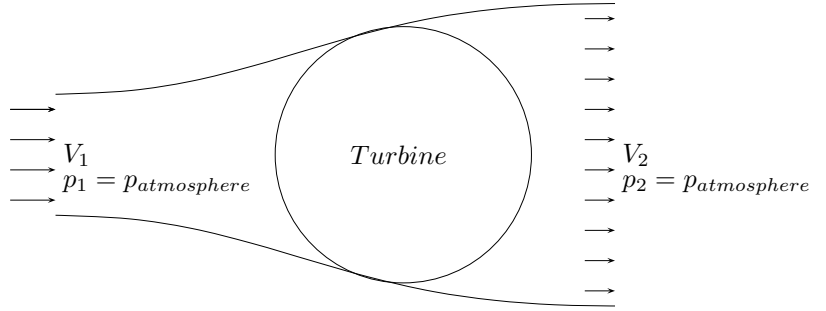
## 6 Betz power coefficient

Maximum wind power (from kinetic energy)

$$E_c = \frac{1}{2} m V^2$$

$$\frac{dE_c}{dt} = \frac{1}{2} \frac{dm}{dt} V^2 + \frac{1}{2} m \frac{dV^2}{dt} \quad \frac{dV}{dt} = 0$$

$$\frac{dE_c}{dt} = \frac{1}{2} \dot{m} V^2 = \frac{1}{2} \rho S V^3$$



**V Wind speed at the turbine level**

**Force applied by the wind on the rotor**

$$F = m \frac{dV}{dt} = \dot{m} \Delta V = \rho S V (V_1 - V_2)$$

$$P = F V = \rho S V^2 (V_1 - V_2)$$

$$P = \frac{\Delta E}{dt} = \frac{\frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2}{\Delta t} = \frac{1}{2} \dot{m} (V_1^2 - V_2^2) = \frac{1}{2} \rho S V (V_1^2 - V_2^2)$$

**From these equalities**

$$V = \bar{V} = \frac{V_1 + V_2}{2}$$

$$F = \rho S V (V_1 - V_2) = \frac{1}{2} \rho S (V_1^2 - V_2^2)$$

$$P = F V = \rho S V^2 (V_1 - V_2) = \frac{1}{4} \rho S (V_1^2 - V_2^2) (V_1 + V_2)$$

defining  $a = \frac{V_2}{V_1}$

$$F = \frac{1}{2}\rho S V_1^2 (1 - a^2) \quad P = \frac{1}{4}\rho S V_1^3 (1 - a^2)(1 + a)$$

The wind power is  $P_{wind} = \frac{1}{2}\rho S V_1^3$

The power coefficient is defined by  $C_p = \frac{P}{P_{wind}} = \frac{\frac{1}{4}\rho S V_1^3 (1 - a^2)(1 + a)}{\frac{1}{2}\rho S V_1^3} = \frac{1}{2}(1 - a^2)(1 + a)$

Search of the maximum power coefficient

$$\frac{dC_p}{da} = 0 = -2a(1 + a) + 1 - a^2 \quad a > 0 \quad a = \frac{1}{3}$$

$$C_p = \frac{16}{27} = 0.593$$

The maximum power coefficient  $C_{p_{maxi}}$  is defined by Betz

$$C_{p_{Betz}} = \frac{16}{27} = 0.593$$

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