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Parallelizing Constraint Solvers for Hard RCPSP Instances*

Technical Report

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Abstract. The Resource-Constrained Project Scheduling Problem (RCPSP) is a well-known scheduling problem aimed at minimizing the makespan of a project subject to temporal and resource constraints. Constraint Programming allows to model and solve RCPSPs in a natural and efficient way, especially when Lazy Clause Generation (LCG) techniques are employed. In this paper we show that hard RCPSPs can be efficiently tackled by a portfolio approach that combines the strengths of different —not necessarily LCG-based— solvers by exploiting their parallel execution on multiple cores. Our approach seeks to predict and run the best solvers for a new, unseen RCPSP instance and also enables the bounds communication between them. This on-average allows to outperform the oracle solver that always chooses the best available solver for any given instance.

1 Introduction

The *Resource-Constrained Project Scheduling Problem* (RCPSP) [9] is the problem of minimizing the makespan (i.e., the total duration) of a project, defined as a collection of tasks subject to precedence relations between the activities and constrained by resource availabilities. This well-known NP-hard problem [8] has countless industrial applications and it is one of the most studied scheduling benchmark.

The *Constraint Programming* (CP) [30] paradigm allows to model and solve hard combinatorial problems. In particular, RCPSP can be naturally and elegantly encoded into a *Constraint Optimization Problem* (COP) where: (i) integer variables are used to track the start time of each task; (ii) constraints over such variables ensure compliance with the precedence relations and resource capabilities; (iii) a special integer variable keeps track of the makespan. The goal is to find an optimal solution, i.e., a consistent assignment of the variables which minimizes the makespan.

An effective technique for solving CP problems consists in using *Lazy Clause Generation* (LCG) [28] techniques. The key idea of LCG is to mimic the Finite Domain propagation by properly generating corresponding SAT clauses during the search. LCG can significantly boost the search for many combinatorial problems, including RCPSP

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and its variants, but obviously it is not a *panacea* and there are cases in which LCG is not fruitful.

To exploit the diverse nature and performance of different solving techniques, a fairly recent trend consists in using a portfolio approach [14]. Basically, given a portfolio $\{s_1, \dots, s_m\}$ of $m > 1$ solvers, a *portfolio solver* seeks to predict the best solver(s) s_{i_1}, \dots, s_{i_k} (with $1 \leq i_j \leq m$ for $j = 1, \dots, k$) for solving a new, unseen problem p and then runs such solver(s) on p . In this context, scheduling $k > 1$ solvers can reduce the risk of selecting only one solver—maybe a bad one for p —and especially enables the knowledge sharing between the scheduled solvers, as well as their parallel execution. Surprisingly, despite their effectiveness, portfolio solvers have been poorly adopted in real-life applications.

In this work we join the aforementioned research streams by showing how CP can be successfully applied for solving hard RCPSP instances by means of a parallel portfolio approach. We first retrieved a fairly large number of non-trivial RCPSPs encoded in *MiniZinc* [26], nowadays one of the most widely used language to define COPs. We then defined some variants of the parallel portfolio solver `sunny-cp` [3], the gold medalist in the open category of MiniZinc Challenge 2015 [34,35], to boost the resolution of the RCPSP instances.

We would like to underline that this is not a “standard” application of a portfolio approach because in this case there is a dominant solver (the LCG solver Chuffed [12], publicly available at [11], best solver of the MiniZinc Challenges 2012—2015 in the free search category) which is able to greatly outperform all the other solvers. This setting is not unusual in many real-life applications. For instance, companies are typically interested in solving only a specific kind of problem emerging from the business process. Conversely, this scenario is not so common in the benchmarks used for evaluating portfolios since the effectiveness of a portfolio approach relies on the diversity of good solvers.

The emphasis of this work is not on the Algorithm Selection problem [23,29] where we are only interested in predicting the best solver for a given problem. Here we show instead how it is possible to combine, leverage, and run simultaneously the predicted solvers on RCPSPs to *outperform* the best available solver and to be on-average better than the *Virtual Best Solver*, i.e., the oracle solver that always chooses the best solver for any given RCPSP instance (or, equivalently, that always runs simultaneously all the available solvers of the portfolio without synchronization and memory contention issues).

Furthermore, we remark that the focus of this work is not on `sunny-cp` tool. Conversely, here we are interested in advancing the state of the art of CP techniques for solving difficult RCPSPs problems. In particular, we show that state-of-the-art LCG solvers can be significantly overcome thanks to other constraint solvers not employing LCG. The message of the paper is twofold: (i) we prove that the belief that portfolios can not be applied in real-life scenarios characterized by a dominating solver is false, (ii) we show that by parallelizing the solvers execution and by enabling the bounds communication between the scheduled solvers we can get an overall better solver which is even greater than the sum of its parts. To the best of our knowledge, we are not aware of similar approaches for efficiently solving hard RCPSP instances.

Paper Structure. Section 2 gives background notions about RCPSP and `sunny-cp`. In Section 3 we explain the evaluation methodology. Section 4 reports the experimental results, while Section 5 draws concluding remarks.

2 Background

In this section we give some background notions about the RCPSP problem and the `sunny-cp` portfolio solver.

2.1 RCPSP

The RCPSP resolution has attracted a lot of attention over the last decades, since this problem emerges from many real-life scenarios. For this reason, several formulations and techniques have been proposed for efficiently solving RCPSP and its variants [15, 16, 21]. The most effective methods make use of hybrid approaches that combine the effectiveness of modern SAT solvers with other solving techniques such as Finite Domain propagation or Linear Programming [17, 31, 32]. To the best of our knowledge, the LCG approach gives the best results for RCPSP [32] and variants like RCPSP/max [33] and RCPSP/max-cal [24].

In this paper we examine a possible CP formulation of RCPSP, as it appears in MiniZinc 1.6 benchmarks,⁴ and we show how state-of-the-art solvers based on LCG techniques can be significantly overcome by taking advantage of a portfolio of constraint solvers not necessarily employing LCG.

The model we consider consists of:

- a set of resources $\mathcal{R} = \{r_1, \dots, r_m\}$;
- a set of tasks $\mathcal{T} = \{t_1, \dots, t_n\}$;
- a set of precedences between tasks $\mathcal{P} = \{P_1, \dots, P_n\}$, such that $P_i \subseteq \mathcal{T} \setminus \{t_i\}$ contains the successors of tasks t_i .

Each resource r_j has a fixed capacity $c_j \in \mathbb{N}$, and each task t_i requires a fixed capacity $q_{i,j} \in [0, c_j]$ of resource r_j . Each task t_i has a constant duration $d_i \in \mathbb{N}$ over time. The objective is to find an optimal schedule, i.e., a function $\sigma : \mathcal{T} \rightarrow \mathbb{N}$ such that:

- $\sigma(t_i)$ is the starting time of t_i ;
- σ meets all the precedence and capacity constraints;
- the makespan, defined as $\max\{\sigma(t_i) + d_i \mid i = 1, \dots, n\}$, is minimal.

The model is already annotated with a default search strategy imposing to first select the variable having smaller domain (i.e., the *min-dom* heuristic) and then try to assign to such variable the smaller value of its domain (*min-value* heuristic). The study of alternative heuristics is outside the scope of this paper and left as future work.

⁴ Available at <https://github.com/MiniZinc/minizinc-benchmarks/blob/master/rcpsp/rcpsp.mzn>

2.2 sunny-cp

sunny-cp is an open-source portfolio solver. While its first implementation was sequential [4], its current version exploits multicore architectures for running more solvers simultaneously by exploiting their *cooperation* via bound sharing and restarting policies. sunny-cp won the gold medal in the open category of MiniZinc Challenge 2015 and, to the best of our knowledge, it is currently the only parallel portfolio solver able to solve generic COPs. A detailed explanation about sunny-cp can be found at [3]. Here we briefly recall how it works.

sunny-cp is built on top of SUNNY algorithm [2] taking a fixed solving timeout T and portfolio Π . SUNNY exploits the k -Nearest Neighbours (k -NN) algorithm to produce a sequential schedule $\sigma = [(s_1, t_1), \dots, (s_k, t_n)]$ where solver $s_i \in \Pi$ has to be run for t_i seconds and $\sum_{i=1}^n t_i = T$. Then, σ is parallelized on the c available cores. The first and most promising $c - 1$ solvers are allocated to the first $c - 1$ cores, while the remaining solvers are assigned to the last available core by linearly widening their allocated times for covering the whole time window $[0, T]$. In the particular case of RCPSPs, the most promising solvers are those that are faster in finding the minimal makespan values in the k -neighborhood.

Solvers are run in parallel and a “*bound-and-restart*” mechanism is used for enabling the bounds sharing between the running solvers. This allows to use the (sub-optimal) solutions found by a solver to narrow the search space of the other scheduled ones. Since sunny-cp treats solvers as black boxes, it does not support the knowledge sharing without their interruption. Therefore, a restarting threshold T_r is used to decide when to stop a solver and restart it with a new bound. A running solver is stopped and restarted if it has not found a solution in the last T_r seconds and its current best bound is obsolete w.r.t. the overall best bound found by another scheduled solver.

sunny-cp solves problems encoded in the *MiniZinc* language [26]. By default, it uses a portfolio Π of 12 state-of-the-art solvers disparate in their nature (viz., Chuffed, CPX, G12/CBC, G12/FD, G12/LazyFD, G12/Gurobi, Gecode, MinisatID, Choco, HafiaCSP, iZplus, and OR-Tools). Some of them are provided in two variants: *fixed* and *free*. The fixed variant is optional, and forces a solver to use the search strategy possibly defined in the MiniZinc input model. The free variant instead allows a solver to use its preferred search strategy, ignoring any other search annotation that appears in the MiniZinc model. By default, in line with what most of its constituent solvers do, sunny-cp uses the fixed variant of a solver (when provided) to exploit the domain knowledge of who wrote the MiniZinc model. However, as later detailed, the free variant may significantly outperform the fixed one and should be taken into account when solving a problem.

3 Experiments Methodology

The RCPSP model described in Section 2.1 is the most represented problem class in the MiniZinc 1.6 benchmarks with 2904 RCPSP instances coming from different scenarios [1, 6, 10, 22, 25, 27]. However, most of these instances are not challenging: often the best solvers of sunny-cp can solve them in a matter of seconds. These instances are

not so interesting since a pre-solving that statically schedules the best solvers for few seconds can solve them without any prediction. For this reason we focus our attention on the “hardest” RCPSP instances of the benchmark. After running both the fixed (when available) and free version of the constituent solvers of `sunny-cp` with a time limit of $T = 900$ seconds,⁵ we discarded all the instances for which at least one solver was able to prove the optimality of a solution (or the problem unsatisfiability) in less than $T/10 = 90$ seconds. In this way we obtained a dataset Δ of 647 RCPSPs for each of which at least one solver of the portfolio is able to find a feasible solution.

Let \mathcal{U} be a universe of solvers and T a solving timeout. To measure the performance of solver $s \in \mathcal{U}$ —whether it be an individual or a portfolio solver—on a problem instance $p \in \Delta$ within T seconds, we consider the following three metrics:

- OPT: measures the optima proven. If s proves the optimality of a solution for p , then $\text{OPT}(s, p) = 1$; otherwise, $\text{OPT}(s, p) = 0$;
- TIME: measures the optimization time. If s proves the optimality of a solution for p in $t < T$ seconds, then $\text{TIME}(s, p) = t$; otherwise, $\text{TIME}(s, p) = T$;
- OBJ: measures the quality of a solution, by normalizing its makespan value in range $[0, 1]$. If s finds no solution, then $\text{OBJ}(s, p) = 0$. Otherwise, if $\text{mkspan}(s, p)$ is the best makespan found by s for problem p and $\mathcal{M}_p = \{\text{mkspan}(s, p) \mid s \in \mathcal{U}\}$ is the set of the best makespans for all the solvers of \mathcal{U} , then:

$$\text{OBJ}(s, p) = 1 - \frac{\text{mkspan}(s, p) - \min \mathcal{M}_p}{\max \mathcal{M}_p - \min \mathcal{M}_p}.$$

In this way, the better the makespan the higher is OBJ. In particular, we have that $\text{OBJ}(s, p) = 1$ if and only if s finds the best solution in \mathcal{U} for p .

Table 1 shows the average performance of the individual solvers of `sunny-cp`, with the exception of G12/CBC that was not used due to its very poor performance. We added as baseline the Virtual Best Solver (*VBS*), the oracle portfolio solver that—for a given problem and performance metric—always chooses the best solver in the portfolio.⁶ The *VBS* is a standard baseline used for evaluating the performance of a portfolio approach and usually the desired goal is to get as close as possible to the *VBS* performance.

As can be seen, for almost 90% of the dataset Δ (578 instances) no solver is able to prove the optimality in 900 seconds. Chuffed clearly dominates all the other solvers, almost reaching the *VBS* performance. In particular its free version turns out to be very effective. This is probably due to the decision heuristics it uses in its free version, e.g., the *Variable State Independent Decaying Sum* (VSIDS) [13] heuristic. The effectiveness of LCG is also confirmed in Table 1 by the performance of the others LCG-based solvers, namely CPX and LazyFD.

Note that—depending on the solver and the evaluation metric—using the free search is not always the best choice. Given the diversity of the results by considering different search strategies, we decided to test three different variants of `sunny-cp`:

⁵ 900 seconds was the timeout of MiniZinc Challenge 2014.

⁶ The *VBS* of a portfolio \mathcal{U} can be seen as a parallel solver that run simultaneously all the $|\mathcal{U}|$ solvers without synchronization and memory contention issues (and without bounds communication).

Solver	OPT (%)		TIME (sec.)		OBJ \times 100	
	Fixed	Free	Fixed	Free	Fixed	Free
Chuffed *	2.63	7.42	887.77	858.64	88.96	96.50
G12/CPX *	1.24	1.08	894.66	894.26	89.03	75.66
G12/LazyFD *	0.15	2.62	899.90	888.48	72.76	75.38
HaifaCSP	-	0.62	-	896.97	-	74.73
Choco *	0	0	900	900	66.75	72.76
OR-Tools *	0.31	0.16	899.05	899.14	65.00	67.11
G12/FD *	0	0	900	900	65.92	15.78
Gecode *	0	0	900	900	64.25	64.20
MinisatID	-	0.31	-	898.32	-	63.64
iZplus *	0	0	900	900	43.43	33.74
G12/Gurobi	-	2.94	-	885.90	-	2.94
<i>VBS</i>	<i>10.67</i>		<i>841.33</i>		<i>100</i>	

Table 1. Average solvers performance. The fixed version is available only for the solvers marked with *. Peak performances are highlighted in bold.

- `sunny-def`: the default version of `sunny-cp`. It uses the portfolio Π_{def} of the 11 solvers listed in Table 1 by always considering the fixed version of a solver when available;
- `sunny-all`: uses a portfolio Π_{all} of $11 + 8 = 19$ solvers which extends Π_{def} by including all the versions of all the available solvers;
- `sunny-stc`: uses a variable sized portfolio $\Pi_{c,\mu}$ of c solvers, where c is the number of available cores and $\mu \in \{\text{OPT}, \text{TIME}, \text{OBJ}\}$ is a performance measure. Specifically, $\Pi_{c,\mu}$ is the subset of the best c solvers of Π_{all} according to the average value of μ over dataset Δ .

`sunny-all` is the natural extension of `sunny-def`, which allows to measure the impact of adding the free versions of solvers to the default portfolio. Both `sunny-all` and `sunny-def` are *dynamic* approaches, since they select the solvers to run on-line according to the instance to be solved. `sunny-stc` follows instead a *static* approach. Indeed, since for each number of cores c its portfolio $\Pi_{c,\mu}$ contains exactly c solvers, no prediction is performed and all its solvers are launched simultaneously regardless of the instance to be solved. `sunny-stc` allows to investigate to what extent the solvers prediction of `sunny-all` is really fruitful and to understand the marginal contribution of the solvers belonging to $\Pi_{\text{all}} \setminus \Pi_{c,\mu}$.

There are clearly many different variants of `sunny-cp` that might be considered. We also tested other plausible `sunny-cp` variants (e.g., by switching the free/fixed search strategy every time a solver is restarted). These however did not lead to major benefits. Clearly, an exhaustive evaluation of all of the variants is a daunting task due to the combinatorial explosion of all the possible approaches and to the fact that a single test on all the RCSPs of Δ requires on average more than 150 computation hours. Following a standard practice for avoiding biasing and overfitting, we validated our

OBJ \times 100	1 core	2 cores	4 cores	8 cores
sunny-def	92.14	94.25	95.83	96.04
sunny-all	94.67	96.36	98.25	98.45
sunny-stc	94.97	95.73	97.57	99.05
<i>VPS</i>	94.97	96.92	97.43	98.21
<i>VBS</i>	98.38			

Table 2. OBJ performance.

experiments by using a *10-fold cross-validation* [5]. We partitioned the dataset Δ in 10 subsets $\Delta_1, \dots, \Delta_{10}$ called folds, treating in turn a fold Δ_i as the test set and the union $\bigcup_{j \neq i} \Delta_j$ of the other folds as the training set. For all the `sunny-cp` approaches, we kept the default value $T_r = 5$ seconds for the restarting threshold. Since `sunny-cp` is parallel, we ran the three versions of `sunny-cp` by considering 1, 2, 4, and 8 cores.

4 Results Discussion

This Section discusses in detail the performance of `sunny-def`, `sunny-all`, and `sunny-stc` in terms of OBJ, OPT, and TIME when exploiting a different number of cores. In addition to the *VBS* we also introduced the *Virtual Parallel Solver* ($VPS_{c,\mu}$), an oracle portfolio solver that for $c \in \{1, 2, 4, 8\}$ and $\mu \in \{\text{OPT}, \text{TIME}, \text{OBJ}\}$ simulates the parallel and independent execution of the solvers of the portfolio $\Pi_{c,\mu}$ introduced in Section 3. For simplicity, where there is no ambiguity, we will use the notation *VPS* or VPS_c instead of $VPS_{c,\mu}$. We recall that *VBS* and *VPS* are “virtual” since they are not implemented but simulated according to the solvers performance on Δ . Since by definition *VPS* is a bound of the performance of `sunny-stc` approach without bounds communication and synchronization overheads, the *VPS* baseline allows to measure the effectiveness of bounds communication. Note also that with this definition the *Single Best Solver* (*SBS*) of the portfolio (i.e., the free version of Chuffed) is equivalent to the VPS_1 and the `sunny-stc` approaches when using a single core ($c = 1$) since $\Pi_{1,\mu} = \{\text{Chuffed/Free}\}$ for every $\mu \in \{\text{OPT}, \text{TIME}, \text{OBJ}\}$. Furthermore, the *VBS* is equivalent to $VPS_{|\Pi_{\text{all}}|}$.

4.1 OBJ Results

Considering the OBJ metric we can say that most of the `sunny-cp` approaches provide high quality solutions ($0.95 < \text{OBJ} < 1$) even when optimality is not proven. Table 2 shows that the demarcation between the different approaches is not so sharp since *SBS* is very close to *VBS* (see Table 1).⁷

The only approach that performs rather poorly is `sunny-def` when exploiting just one core. However, we remark that this approach contains only the fixed versions of the

⁷ Note that the OBJ values of the *SBS* and *VBS* of Table 1 differ from those of Table 2, since introducing `sunny-cp` in the universe \mathcal{U} allowed to find new and better makespan values, thus reducing the average OBJ of *SBS* and *VBS*.

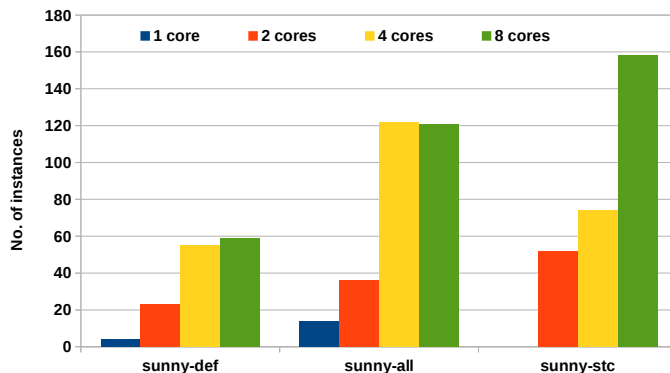


Fig. 1. Number of times `sunny-cp` outperforms the `VBS` in terms of OBJ.

solvers: if we add the free versions to the portfolio we can see a clear OBJ improvement (from the 92.14 of `sunny-def` to the 94.67 of `sunny-all`). Unfortunately, with $c = 1$ core Chuffed/Free is still better (94.97). This means that the solvers prediction is not so accurate in a sequential setting. Nevertheless, the benefits of solvers prediction and bounds communication become more and more evident when increasing the number of available cores. When $c = 2$, `sunny-all` overtakes `sunny-stc` and comes close to VPS_2 , while for $c = 4$ it is better than VPS_4 and almost as good as the `VBS`. In particular, for $c \geq 4$ the effectiveness of bounds communication becomes clear. `sunny-stc` with 4 cores is better than VPS_4 , i.e., its corresponding version without bounds communication and synchronization issues. With 8 cores `sunny-stc` outperforms not only VPS_8 , but also the `VBS`. In other terms, 8 cores are enough for providing better solution than a “magic solver” that runs simultaneously —without synchronization issues— all the 19 solvers of I_{all} . Also `sunny-all` is able to outperform `VBS` with $c = 8$, even if its performance is worse than `sunny-stc`. This means that the impact of `sunny-all` on-line selection is reduced when a high number of cores can be used. This is reasonable since the more algorithms can be selected, the less is the risk of not selecting a good one. Despite a growing performance when increasing the number of cores, `sunny-def` is instead always worse than the other approaches. This reflects the importance of using the free search for finding high quality solutions.

For the OBJ metric, apart from the free search, the main benefits appear coming from the solvers parallelization and the bounds communication, rather than to the solvers selection and scheduling. The `VBS` is however overtaken on-average by both `sunny-all` and `sunny-stc` by using 8 cores (i.e., less than half of the available solvers). Figure 1 shows how many times `sunny-cp` is able to produce better solutions than the `VBS`. `sunny-all` outperforms the `VBS` 121 times with 8 cores (18.7% of Δ). Surprisingly, with 4 cores it outperforms the `VBS` for one problem more (122 times). This is however not contradictory because the side-effects of restarting the solvers and the race conditions between their concurrent execution do not guarantee the

OPT (%)	1 core	2 cores	4 cores	8 cores
sunny-def	2.01	4.64	9.58	10.36
sunny-all	6.65	9.58	12.52	15.46
sunny-stc	7.42	7.88	9.12	9.58
<i>VPS</i>	7.42	<i>8.04</i>	8.35	<i>8.50</i>
<i>VBS</i>	<i>10.66</i>			

Table 3. OPT performance.

performance monotonicity as c increases. `sunny-stc` reaches the peak performance with 8 cores: the *VBS* is outperformed 158 times (24.42% of Δ), meaning that almost one time out of four it finds a better solution than *VBS*.

4.2 OPT and TIME Results

The OPT performances are depicted in Table 3. This metric is challenging in our context: we are dealing with hard RCPSP instances for which no solver is able to prove the optimality in less than 90 seconds and the *VBS* can prove only 69 optimum (10.66% of Δ).

`sunny-def` does not perform well with $c \leq 2$ cores, but then gradually improves by proving almost the same number of optima of *VBS* even without the free search variants. This means that `sunny-def` compensates for the lack of solvers belonging to $\Pi_{\text{all}} \setminus \Pi_{\text{def}}$ by properly combining the solvers of its portfolio Π_{def} . For $c \geq 2$ the best approach is `sunny-all`, which is able to outperform the *VBS* when using 4 or more cores. In particular, the gain with 8 cores is somewhat impressive: 4.8% optima proven more than *VBS*. Here the performance difference is not only due to parallelism and bounds communication, but especially to the solver selection. Indeed, the gap with `sunny-stc` becomes larger as the number of cores increases. Furthermore, contrary to what happens for the OBJ metric, it is interesting to see how for $c \geq 4$ the default approach `sunny-def` is better than `sunny-stc`. In other terms, adding new solvers without an adequate per-instance selection of them can be fruitful in terms of solution quality, but ineffective for what concerns the optima proven.

Figure 2 reports the number of times `sunny-cp` is able to outperform the *VBS* in terms of OPT, i.e., how many times it can prove a new optimality not provable by any other version of any given solver. The performance difference between `sunny-stc` and the other approaches is clear, especially for $c \geq 4$. Given the low number of instances solved to optimality by the *VBS*, the performance of `sunny-def` with 4 cores (*VBS* outperformed 28 times) and `sunny-all` with 8 cores (34 times) are quite significant.

Table 4 shows the average TIME performances. Being the majority of the instances of Δ very hard to solve, it is not surprising that the TIME values are very close to the timeout $T = 900$. All the tested approaches perform well, since they are very close to or better than the *VBS*. On average, `sunny-def` is able to outperform the *VBS* with $c \geq 4$ cores. For $c \in \{1, 2, 4, 8\}$, the average TIME of `sunny-stc` is lower than the

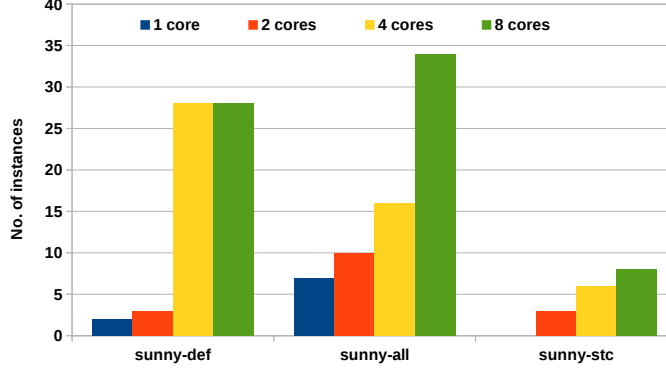


Fig. 2. Number of times `sunny-cp` outperforms the `VBS` in terms of OPT.

TIME (sec.)	1 core	2 cores	4 cores	8 cores
<code>sunny-def</code>	889.33	869.42	830.71	825.55
<code>sunny-all</code>	858.20	838.41	823.18	797.53
<code>sunny-stc</code>	858.63	853.55	849.45	844.69
<i>VPS</i>	858.63	854.13	852.16	851.17
<i>VBS</i>	841.33			

Table 4. TIME performance.

corresponding VPS_c . This means that, even if `sunny-stc` never overtakes the `VBS`, the bounds communication allows to reduce the optimization time for every number of cores c . The best approach results to be `sunny-all`: for every $c \in \{1, 2, 4, 8\}$ it is better than the other `sunny-cp` approaches and the corresponding VPS_c . Moreover, it can outperform on-average the `VBS` with only two cores. Its effectiveness in reducing the optimization time is also corroborated by the fact that for 41 instances (6.33% of Δ) `sunny-all` can prove the optimality of a solution in less than 90 seconds.

4.3 Summary

Summarizing, we can say that all the `sunny-cp` variants we tested can be effective on the RCPSP instances of Δ , especially when more than one core is used. The solvers parallelization is not the only key for the success of such approaches. Firstly, the use of the free search (when available) allowed us to improve the solving process. Moreover, the bounds communication between the scheduled solvers enables to outperform the `VBS`. Furthermore, we realized that it is also important —especially for OPT and TIME metrics— to properly schedule a subset of solvers dynamically, i.e., according to the instance to be solved. A simpler static approach can be instead useful for finding good solutions as witnessed by the OBJ results. In general we might say that `sunny-`

`all` is the best approach while `sunny-def` is the worse one, proving that it is important to use solvers able to adopt their own preferred search strategy. However, we should be careful to draw hasty conclusions. Let us show a concrete (counter)example showing the potential and the differences between the tested approaches.

Solver	mkspan		TIME (sec.)	
	Fixed	Free	Fixed	Free
G12/Gurobi	N/A	—	T	T
Choco	600	597	T	T
Chuffed	593	594	T	T
G12/CPX	593	631	T	T
G12/FD	600	672	T	T
G12/LazyFD	—	—	T	T
Gecode	600	600	T	T
HaifaCSP	N/A	657	T	T
iZplus	1446	1446	T	T
Minisatid	N/A	—	T	T
OR-Tools	600	600	T	T
sunny-def	593		9.90	
sunny-all	593		31.55	
sunny-stc	593		T	

Table 5. RCPSP Example. T is the timeout, N/A means not available, while — means no solutions found in T seconds.

Table 5 shows the performance of the different solvers on a RCPSP instance of Δ .⁸ As can be seen, none of the single solvers of \mathcal{I}_{all} can conclude the search within $T = 900$ seconds. G12/Gurobi, G12/LazyFD, and Minisatid can not find any solution within the timeout. The fixed search variant here is better for Choco, Chuffed, G12/CPX, and G12/FD while for all the other solvers it is equivalent to the free one. For this instance, the VBS chooses (indifferently) one between the fixed versions of Chuffed and G12/CPX since they found the best makespan value 593, without however proving its optimality ($OPT = 0$ and $TIME = T$).

`sunny-def` greatly outperforms VBS , being able to prove the optimality of 593 in less than 10 seconds ($OPT = 1$ and $TIME = 9.90$). This is because G12/CPX finds the bound 593 in few seconds and the other scheduled solvers are restarted with such new bound. Now, Gecode can prove almost instantaneously that 593 is the optimal makespan, while without this help the best value it finds in half an hour of computation is 600. `sunny-all` works similarly, but more slowly. This time difference is due to the SUNNY algorithm, which computes instantaneously the solvers scheduled with $|\mathcal{I}_{def}| = 11$ solvers, while with $|\mathcal{I}_{all}| = 19$ it takes about 23 seconds for

⁸ The model is `rcpsp.mzn` while the data is in `1a05.x2.dzn`

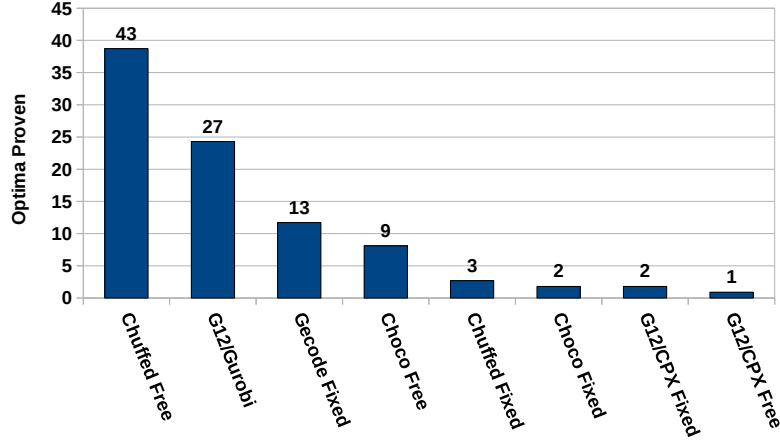


Fig. 3. No. of optima proven by individual solvers of `sunny-all` with 8 cores.

computing the schedule of the solvers. Finally, `sunny-stc` cannot prove the optimality of 593 since it schedules G12/CPX but not Gecode. Indeed, $\Pi_{8,OPT}$ contains Chuffed (Free/Fixed), G12/CPX (Free/Fixed), G12/LazyFD (Free/Fixed), HaifaCSP, iZplus (Fixed).

The behavior depicted in Table 5 is not an isolated case. As an example, Figure 3 shows how many times a constituent solver of `sunny-all` proves the optimality when 8 cores are available. In total `sunny-all` proves the optimality of 100 instances (15.46% of Δ), 31 instances more than the *VBS*. Despite Chuffed Free completes the search the largest number of times (43/100), the majority of the instances are solved to optimality by other solvers. Moreover, we can not say that in this case `sunny-all` is enhancing Chuffed Free, since when it is executed alone it can prove the optimality of more instances (48 instances, i.e., 7.42% of Δ as reported in Table 1). The same applies to its fixed version and G12/CPX. Conversely, `sunny-all` can significantly boost the individual OPT performance of non-LCG solvers like G12/Gurobi, Choco and Gecode. G12/Gurobi proves 27 optima, 8 more than its individual performance. Particularly interesting it is also the difference for Choco (both versions) and Gecode (fixed version) since none of such solvers is able to prove any optima when is run alone. By exploiting instead the bounds found by other solvers they can conclude the search in a not negligible number of times: Gecode proves 13 optima and Choco 11 optima (9 with the free version, and 2 with the fixed one).

5 Conclusions

The Resource-Constrained Project Scheduling Problem (RCPSPs) is a well-known scheduling problem applicable in many real-life scenarios. A state-of-the-art method-

ology consists in solving RCPSPs through Lazy Clause Generation (LCG) techniques, which allow to combine the strengths of SAT solving and Finite Domain propagation.

In this paper we show how it is possible to integrate Constraint Programming, Algorithm Selection, Multicore Computing and other search heuristics for solving hard RCPSP instances. We improve the state of the art on RCPSP solving by showing how the performance of state of the art LCG solvers can be significantly overcome by using a portfolio approach. In particular, our methodology lies in the use of a portfolio of different constraint solvers for selecting and running a subset of them on multiple cores. We not only seek to predict which are the best solvers for a new, unseen RCPSP instance, but we also enable the bounds communication between the scheduled solvers. To the best of our knowledge, in the literature there are no similar approaches for solving RCPSPs and variants.

Empirical evaluations conducted on hundreds of non-trivial instances show that, by exploiting different versions of the `sunny-cp` solver, it is possible to get parallel portfolio solvers able to outperform on-average the oracle solver that always chooses the best available solver for any given instance. Improvements are manifold in terms of both solution quality, optima proven and optimization time. We noticed in particular significant performance gains in quickly proving more optima.

We believe that this work may open the way to several extensions. Probably the most interesting ones concern the analysis of how to properly integrate and combine the (concurrent) execution of different constraint solvers. In particular, a major scientific challenge consists in understanding the interaction between constraint solvers. There are many open questions. Which information should be shared (e.g., only bounds, or other knowledge such as nogoods or snippets of the search tree)? When and how this information should be exchanged? When restarting a solver is useful and when is instead harmful?

In this work we focused on system having less cores than available solvers. Clearly, different techniques (e.g., running the same, randomized solver in parallel, search space splitting techniques) can be adopted with powerful clusters where the number of cores exceeds the portfolio size. The evaluation of portfolio approaches in such a setting is left as a future work.

Concerning the RCPSP problem, it would be interesting to compare and integrate our approach with Operations Research techniques. Moreover, it is certainly worth to evaluate and combine other search heuristics for improving the RCPSP resolution (e.g., precedence-setting searches, texture-based heuristics [7]).

From the portfolio perspective, we hope that this work can stimulate the utilization of portfolio solvers also in real-life scenarios where typically a single, dominant solver is used for solving different instances of the same problem. It would therefore be interesting to try to apply our approach also to other variants of RCPSP (e.g., RCPSP/max, RCPSP/max-cal, or Cyclic/RCPSP) and to other classes of problems (e.g., job shop scheduling or bin packing).

Finally, recalling that `sunny-cp` was used only since it was the only parallel portfolio solver for COPs, in the future it would be interesting to adapt and test other portfolio approaches or related techniques such as Algorithm Configuration [18–20] for the automatic tuning of solvers parameters.

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