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# Distributed RSS-based Localization in Wireless Sensor Networks with Asynchronous Node Communication

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**Abstract.** In this paper we address the node localization problem in large-scale wireless sensor networks (WSNs) by using the received signal strength (RSS) measurements. According to the conventional path loss model, we first pose the maximum likelihood (ML) problem. The ML-based solutions are of particular importance due to their asymptotically optimal performance (for large enough data records). However, the ML problem is highly non-linear and non-convex, which makes the search for the globally optimal solution difficult. To overcome the non-linearity and the non-convexity of the objective function, we propose an efficient second-order cone programming (SOCP) relaxation, which solves the node localization problem in a completely distributed manner. We investigate both synchronous and asynchronous node communication cases. Computer simulations show that the proposed approach works well in various scenarios, and efficiently solves the localization problem. Moreover, simulation results show that the performance of the proposed approach does not deteriorate when synchronous node communication is not feasible.

**Keywords:** Wireless localization, wireless sensor network (WSN), received signal strength (RSS), second-order cone programming problem (SOCP), cooperative localization, distributed localization.

## 1 Introduction

Wireless sensor network (WSN) consists of a large number of low-power sensor nodes that have some sensing, processing and communication capabilities. WSNs find application in various areas, and the capability to accurately locate all sensor nodes in the network is essential for many of them (e.g. monitoring, military operations, rescue missions, etc.). In general, sensor nodes can be classified as anchor and target (source) nodes [1]. The positions of anchor nodes are known *a priori* (usually measured by global positioning system (GPS) or manually), while the positions of target nodes are yet to be determined. For economic or other practical reasons, only a small fraction of

nodes are set to be anchor nodes, hence, an efficient algorithm for node localization is necessary for WSNs.

Instead of the use of GPS system, which is very expensive and limited to outdoor environments, algorithms that rely on distance measurements between neighboring nodes are emerging recently [2], [3], [4], [5], [6]. Depending on the available hardware, current distance-based algorithms extract the distance information from time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA) or received signal strength (RSS) measurements [7]. Localization based on RSS measurements requires the least processing and communication (the least energy), and no specialized hardware [8], which makes it an attractive low-cost solution for the localization problem.

Localization algorithms can be executed in a centralized or a distributed fashion. The former approach assumes the existence of a fusion center which coordinates the network and performs all computational processing. This approach leads to large energy and bandwidth consumption, with a bottleneck around the fusion center and the computational complexity of such an approach grows with the increase of a number of nodes in the network. In many practical scenarios, it is not efficient or not possible for the nodes to share their private objective functions with a central processor or with each other [9], which makes the distributed localization a more preferable solution. Even though it is sensitive to error propagation, distributed concept is energy-efficient, has low-computational complexity and high-scalability [1]. In such an approach, the communication is possible only between the neighboring nodes, and the data associated with each node is always processed locally.

Recent RSS-based localization algorithms use a centralized concept to solve the sensor nodes localization problem [2], [3], [4], [5], [6]. A distributed approach for solving the RSS localization problem with synchronized node communication was introduced in [10], [11]. However, synchronized node communication requires the use of more sophisticated hardware in the network, which raises the cost of the network implementation.

In this work, we consider a large-scale RSS-based target localization problem, and we provide a solution that is completely based on a distributed approach. We investigate both the synchronous and asynchronous node communication scenarios, and propose a novel algorithm based on second-order cone programming (SOCP) relaxation. We first formulate the maximum likelihood (ML) optimization problem, which is highly non-linear and non-convex. To overcome these difficulties, we propose a SOCP relaxation approach to transform the original ML problem into a convex one, which can then be efficiently solved by interior-point algorithms [12].

## 2 Relationship to Collective Awareness Systems

Sensor nodes in WSNs are deployed over a monitored area in order to acquire the desired information (such as temperature, wind speed, pressure, etc.). Nowadays, the basic concept of WSNs is used for the Internet-of-Things (IoT) in which all devices, objects and environments are connected through Internet to form the so-called smart

environments [13]. Such systems are capable of harnessing collective intelligence for promoting innovation and taking better-informed and sustainability-aware decisions.

Being able to accurately estimate object's position is a key factor in a number of sensor network applications (such as energy-efficient routing, target tracking and detection). Combining the location information with other information collected inside the network enables us to link objects, people and knowledge and develop intelligent systems (collective awareness systems). Such systems may improve safety and efficiency in everyday life, since each individual device can make better-informed and substantially-aware decisions to respond faster and better to the changes in dynamical environments (search and rescue missions, logistics in warehouses, etc.). Furthermore, such systems enhance sustainable growth, since they support new forms of social and business innovation.

### 3 Problem Formulation

We consider a  $p$ -dimensional ( $p \geq 2$ ) WSN with  $M + N$  nodes, where  $M$  and  $N$  are the number of target and anchor nodes, respectively. The locations of the nodes are denoted as  $\mathbf{x}_1, \dots, \mathbf{x}_M, \mathbf{x}_{M+1}, \dots, \mathbf{x}_{M+N}$ . The considered WSN can also be seen as a connected graph,  $G = (V, E)$ , where  $V$  and  $E$  represent the set of nodes (vertices) and the set of node connections (edges) in the graph, respectively. Due to lifetime of the network or other physical limitations, each node has a limited communication range,  $R$ . An edge exists between two nodes,  $i$  and  $j$ , if and only if they are within the communication range of each other, i.e.  $E = \{(i, j) : \|\mathbf{x}_i - \mathbf{x}_j\| \leq R\}$ . The set of neighbors of a target node  $i$  is defined as  $\Omega_i = \{j : (i, j) \in E\}$ , and each neighboring node  $j$  is seen as an anchor node in the localization process by the  $i$ -th target node.

We assume that each target node  $i$  is given an initial estimation of its position,  $\mathbf{x}_i^{(0)}$ . For ease of expression, we define  $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]$  as the  $2 \times M$  matrix of all target positions in the WSN; hence,  $\mathbf{X}^{(0)}$  contains all initial estimations of the target positions.

Node  $i$  measures the received power from the signal transmitted by its neighboring node  $j$ ,  $P_{ij}$ , which, under the log-normal shadowing, can be modeled as (in dBm) [14], [15]

$$P_{ij} = P_0 - 10\gamma \log_{10} \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{d_0} + v_{ij}, \quad \forall (i, j) \in E, \quad (1)$$

where  $P_0$  is the power measured at a short reference distance  $d_0$  ( $\|\mathbf{x}_i - \mathbf{x}_j\| \geq d_0$ ),  $\gamma$  is the path loss exponent, and  $v_{ij}$  is the log-normal shadowing term between the  $i$ -th target node and its neighbor node  $j$ , modeled as a zero-mean Gaussian random variable with variance  $\sigma_{ij}^2$ , i.e.  $v_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$ .

Based on the measurements from (1), we derive the maximum likelihood (ML) estimator as

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\operatorname{argmin}} \sum_{(i,j) \in E} \frac{1}{\sigma_{ij}^2} \left[ P_{ij} - P_0 + 10\gamma \log_{10} \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{d_0} \right]^2. \quad (2)$$

The least squares (LS) problem defined in (2) is non-linear and non-convex, hence, finding the globally optimal solution is difficult, since there may exist multiple local optima. Following [5], we show that the RSS measurement model in (1) can be approximated into a convex optimization problem, which can be solved by interior point algorithm [12], and obtain the global solution.

For the sake of simplicity, in the further text we assume that  $\sigma_{ij}^2 = \sigma^2, \forall (i, j) \in E$ . Furthermore, we assume that the transmit power,  $P_T$ , of each node in the network is equal, i.e. all nodes have identical  $P_0$  and  $R$ .

#### 4 Distributed Localization Using SOCP Relaxation

Note that the objective function in (2) depends only on the positions and pairwise measurements between the neighboring nodes. This means that we can portion the objective function in (2) and perform the minimization independently by each target node, using only local information gathered from its neighbors. Instead of having a sink, which collects and processes the information from all nodes in WSN, we can divide the optimization problem into smaller sub-problems which can be carried out locally by each target node. This kind of problem execution is particularly suitable for large scale networks, since the number of nodes inside the network has no major impact on the neighborhood fragments, and hence, the computational complexity remains the same (no significant changes) as more nodes are added in the network [1]. The price to pay for using this kind of approach is the increased energy consumption due to higher node communication, since distributive localization algorithms require repetition of the following phases:

- *Communication phase*: nodes in the network transmit their estimated position to their neighboring nodes.
- *Computation phase*: each target node computes its position estimation based on the information gathered in the communication phase.

If the information exchange between nodes is always performed at the beginning of each iteration, we say that the node communication is *synchronous* [16]. However, due to some physical imperfections (e.g. internal clock of the nodes), synchronous node communication may not be feasible in practice. This is the reason why we also investigate *asynchronous* node communication. In such communication, a randomly chosen target node performs an update of its position estimation based on the available information from its neighbors at that particular moment (computation phase) and transmits the updated information to its neighbors thereupon (communication phase). This process is then repeated until each node reaches the maximum number of iterations,  $K_{max}$ .

In the  $k$ -th iteration of the computation phase of our algorithm, each target node  $i$  solves a SOCP relaxation of the following problem:

$$\hat{\mathbf{x}}_i^{(k)} = \underset{\mathbf{x}_i}{\operatorname{argmin}} \sum_{(i,j) \in E} \frac{1}{\sigma^2} \left[ P_{ij} - P_0 + 10\gamma \log_{10} \frac{\|\mathbf{x}_i - \hat{\mathbf{x}}_j^{(k-1)}\|^2}{d_0} \right], \quad (3)$$

where  $\hat{\mathbf{x}}_j^{(k-1)}$  denotes the estimated position of the neighboring node  $j$  in the  $(k-1)$ -the iteration. In the following text we will describe a SOCP relaxation method which approximates the problem in (3) into a convex one.

#### 4.1 SOCP Relaxation

Approximating (1) as  $P_{ij} \approx P_0 - 10\gamma \log_{10} \frac{\|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|}{d_0}$ ,  $\forall (i, j) \in E$ , we get

$$\alpha_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 \approx d_0^2, \quad (4)$$

where  $\alpha_{ij} = 10^{\frac{P_{ij}-P_0}{5\gamma}}$ . According to (4), the following LS estimation problem can be formulated<sup>1</sup>:

$$\mathbf{x}_i^{(k)} = \underset{\mathbf{x}_i}{\operatorname{argmin}} \sum_{(i,j) \in E} \left( \alpha_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 - d_0^2 \right)^2. \quad (5)$$

Define auxiliary variables  $\lambda_{ij} = 10^{\frac{P_{ij}}{5\gamma}}$ ,  $\rho = 10^{\frac{P_0}{5\gamma}}$ ,  $y_i = \|\mathbf{x}_i\|^2$ , and  $\mathbf{z} = [z_{ij}]$ , where  $z_{ij} = \lambda_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 - \rho d_0^2$ ,  $\forall (i, j) \in E$ . Introduce an epigraph variable  $t$ , and apply second-order cone constraint (SOCC) to obtain a convex optimization problem:

$$\underset{\mathbf{x}_i, y_i, \mathbf{z}, t}{\operatorname{minimize}} \quad t$$

subject to

$$z_{ij} = \lambda_{ij} \left( y_i - 2\mathbf{x}_j^{(k-1)T} \mathbf{x}_i + \|\mathbf{x}_j^{(k-1)}\|^2 \right) - \rho d_0^2, \quad \forall (i, j) \in E,$$

$$\|2\mathbf{z}; t - 1\| \leq t + 1, \quad \|2\mathbf{x}_i; y_i - 1\| \leq y_i + 1. \quad (6)$$

Problem in (6) is a SOCP problem, which can efficiently be solved by the CVX package [16] for specifying and solving convex programs. We will refer to (6) as ‘‘SOCP’’ in the further text.

<sup>1</sup> We can rewrite (1) as  $\frac{P_{ij}-P_0}{5\gamma} + \log_{10} \frac{\|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|}{d_0} = \frac{v_{ij}}{5\gamma}$ , which corresponds to  $\alpha_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 = d_0^2 10^{\frac{v_{ij}}{5\gamma}}$ . For sufficiently small noise, we can apply the first-order Taylor series expansion to the right-hand side of the previous expression to obtain  $\alpha_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 = d_0^2 \left( 1 + \frac{\ln 10}{5\gamma} v_{ij} \right)$ , i.e.  $\alpha_{ij} \|\mathbf{x}_i - \mathbf{x}_j^{(k-1)}\|^2 = d_0^2 + \varepsilon_i$ , where  $\varepsilon_i = d_0^2 \frac{\ln 10}{5\gamma} v_{ij}$  is a zero-mean Gaussian random variable with variance  $\frac{(\ln 10)^2 d_0^4 \sigma^2}{25\gamma^2}$ , i.e.,  $\varepsilon_i \sim \mathcal{N} \left( 0, \frac{(\ln 10)^2 d_0^4 \sigma^2}{25\gamma^2} \right)$ . Clearly, the corresponding LS estimator is given by (5).

## 5 Complexity Analysis

The trade-off between the computational complexity and the estimation accuracy is the most important feature of any localization algorithm, since it determines its applicability potential. Here, we consider only the worst case asymptotic complexity of an algorithm.

According to [18], the worst case complexity of the proposed ‘‘SOCP’’ approach is  $M \cdot K_{max} \cdot \mathcal{O}((\max\{|\Omega_i|\})^3)$ , where  $|\Omega_i|$  represents the cardinality of the set  $\Omega_i$ , for  $i = 1, \dots, M$ .

As we can see from the above result, the worst case complexity of a distributive localization algorithm mainly depends on the neighborhood fragments (the biggest one). If the number of nodes in WSNs is increased, it will not significantly affect the size of the neighborhood fragments, which makes the distributed algorithms a desirable solution in highly-dense or large-scale networks.

## 6 Simulation Results

In this section, we present the computer simulation results in order to evaluate the performance of the proposed approach. The considered algorithms were solved by using the MATLAB package CVX [17], where the solver is SeDuMi [19].

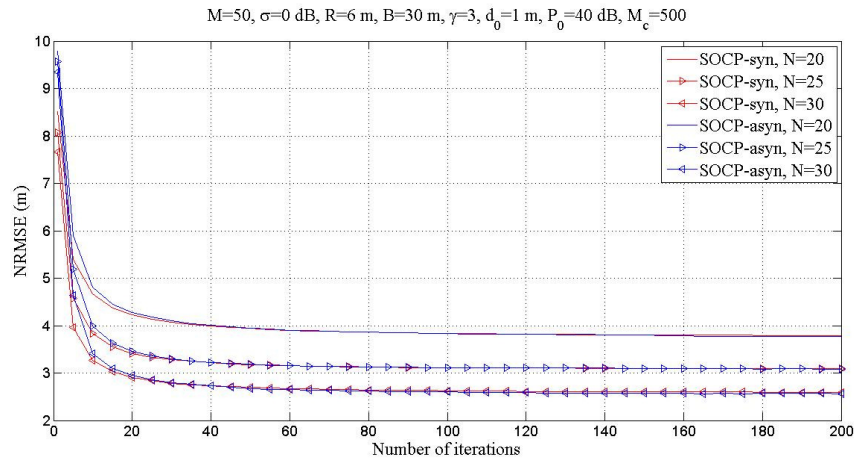
Nowadays, flexibility and adaptability of a network are very important features in practical applications; hence we consider a random deployment of the nodes. All nodes were randomly deployed inside a square region of length  $B = 30$  m in each Monte Carlo ( $M_c$ ) run. In order to make the comparison fair, we first obtained  $M_c = 500$  nodes positions, and we applied the proposed approaches for those scenarios. In each  $M_c$  run, we made sure that the network graph is connected. The path loss exponent is  $\gamma = 3$ , the reference distance  $d_0 = 1$  m, the reference power  $P_0 = -10$  dBm, and the communication range of a node  $R = B/5$  m. We assumed that the initial estimation of the target positions,  $\mathbf{X}^{(0)}$ , is in the intersection of the diagonals of the square area, and that  $K_{max} = 200$ . We assume that one iteration step is completed after  $M$  nodes compute and transmit their position estimations, for both synchronous and asynchronous node communication. As the performance metric we used the normalized root mean square error (NRMSE), defined as

$$\text{NRMSE} = \sqrt{\frac{1}{M M_c} \sum_{i=1}^{M_c} \sum_{j=1}^M \|\mathbf{x}_{ij} - \hat{\mathbf{x}}_{ij}\|^2}, \quad (6)$$

where  $\hat{\mathbf{x}}_{ij}$  denotes the estimate of the true location of the  $j$ -th target in the  $i$ -th Monte Carlo run,  $\mathbf{x}_{ij}$ .

The NRMSE performance of the proposed method versus number of iterations for synchronous and asynchronous node communication, and variable  $N$  is depicted in Fig. 1. From Fig. 1 we can see improvement of NRMSE performance as the number of iterations is increased, as expected. Further, we can see that NRMSE performance improves as  $N$  is increased, as expected. Lower estimation accuracy is achieved with asynchronous than with synchronous node communication in the early phase of the algorithm. However, we can see that the asymptotical performance of the proposed

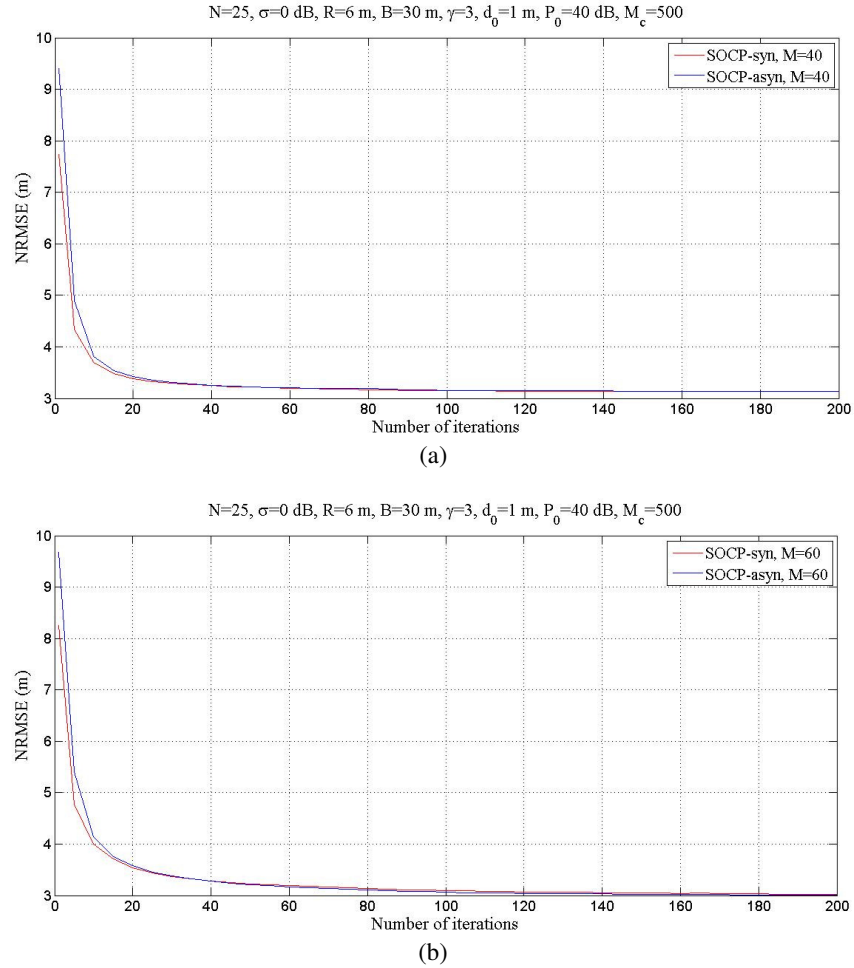
approach is the same for both asynchronous and synchronous node communication. Furthermore, Fig. 1 exhibits that all major improvements in the performance occur until approximately 80 iterations, and we can conclude that our algorithm converges after this number of iterations, for the considered scenarios.



**Fig. 1.** Comparison of the performance of the proposed method for synchronous and asynchronous node communication: NRMSE versus number of iterations for variable  $N$ .

The NRMSE performance of the proposed method versus number of iterations for synchronous and asynchronous node communication, and variable  $M$  is depicted in Fig. 2. We can see from Fig. 2 that NRMSE performance of the proposed approach does not degrade as more target nodes are added in the network. As it was anticipated, estimation accuracy is weaker in the early phase of the algorithm for higher  $M$ , and the proposed algorithm converges slower for this setting. Fig. 2 exhibits better NRMSE performance of the proposed approach with synchronized than with asynchronous node communication after the first few iterations. However, asymptotical performance is the same for both types of node communication.





**Fig. 2.** Comparison of the performance of the proposed method for synchronous and asynchronous node communication: NRMSE versus number of iterations: (a)  $M = 40$ , (b)  $M = 60$ .

## 7 Conclusions

In this work, we investigated the RSS-based sensor localization problem in large-scale WSNs, which we solved in a completely distributed fashion. We considered both synchronous and asynchronous node communication scenarios, and we proposed a distributed algorithm that is based on SOCP relaxation technique. Due to practical

demands, such as flexibility and adaptability of the network, randomly generated WSN were taken into consideration. Simulation results show that the proposed approach efficiently solves the sensor localization problem for different settings. Moreover, simulation results show that, even though synchronous node communication is preferred over asynchronous when only few iteration steps are allowed, the asymptotical performance of the proposed approach does not suggest any preference between the mentioned types of node communication.

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