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## **L2-Orthogonal ST-Code Design for Multi-h CPM with fast Decoding**

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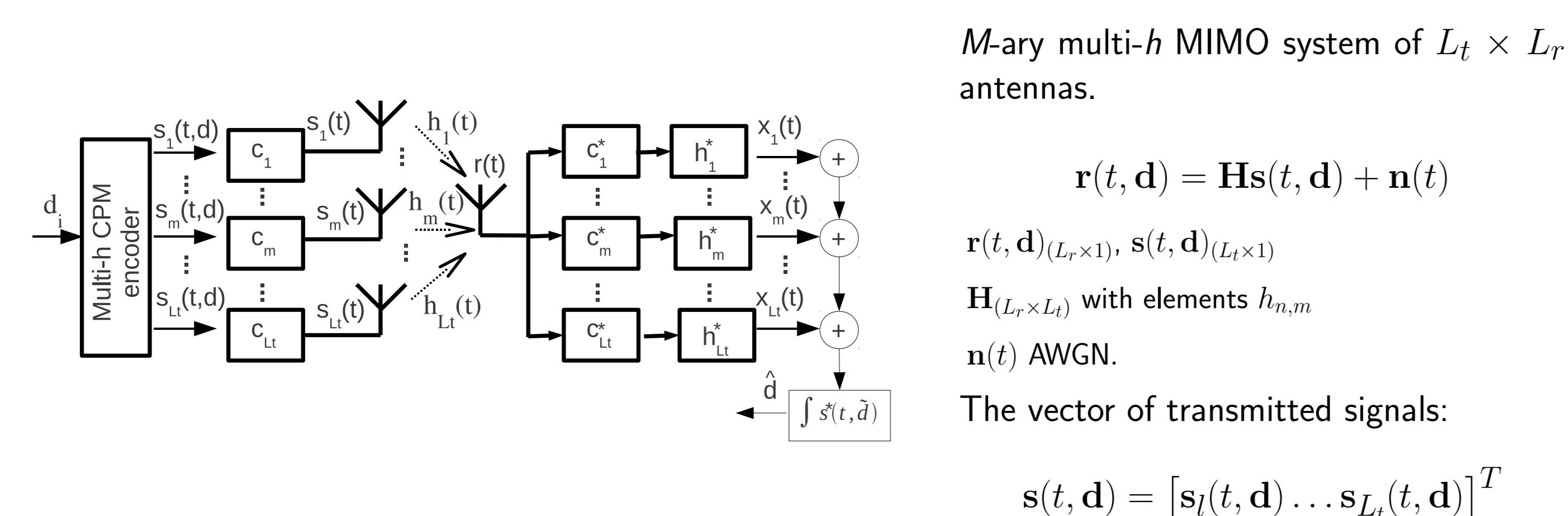
## Abstract

- ⊕ CPM: favorable trade-off between power and bandwidth efficiency.
- ⊕ Multi-h CPM: generalization to further decrease the need for bandwidth.
- ⊖ Difficult decoding in multi-path environments with no diversity.

How to overcome these limitations:

- ⊕ **IDEA:** To combine CPM with Space-Time Block Coding (STBC).
- Non trivial extension to  $L^2$ -orthogonal Space-Time codes provides full diversity and better spectral compactness.[1][2]
- Decoding complexity greatly decreased.[1][2]

## The System Model



## The baseband general form [3]

$$s(t, \mathbf{d}) = \sqrt{\frac{E_s}{T}} \exp(j\phi(t, \mathbf{d}))$$

The information-carrying phase function

$$\phi(t, \mathbf{d}) = 2\pi \sum_{k=0}^{K-1} h_{[k]} d_k q(t - (k-1)T)$$

modulation indices  $h_1, \dots, h_H$ , cycle in time with period  $H$  as:

$$[k] = \text{mod}(k, H) + 1$$

$h_{[k]}$  quotient between two relative prime integers.

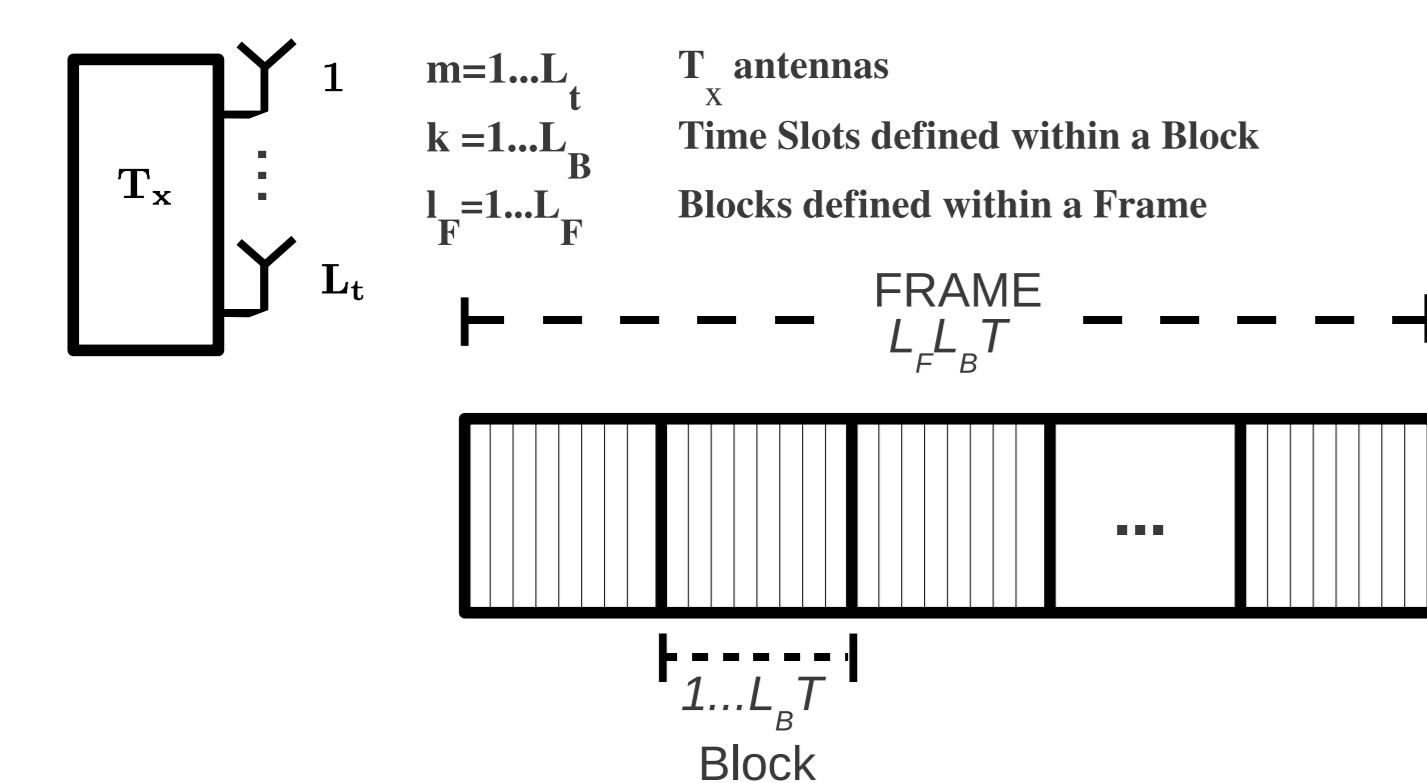
$$H_k = (h_1, h_2, \dots, h_k) = \left( \frac{p_1}{q}, \frac{p_2}{q}, \dots, \frac{p_k}{q} \right)$$

The phase continuity is ensured by the phase pulse  $q(t)$ ,

$$q(t) = \begin{cases} 0 & t < 0 \\ \bar{q}(t) & \text{elsewhere} \\ 1/2 & t \geq \gamma T \end{cases}$$

$\bar{q}(0) = 0$ ,  $\bar{q}(\gamma T) = 1/2$  and  $\lim_{t \rightarrow \gamma T} \bar{q}(t) = \bar{q}(\gamma T)$  for  $0 \leq \tau < \gamma T$ .  $\gamma$  is the overlapping factor and the symbol index is given by  $k = \lfloor t/T \rfloor$

## $L^2$ Orthogonality



The phase continuity is ensured by an antenna dependent phase memory:

$$\theta_{m,k} = \theta_{m,k-1} + \frac{h_{[k-\gamma]}}{2} d_{m,k-\gamma} + c_{m,k}(T) - c_{m,k}(0).$$

Auto-correlation coefficients cancel and cross-correlation coefficients between antennas put to 0

$$\sum_{k=1}^{L_B} \int_{(k-1)T}^{kT} \exp(j2\pi[\theta_{m,k} + \sum_{i=k-\gamma+1}^k h_{[i]} d_{m,i} q(t - (i-1)T) + c_{m,k}(t) - \theta_{m',k} - \sum_{i=k-\gamma+1}^k h_{[i]} d_{m',i} q(t - (i-1)T) - c_{m',k}(t)]) dt = 0$$

To ease the design, two assumptions are introduced,

1.  $c_{m,k}(t) = c_{m,k'}(t)$
2.  $d_{m,k} = d_{m',k}$

For any arbitrary number of transmit antennas, we introduce correction functions  $c_{m,k}$  as:

$$c_m^{lin}(t) = \frac{m-1}{L_B T} t \quad \text{for } (k-1)T < t < kT$$

For  $m = 1, \dots, L_t$  and the transmitted signal takes the form of

$$s_m(t, \mathbf{d}) = s(t, \mathbf{d}) \exp(j2c_m^{lin}(t))$$

## $L^2$ Decoding

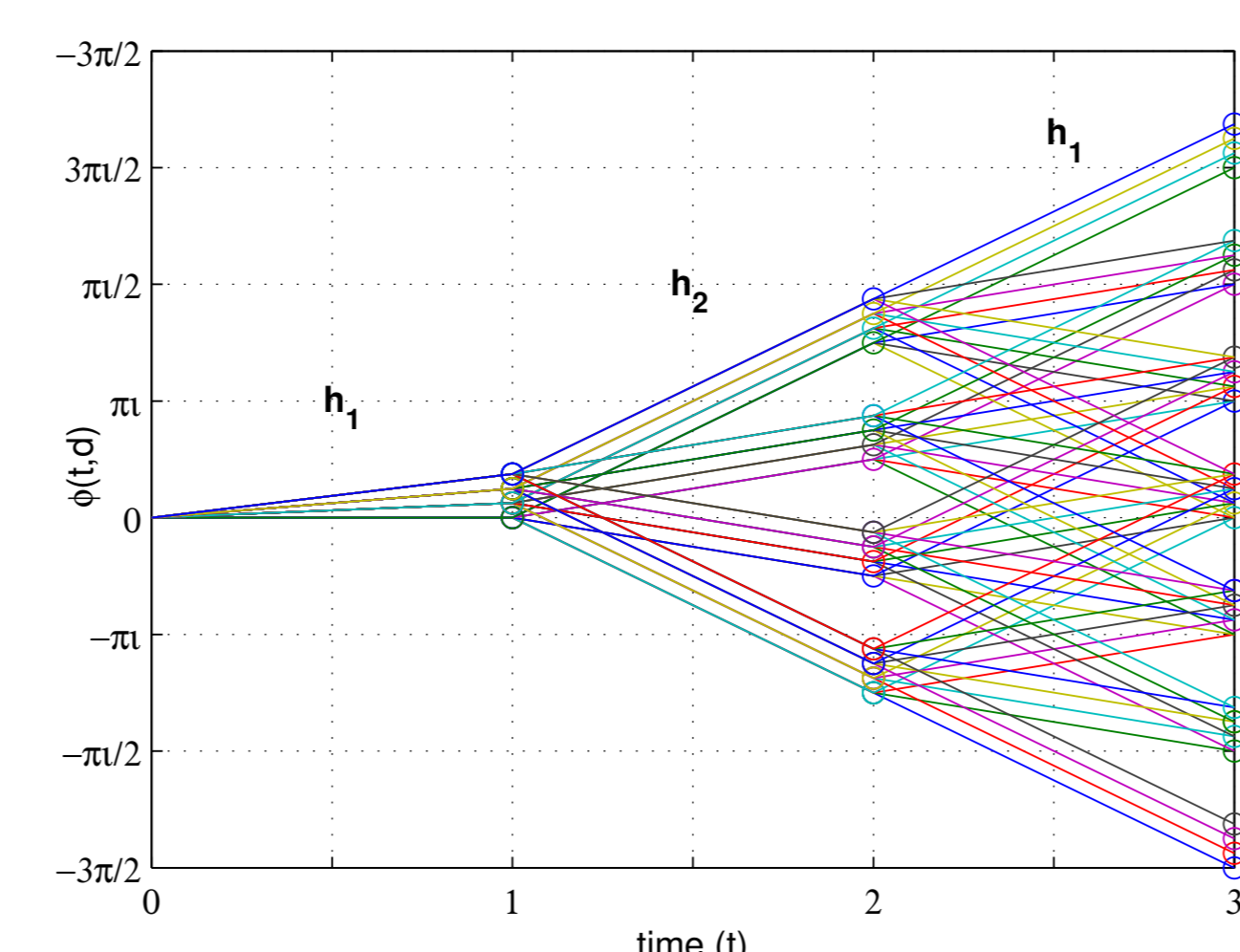


FIG. 1: Inner-block phase transition for  $h_1 = \frac{1}{4}$  and  $h_2 = \frac{3}{4}$ .

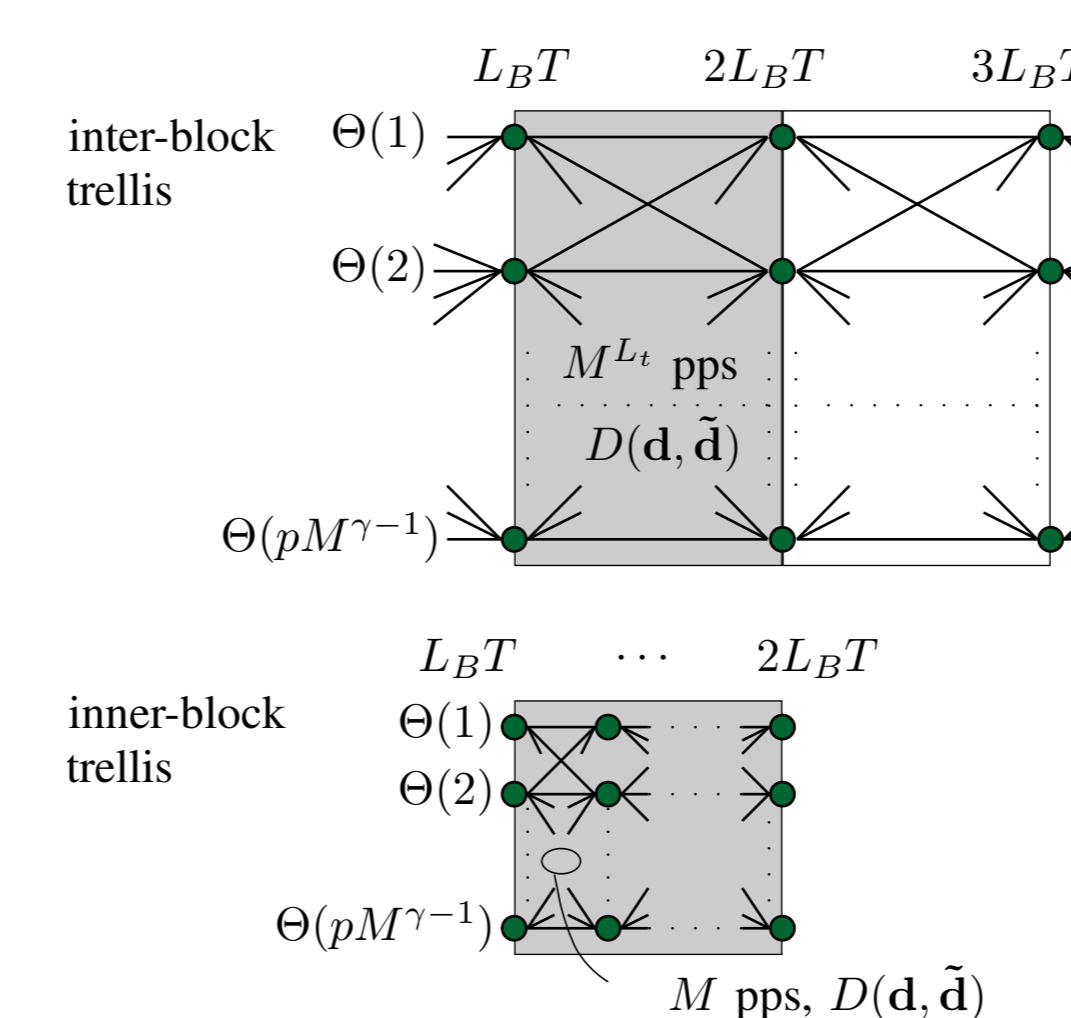


FIG. 2: Simplified detection with  $k = 1$  ( pps-paths per state).

Classical correlation based multi-h detector expressed blockwise:

$$D(\mathbf{d}, \tilde{\mathbf{d}}|\Theta(k)) = \sum_{k=1}^{L_F} \sum_{n=1}^{N_s} \int_{kT L_B}^{(k+1)T L_B} \text{Re} \left\{ r(t, \mathbf{d}_{[n]}) \dots \mathbf{d}_{L_B[n]} \cdot \left( \sum_{m=1}^{L_t} h_m^* s_m^*(t, \theta, \tilde{\mathbf{d}}_{[n]}) \right) \right\} dt$$

⊖ **Impractical:**

- $pM^{(L_t L_B N_s + \gamma - 1)}$  matched filters of length  $N_s L_B T$  employed  $pM^{\gamma-1}$  times.
- $pM^{L_B-1}$  states and  $M^{L_B T}$  symbols to decode with  $N_s$  samples each.

⊕ **Orthogonality:** the cross correlation terms are canceled out with blockwise decoding.

$$D_B(\mathbf{d}, \tilde{\mathbf{d}}|\Theta(k)) = \sum_{m=1}^{L_t} \int_{(l-1)T}^{lT} \text{Re} \{ r(t, \mathbf{d}) h_m^* c_m^*(t) s^*(t, \tilde{\mathbf{d}}) \} dt.$$

To get decoding with complexity **growing linearly** w.r.t number of transmit antennas, we introduce

$$x(t, \mathbf{d}) = r(t, \mathbf{d}) \sum_{m=1}^{L_t} h_m^* c_m^*(t).$$

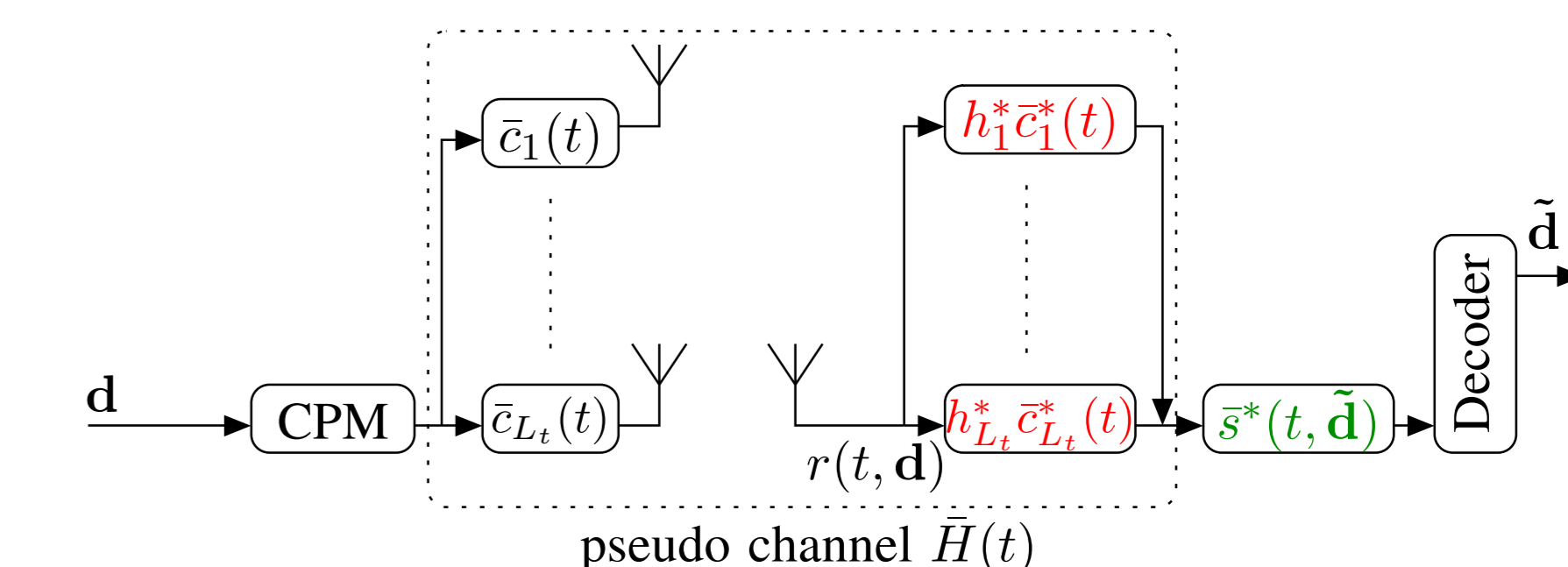
A simplified expression for a classical multi-h decoder [4] then is given by

$$D(\mathbf{d}, \tilde{\mathbf{d}}|\Theta(k)) = \int_{(l-1)T}^{lT} \text{Re} \{ x(t, \mathbf{d}) \cdot s(t, \tilde{\mathbf{d}})_{\Theta(k)} \} dt$$

Hence our detector based on the correlations gives us an expression which maximizes the minimum distance as

$$D_r(\mathbf{d}, \tilde{\mathbf{d}}|\Theta(k)) = \arg \max_{\Theta(1) \rightarrow \Theta(pM^{\gamma-1})} \left\{ \int_{(l-1)T}^{lT} \text{Re} \{ x(t, \mathbf{d}) s(t, \tilde{\mathbf{d}}) \} dt \right\}.$$

The simplified detector can now be represented in a similar form as in [5],



## Simulation Results

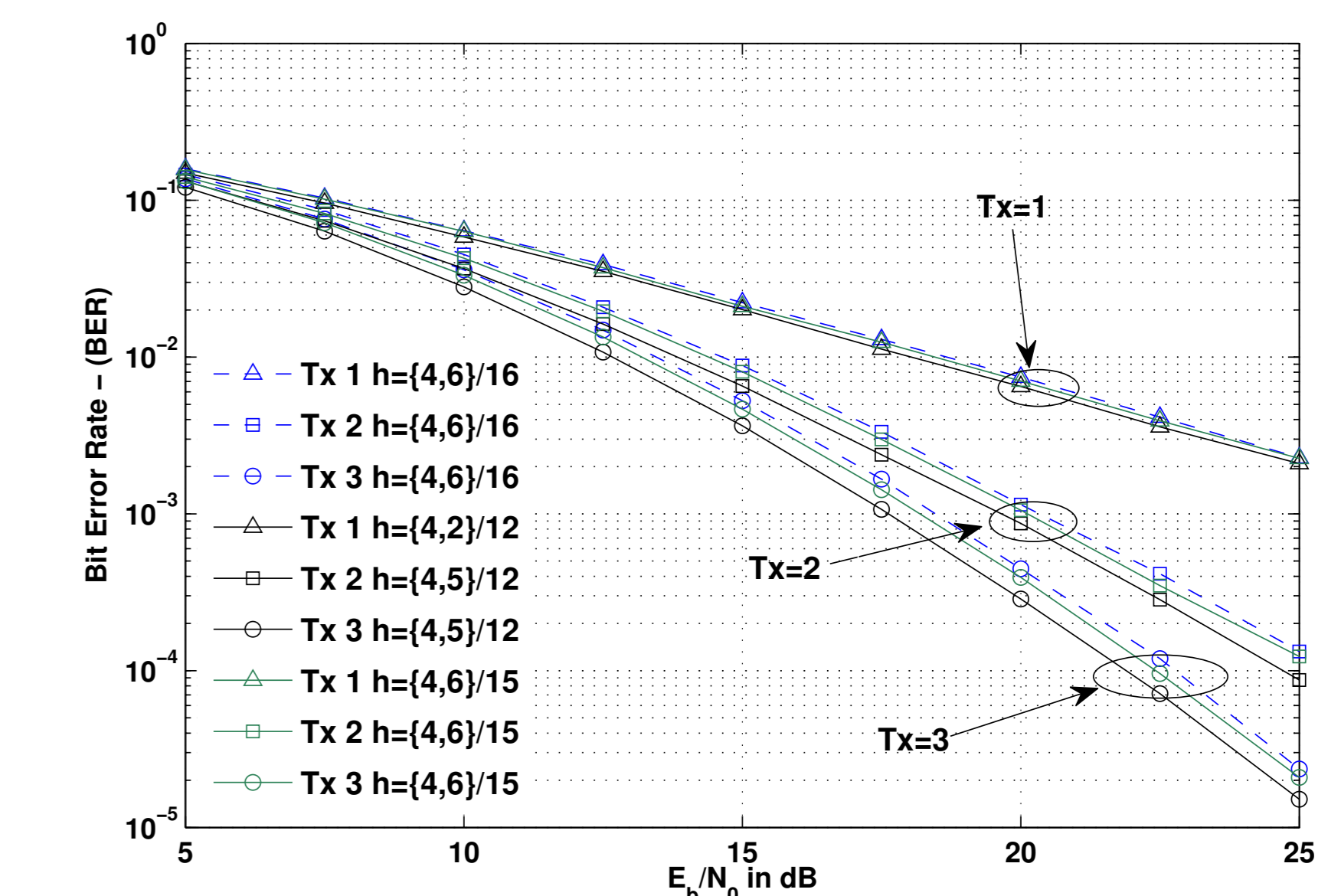


FIG. 3: BER for  $T_x = 1, 2, 3$ ,  $L_B = 2$  and  $h_i = \{4, 5\}/12$ ,  $h_i = \{4, 6\}/16$ ,  $h_i = \{4, 6\}/15$ .

## Conclusions

- ✓  $L^2$ -orthogonal STBC provide full diversity by the construction of orthogonal waveforms.
- ✓ We have shown that the inner trellis is equivalent to the inter-trellis for multi-h CPM.
- ✓ By choosing  $H \leq L_B$  we get the largest minimum distance in a block.
- ✓  $L^2$ -orthogonal STBC satisfies the needs for energy and spectral efficiency.
- ✓ Decoding complexity increases as  $PM$  instead of  $PMM^{L_T}$ .

## References

- [1] M. Hisojo, J. Lebrun, and L. Deneire. Wireless robotics: Generalization of an efficient approach with multi-h CPM signaling and  $L^2$ -Orthogonal space-time coding. *Wireless Personal Communications*, 2013.
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- [4] J. B. Anderson and D. P. Taylor. A bandwidth-efficient class of signal-space codes. *IEEE Trans. Inf. Theory*, 1978.
- [5] M. Hesse, J. Lebrun, and L. Deneire.  $L^2$ -Orthogonal ST-Code design for CPM. *IEEE Trans. Commun.*, 2011.