Universal lossless coding with random user access: the cost of interactivity
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ABSTRACT
We consider the problem of video compression with free viewpoint interactivity. It is well believed that allowing the user to choose its view will incur some loss in terms of compression efficiency. Here we derive the complete rate-storage region for universal lossless coding under the constraint of choosing the view at the receiver. This leads to a counterintuitive result: freely choosing its view at the receiver incurs a loss in terms of storage only and not in the transmission rate. The gain of the optimal scheme with respect to interactive schemes proposed so far is derived and a practical scheme that achieves this gain is proposed.

Index Terms— Free viewpoint, Source coding with side information, Distributed video coding.

1. INTRODUCTION
Free viewpoint television [1] is a new paradigm, in which users can interact with the server and request in real-time a desired viewpoint. The targeted applications are numerous, especially when 3D scenes contain some localized points of interest such as sport, concert or cultural events. Enabling users to interactively navigate through different viewpoints of a static scene is thus a new interesting functionality that however imposes new challenges for 3D streaming systems. In particular, the encoder must prepare a priori a compressed media stream that is flexible enough to enable the free selection of the viewpoint by the users.

A very first approach to offer interactivity, would be to encode each frame of each view independently (in an Intra mode). However, this would be inefficient in terms of compression since the correlation between successive frames and parallel views would not be exploited. On the other hand, non-interactive compression schemes (traditional, scalable or multiview), which do exploit these correlations, can not be used in an interactive scenario. Indeed, all share the property that the encoder knows perfectly the status of the decoder, i.e. the less correlated frame is available at the decoder. This raises an interesting question: does this worst case scheme achieve the best compression/storage performance?

To answer this question, we introduce two distinct criteria to measure the efficiency of an interactive compression scheme. First, the storage corresponds to the average number of bits per source symbol needed to be stored at the server. Second, the rate corresponds to the average number of bits per source symbol sent to the user upon request. We derive the whole achievable rate-storage region, and show that a frame has to be stored with the worst case assumption, but can be sent according to the best rate i.e. as if the encoder would know, which frames are available at the decoder. Therefore, it is possible to further improve [6]. In fact, interactivity incurs a loss, with respect to non-interactive schemes, in the storage only but not in the transmission rate, as intuition might suggest.

2. PROBLEM STATEMENT AND SOURCE MODEL
In free viewpoint video coding, M views \( \{X_i\}_{i \in [1,M]} \) are available and can be chosen freely by the user, as shown in Figure 1. The source \( X_i \) stands for a frame, while a realization of this source corresponds to a pixel value. At a given time
Definition 1. (Static source). A static source \((X, Y)\) is a discrete sequence of i.i.d. random variables drawn according to a distribution chosen randomly in the finite set of probability distributions \(\{P(X, Y)\} \in \{1, N\}\). The choice of one distribution, say \(P(X, Y)\), is made with probability \(P(J = j) = p_j\). The source is completely determined by \(P = \{P(X, Y), p_j\} \in \{1, N\}\).

The static source is the static source with prior defined in [7] to model SI uncertainty at the encoder. This source differs from a mixture model, since the probability chosen, will remain the same for the whole sequence. Therefore, the static source is stationary but not ergodic.

Definition 2. (Universal lossless source coding with random \(Y_i\)). Let \((X, Y)\) be a static source determined by \(P = \{P(X, Y), p_j\} \in \{1, N\}\).

Lossless source coding with random user access (see Figure 2) is the problem of compressing the source \(X\) knowing that some SI \(Y\) will be available at the decoder, storing the compressed bitstream with \(S\) bits per source symbol, and sending a bitstream extracted from the stored one, at a rate of \(R\) bits per source symbol, once the SI has been revealed to the encoder. The encoder knows the set of possible SIs i.e. for each \(j\), the sequence of realizations for the source \(Y_j\). This set of sequence realizations is denoted \(\{Y_j\} \in \{1, N\}\). Moreover, the scheme is universal in the sense that the source statistics \(P)\) are neither known at the encoder nor at the decoder.

Definition 3. (Achievable rate-storage pair). Let \((X, Y)\) be a static source determined by \(P = \{P(X, Y), p_j\} \in \{1, N\}\). A rate-storage pair \((R, S)\) is said to be achievable for universal lossless source coding with random user access, if, for a request \(j \in [1, N]\), there exists a sequence of encoder, bit extractor and decoder (indexed by the length of the source \((X, Y)\)) that can reproduce the source \(X\) (i.e. \(\hat{X} = X\)), as the sequence length of the source \((X, Y)\) goes to infinity.

Definition 4. (The rate-storage region). The rate-storage region of the universal lossless source coding problem under random user access is the closure of the set of achievable rate-storage pairs \((R, S)\).

3. COST OF INTERACTIVITY

In this section, we derive the rate storage region as defined in the above sections. This allows us to characterize the true cost of interactivity for lossless coding. Then, we compare some existing schemes with the optimal scheme, that achieves the best rate-storage pair.

Theorem 1. (The rate-storage region). Let \((X, Y)\) be a static source determined by \(P = \{P(X, Y), p_j\} \in \{1, N\}\). We consider universal lossless source coding under random user access as in Figure 2. For a request \(j\), the region of achievable (rate-storage) pair \((R, S)\) is

\[
R \geq H(X|Y_j) \quad (1)
\]

\[
S \geq \max_{k: p_k > 0} H(X|Y_k) \quad (2)
\]

where \(H(X|Y_j)\) stands for the conditional entropy of the source \(X\) given \(Y_j\). The region of achievable rate-storage is shown in Figure 3 and corresponds to the white area.

Proof: Achievability: inner bound. Non universal case. Let us first assume that the statistics \(P\)
are known at the encoder and decoder. [8] proposes an incre-
mental coding strategy for broadcasting a source $X$ losslessly
to a number of receivers with different qualities of SI, denoted
$(Y_1, \ldots, Y_N)$. For each sequence of realizations of the source
$X$, the encoder sends an index (from the least to the most
significant bit), whereas the receivers simply “tune out” once
they can decode. More precisely, receiver with SI $Y_j$ tunes
out when it has received $H(X|Y_j)$ bits per symbol.
This scheme can be used in our setup: the tuning out is
performed by the bit extractor. This is possible since the
SI might be $S \leq H(X)$, whereas the receivers simply “tune out” once
they can decode. More precisely, receiver with SI $Y_j$ available at the decoder. It therefore knows how
many bits are required for the decoder to succeed. Therefore,
$R = H(X|Y_j)$ is achievable. Moreover, for this
code, the total number of bits to be stored corresponds to
the worst case i.e. $S = \max_{k; p_k > 0} H(X|Y_k)$. Therefore
$(R, S) = (H(X|Y_j), \max_{k; p_k > 0} H(X|Y_k))$ is achievable.
This point is labeled (1) in the rate-storage region of Fig. 3.
Generalization of the scheme [8] to the universal case: The
encoder first sends the empirical conditional distribution
$P(X)$ (usually referred to as type). Then, the encoder uses
the incremental scheme described above. The type scales
logarithmically with the source sequence length, whereas the
data grows linearly. Therefore, the universal scheme will
asymptotically achieve the same rate-storage pair as the non-
universal scheme.

Converse: outer bound. Here, we use the converse of less
stringent theorems, where more information is available at
the encoder and/or decoder.

- $R \geq H(X|Y_j)$. Proof by contradiction, by invoking the

 converse of the Slepian-Wolf theorem [9] with perfectly

 known SI statistics at the encoder.

- $S \geq \max_{k; p_k > 0} H(X|Y_k)$. Proof by contradiction. If

 $S < \max_{k; p_k > 0} H(X|Y_k)$, then with non zero probability,

 the SI might be $Y_k$, where $k = \arg \max_{k; p_k > 0} H(X|Y_k)$. From

 the converse of the Slepian Wolf theorem [9], this source

 can not be recovered at the decoder.

It is interesting to compare the optimal scheme for uni-
versal lossless coding with random user access (point (1) in
Figure 3) to other existing schemes. Let us first consider the
scheme without interactivity. This will allow us to derive the
cost of interactivity.

Corollary 1. (Cost of interactivity). The cost of inter-
activity that allows each user to control the focus of at-
tention rather than receiving the one decided by a direc-
tor at the server, is in the storage only and is limited to
$\Delta S = \max_{k; p_k > 0} H(X|Y_k) - H(X|Y_j)$.

Proof: In the case without interactivity, the encoder knows
the SI $Y_j$ at the encoder and the source $X$ can be encoded
losslessly at rate $H(X|Y_j)$ [9], even in the case where the
statistics are neither known at the encoder nor at the decoder
[10]. Therefore, the achievable storage-rate pair satisfies
$R = S = H(X|Y_j)$.

The case without interactivity is labeled (3) in Figure 3.

4. PRACTICAL SCHEME
We now propose a practical scheme to solve the interactive
lossless source coding problem. First note that, to design an
optimal interactive lossless source coding scheme, it is nec-
essary to split the encoder into a compression and a transmis-
sion phase. If not, the sent bitstream is the stored one, and
the transmitted rate equals the storage, which corresponds to
the worst case (point (2) in Figure 3). The incremental code
proposed in [8] satisfies this splitting. However, it cannot be
used in practice since it relies on the random generation of
the code. But, as noticed in [8], our problem is equiva-

lent to a scheme where the source $X$ is sent unprotected on a
broadcast channel, which produces a set of outputs $\{Y_j\}$ (one
output per receiver). Then, some additional bits are sent on a
perfect channel. Each receiver stops gathering bits once it can
reconstruct $X$. Moreover, there exists codes known as Digital
Fountain codes [12] that can solve this problem.
ENCODER: compression
Data: $X, Y_1, \ldots, Y_J, \ldots, Y_N$
Result: stored bitstream (S)
order all $Y_j$ s.t. $H(X|Y_1) < \ldots < H(X|Y_N)$;
choose a systematic channel code of rate $r = \frac{1}{1+e_N}$, where $e_N$ is the proportion of erased symbols in the largest occlusion area;
for each bitplane $B_j$ of $X$
    encode $B_j$ with rate $r$ and keep only the parity bits;
    for each SI $Y_j$
        determine the number of parity bits $b_j$ s.t. $Y_j$
        and $b_1 + \ldots + b_j$ parity bits are sufficient to
        recover $X$;
        store $b_j$ bits plus a marker EOS;
end
ENCODER: transmission
Data: stored bitstream (S), $j$
Result: sent bitstream (R)
for each bitplane do
    send $b_1 + \ldots + b_j$ parity bits;
end
DECODER:
Data: sent bitstream (R)
Result: $X$
for each bitplane do
    estimate $X$ from $Y_j$ and the $b_1 + \ldots + b_j$ parity bits;
end
Algorithm 1: Practical scheme for lossless source coding with random user access

These Fountain codes were first proposed for the erasure channel. Interestingly, [13] shows that the correlation between the views can be modeled by an $m$-ary erasure channel. The erased symbols correspond in fact to the disoccluded pixels after view prediction. Therefore, the SI $Y_j$ can be seen as the output of an erasure channel with erasure probability $e_j$ and input, the source $X$. Let us assume that the source $X$ is i.i.d. uniformly distributed, and can take $m = 2^L$ values. The optimal transmitted rate is $H(X|Y_j) = e_j H(X) = e_j L$ i.e. exactly the amount of erased symbols.

Our scheme (see details in Algorithm 1) encodes the source $X$ with a systematic code and stores the necessary parity symbols. The rate code is chosen in order to deliver the number of parity bits for the maximal erasure probability (worst case). Then, upon receiver request, some parity bits, extracted from the parity bit sequence, are sent. If the number of parity bits extracted, exactly matches the bound $H(X|Y_j)$ for all $j$, then the code is said to be optimal for the source coding problem with random user access. For the erasure channel, there exist such codes, which are called Maximum Distance Separable codes. However, there decoding complexity is prohibitive. Instead, we choose the standardized LDPC-Staircase codes (RFC 6816) [14], that achieve a very good tradeoff between encoding/decoding complexity and performance. This choice is motivated by the fact that low decoding complexity is a key issue in interactive communication. Moreover, we choose binary codes since [15] shows the equivalence between the $m$-ary erasure channel and $L$ parallel binary erasure channels, where $m = 2^L$. Therefore, for our problem, codes for the binary erasure channel are sufficient to achieve the optimal rate-storage pair. An implementation is made available by the authors of the RFC at [16]. Furthermore, for complexity reason, and in order to allow parallel decoding of a frame, we split a frame into blocks of size 200x200. For instance, an HD frame (1920x1080) will result into 52 blocks.

Table 1 compares the theoretical transmission rate and the one obtained with our scheme based on the LDPC-Staircase codes (RFC 6816) [14]. The table shows that the overhead is very limited: only 0.275% (resp. 0.23%) more symbols is needed with respect to the optimal performance, when the occlusion area is up to 10% (25% resp.) of the image size.

<table>
<thead>
<tr>
<th></th>
<th>$e_1 = 1%$</th>
<th>$e_2 = 5%$</th>
<th>$e_3 = 10%$</th>
<th>Number Stored</th>
</tr>
</thead>
<tbody>
<tr>
<td>theoretical</td>
<td>400</td>
<td>2.000</td>
<td>4.000</td>
<td>4.000</td>
</tr>
<tr>
<td>LDPC-Staircase</td>
<td>411</td>
<td>2.011</td>
<td>4.011</td>
<td>4.011</td>
</tr>
<tr>
<td>r=.909</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of symbols sent</td>
<td>Number Stored</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>theoretical</td>
<td>$e_1 = 10%$</td>
<td>$e_2=20%$</td>
<td>$e_3 = 25%$</td>
<td>10.000</td>
</tr>
<tr>
<td>LDPC-Staircase</td>
<td>4.023</td>
<td>8.023</td>
<td>10.023</td>
<td>10.023</td>
</tr>
</tbody>
</table>

Table 1. $X$ is a block of $200 \times 200 = 40.000$ symbols; the SI $Y_j$ can predict all $X$ except for a proportion of $e_j$ symbols. In the practical scheme, the number of sent and stored symbols, to recover $X$ from $Y_j$, is very close to the theoretical bound: $H(X|Y_j)$.

5. CONCLUSION

In this paper, we studied video compression with free viewpoint interactivity. For lossless compression, we derived the optimal performance under the assumption that the statistics of the source are neither known at the encoder nor at the decoder. We showed that the optimal scheme needs to store the data according to the worst case (as if the transmitter would not use the previous user requests) but sends the data at the same rate as a non-interactive scheme. Therefore, interactivity incurs a loss in terms of storage only. The gain with respect to interactive schemes proposed so far was derived. Finally, a practical scheme that achieves this gain was proposed in the case of i.i.d. sources. Future work will include the design of codes for multiview videos.
6. REFERENCES


