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Layer-averaged Euler and Navier-Stokes equations

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Abstract

In this paper we propose a strategy to approximate incompressible free surface Euler and Navier-Stokes models. The main advantage of the proposed models is that the water depth is a dynamical variable of the system and hence the model is formulated over a fixed domain.

The proposed strategy extends previous works approximating the Euler and Navier-Stokes systems using a multilayer description. Here, the needed closure relations are obtained using an energy-based optimality criterion instead of an asymptotic expansion. Moreover, the layer-averaged description is successfully applied to the Navier-Stokes system with a general form of the Cauchy stress tensor.

Keywords : Incompressible Navier-Stokes equations, incompressible Euler equations, free surface flows, newtonian fluids, complex rheology

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1 Introduction

Due to computational issues associated with the free surface Navier-Stokes or Euler equations, the simulations of geophysical flows are often carried out with shallow water type models of reduced complexity. Indeed, for vertically averaged models such as the Saint-Venant system [7], efficient and robust numerical techniques (relaxation schemes [9], kinetic schemes [23, 2],...) are available and avoid to deal with moving meshes.

In order to describe and simulate complex flows where the velocity field cannot be approximated by its vertical mean, multilayer models have been developped [1, 3, 4, 8, 13, 12]. Unfortunately these models are physically relevant for non miscible fluids.

In [14, 6, 5, 24], some authors have proposed a simpler and more general formulation for multilayer model with mass exchanges between the layers. The obtained model has the form of a conservation law with source terms, its hyperbolicity remains an open question. Notice that in [5] the hydrostatic Navier-Stokes equations with variable density is tackled and in [24] the approximation of the non-hydrostatic terms in the multilayer context is studied.

The objective of the paper is twofold. First we want to present another derivation of the models proposed in [6, 5, 24], no more based on an asymptotic expansion but on an energy-based optimality criterion. Such a strategy is widely used in the kinetic framework to obtain kinetic descriptions e.g. of conservations laws [18, 23]. Second, we intend to obtain a multilayer formulation of the Navier-Stokes system with a rheology more complex than the one arising when considering newtonian fluids.

The paper is organized as follows. In Section 2 we recall the incompressible hydrostatic Navier-Stokes equations with free surface with the associated boundary conditions. In Section 3 we detail the layer averaging process for the Euler system and obtained the required closure relations. The proposed layer-averaged Euler system is given in Section 4 and its extension to the Navier-Stokes system with a general rheology is presented in Section 5.