

Abstract

Cloth dynamics plays an important role in the visual appearance of moving characters. Properly accounting for frictional contact is of utmost importance to avoid cloth-body penetration and to capture folding behavior due to dry friction. We present here the first method able to account for contact with **exact Coulomb friction** between a cloth and the underlying character. Our key contribution is to formulate and solve the frictional contact problem merely on velocity variables, by leveraging some tools of convex analysis. Our method is both fast and robust, allowing us to simulate full-size garments with more realistic body-cloth interactions compared to former methods, while maintaining similar computational timings.



Cloth design and body animation by Laurence Boissieux

2. Primal formulation of the constrained dynamics

Hypothesis: **body-cloth contacts occur at cloth vertices only.**

Then each nonzero block of J related to a vertex i is a rotation matrix E_i describing the local contact basis. We can build the square block-diagonal matrix G with

$$G_{i,i} := \begin{cases} E_i & \text{if } i \text{ in contact} \\ \mathbb{I}_{3 \times 3} & \text{otherwise.} \end{cases}$$

G can be inverted trivially as $G^{-1} = G^\top$. Augmenting u and r so that their size matches that of v , we can write (1) as

$$\begin{cases} GMG^\top u + Gf = r \\ v = G^\top u \\ (u_i, r_i) \in \mathcal{C}(\mu_i) & \text{if } i \text{ in contact} \\ r_i = 0 & \text{otherwise.} \end{cases} \quad (2)$$

Denoting $\widehat{W} := GMG^\top$ and $b := Gf$, we obtain a system that is similar to the dual formulation of Daviet et al. (2011), except that u and r have reversed roles. Unlike the Delassus operator W , our new operator \widehat{W} is **sparse and easy to assemble.**

1. Related work

Contact and friction are commonly accounted for in cloth simulations through an iterative **velocity filtering process** (Bridson et al., 2002).

At each time step:

- ▶ the cloth dynamics are first resolved as if no contact were applied by solving $Mv + f = 0$
- ▶ each cloth vertex has its velocity sequentially modified so as to satisfy non-penetration and Coulomb frictional constraints locally.

Upon convergence, this method ensures a contact-free configuration at the end of each time step, but **fails to yield a consistent mechanical state w.r.t. both internal energy and Coulomb constraints:**

- ▶ excessive bending energy may be accumulated around contacting vertices
- ▶ vertices may be artificially constrained to stick when they should slide.

Treating frictional contact implicitly alleviates such issues, since internal dynamics and frictional contacts are solved **simultaneously.**

With μ the coefficient of friction, the dynamics read

$$\begin{cases} Mv + f = J^\top r \\ Jv = u \\ (u, r) \in \mathcal{C}(\mu) \quad \text{(the Coulomb law).} \end{cases} \quad (1)$$

The classical way to solve this implicit formulation is to eliminate the velocities v (**primal variables**) by introducing the Delassus matrix $W := J^\top M^{-1} J$. This yields equations where the only remaining variables are the contact forces r (**dual variables**). In the case of fiber models, Daviet et al. (2011) introduced an efficient solver for this dual problem. **For cloth however, such a method is computationally inefficient**, as the inverse of the matrix M is dense, and therefore W as well.

3. Fixed-point algorithm

1. Get initial guess v^0 from unconstrained dynamics, $v^0 := -M^{-1}f$
 - Use iterative solver (GMRES)
2. Initialize $u^0 := Gv^0$ and $s_i^0 := \mu_i \|u_{i|_r}^0\| e_i \forall i$
3. For $k \geq 0$
 - a. Solve the minimization problem (3) at s^k
 - Leverage the solver from (Daviet et al., 2011)
 - Set the optimum as \tilde{u}^{k+1}
 - b. Compute $s_i^{k+1} := \mu_i \|\tilde{u}_{i|_r}^{k+1}\| e_i \forall i$
 - c. Break if $\|s^{k+1} - s^k\|$ is small enough
4. Compute $u := \tilde{u}^{k+1} - s^{k+1}$ and $v := G^\top u$
5. Proceed to next time-step

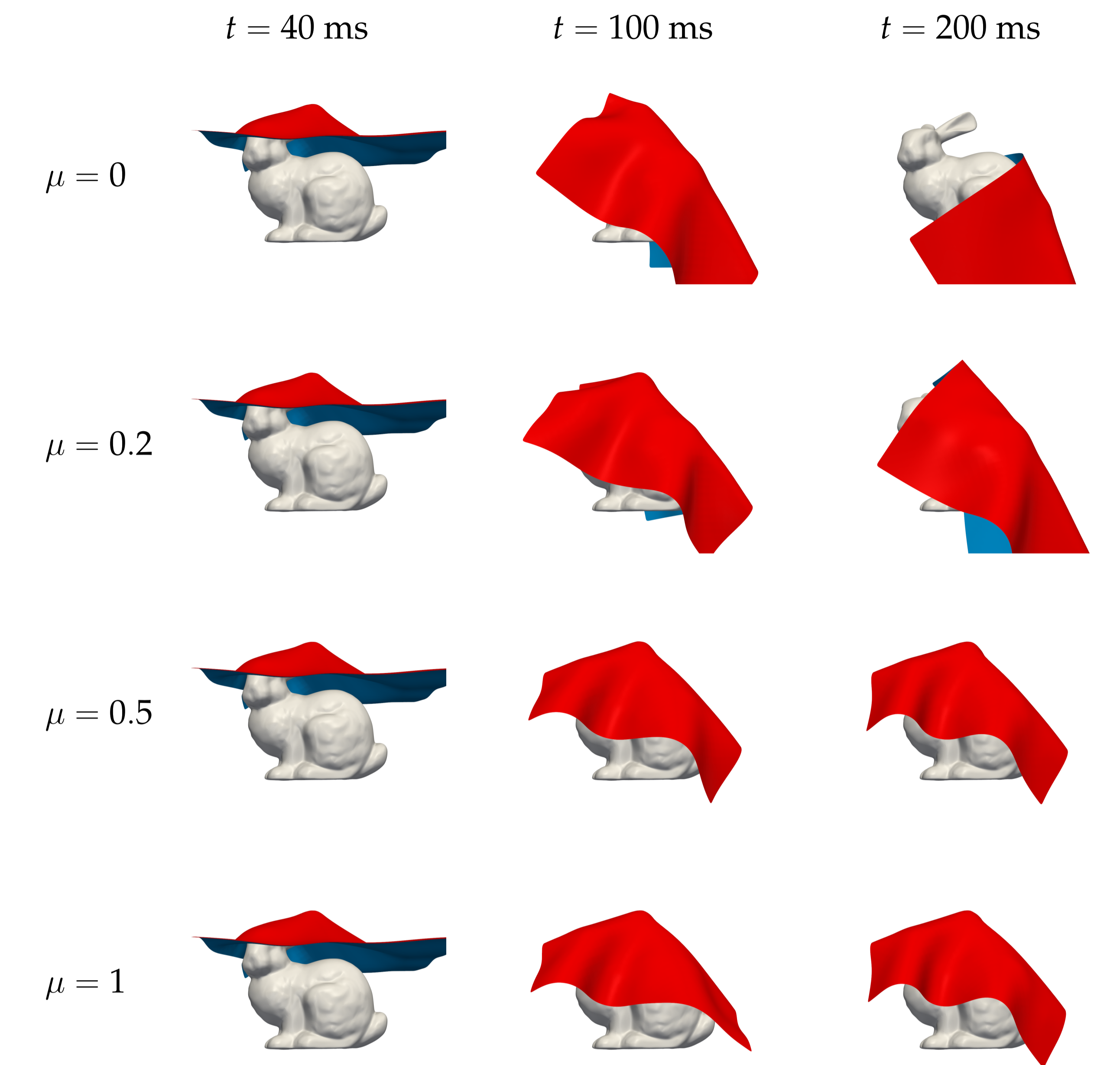
4. Results: Timings

Scene	m	n	Dual		Our method	
			Build	Solve	Build	Solve
Bunny	6500	169	2.25	0.017	0.02	0.074
Skirt	5208	741	76.7	0.23	0.074	0.14
Dress	6060	1086	157	0.19	0.077	0.048

m : number of vertices, n : mean number of contacts.

Times are in seconds.

5. Results: Influence of Coulomb friction



Perspectives

Direct solver

The dual formulation can also be decomposed as a sequence of convex optimization problems, but directly tackling (1) with a Gauss–Seidel algorithm turns out to be generally faster. Adapting the latter strategy to our primal formulation by devising an efficient one-contact solver for the primal case could lead to similar speedups.

Arbitrary contact points

An open question deals with the construction of an analogous matrix G with good properties:

- ▶ when the contact points have arbitrary positions on the mesh, i.e., are not located at vertices
- ▶ when taking into account contact between two simulated vertices, e.g., for self-contacting cloth.

References

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