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# Universal Robust Adaptive Control of Robot Manipulators Using Real Time Estimation

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**Abstract:** This paper proposes an universal adaptive control structure for robot manipulators, without knowing the dynamic model of the system, as well it is robust to corrupt payload change and initial conditions. It considers a simplified model to describe the robot dynamics, instead of the commonly used explicit dynamic model. The simplification allows to largely reduce the number of parameters to be updated. Moreover the simplified model should represent the current system dynamics, which can be ensured by a real time estimation of the model parameters. In this case, the corrupt change of payload will be detected within short time window such as 0.1 second, and the system dynamics will be adjusted quickly to real values. Modulating functions techniques are also applied on the real time estimation process, to decrease the order of input via integration by part method, which avoids using joint velocities and accelerations. Meanwhile the filtering property of modulating functions are studied so that groups of modulating functions are selected in order to eliminate the high frequency noise influence. In the end simulation results on a two degrees of freedom planar robot prove the control structure efficient.

*Keywords:* Adaptive control, real time estimation, modulating functions, frequency analysis.

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## 1. INTRODUCTION

Adaptive control theory has been investigated extensively in the past decades as an interesting approach to estimate or adjust on-line the dynamic parameter values used in the control, in case of inaccuracy in the dynamic parameters of the robot, high frequency unmodeled dynamics, variation in payload and mass of links and so on. This control scheme is effective for precise tracking task of robot manipulator in presence of parametric uncertainty, and there exist different approaches such as:

- simplification of dynamic model, see Morikazu and Suguru (1981);
- adaptive techniques designed for linear systems, see Hsia (1986);
- nonlinear linearizing adaptive control, see Craig et al. (1987);
- passivity-based adaptive control, see Slotine and Weiping (1988);
- adaptive fuzzy control, see Khalate et al. (2011).

However these methods are somehow based on the awareness of robot model and updating numerous parameters, which is complex and hard to be implemented in real time. In this paper we proposed an extremely simplified model to represent the manipulator's dynamics, where the

model parameters are time-varying. Compared with robot explicit dynamic model including inertia, first moments, masses and friction parameters, the number of parameters to be updated in this model is much reduced. This gives simplicity to multi-link case and advantages in parameters estimation, because with less parameters it requires less time consumption to get robust estimation. The validity of the simplified model is ensured by real time parameters estimation, where the time-varying parameters are approximated as constant or linear varying component in short time interval, according to their varying speed. The reconstruction of system dynamics is ensured once the estimation time is reduced to 0.1s as it is tested in simulation, so that it allows the estimation responds quickly to the dynamics variation and makes the adaptive control robust to corrupt change and initial conditions.

In the estimation process we utilize modulating function approach to avoid using joint velocities and accelerations. Commonly these two derivatives are computed from joint position, which causes problem in robot identification process because small error in measurement can induce large error in the computed derivatives, specially for high order derivatives. Thus it is better to use only joint position. The modulating function property plays an important role in reducing the order of inputs in the estimation model as in Liu et al. (2013). In robotics identification field, the mod-

ulating function technique is new and similar approach can be found in Guo et al. (2014). There exist all kinds of modulating functions, we will study their filtering property and select some groups of modulating functions which have low-pass filtering property.

This paper is organized as follows: section 2 deduces the simplified robot model from the robot explicit dynamic model and presents the design of the adaptive controller; section 3 gives precise description on real time estimation of model parameters using modulating functions, as well as the frequency analysis on modulating function filtering property, and different modulating functions are discussed; in section 4 simulation is carried out with a two degrees of freedom planar robot model, the simulation result shows that the adaptive control structure has good tracking precision and is robust to high frequency noise, corrupt change of system dynamics and initial conditions; and in last section it comes to a conclusion.

## 2. SIMPLIFIED MODEL AND ADAPTIVE CONTROLLER

In this section, we first provide the rigid-body dynamic model of manipulator and extend it to a simplified model with time-varying parameters. Then an adaptive controller is designed for this model.

The general form of the dynamic model can be deduced from Lagrangian formulation:

$$\tau = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) + \tau_f, \quad (1)$$

where  $\mathbf{q}$  is the  $n \times 1$  vector of joint position,  $\tau$  is the  $n \times 1$  vector of applied joint torques or force,  $\mathbf{M}(\mathbf{q})$  is the  $n \times n$  symmetric and positive definite inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$  is the  $n \times 1$  vector of Coriolis and centrifugal torques,  $\mathbf{Q}(\mathbf{q})$  is the  $n \times 1$  vector of gravity torques, and  $\tau_f$  is the  $n \times 1$  friction torques which is usually modelled at non zero velocity as a combination of Coulomb friction, viscous friction and an offset friction part which regroups the amplifier offset and the asymmetrical Coulomb friction. The dynamic parameters  $\mathbf{X}_{\text{dyn}}$  are linear with respect to dynamic model and the model can be reformulated as  $\tau = \mathbf{A}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{X}_{\text{dyn}}$ , where  $\mathbf{A}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  is the observation matrix. Without considering the regrouping rule, there exist 13 dynamic parameters for each link, 6 inertial parameters, 3 first moment parameters, 1 mass parameter and 3 friction parameters.

### 2.1 Simplified Model

The analytical expression of dynamic model is complex and the unknown dynamic parameters are numerous. This brings difficulty to estimation because it need rich measurements to well identify each value of the parameters, which usually cannot be implemented on-line. In Craig et al. (1987) the authors proposed to linearize of the robot dynamic model and implement the classic Lyapunov based adaptive controller to compute the variations of model unknowns. This paper will use the same model but update the model unknowns by estimating them on-line. The simplified model with fewer parameters writes as follows:

$$\tau = \mathbf{M}(\mathbf{t})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{t}). \quad (2)$$

The equivalence of these models exists when it satisfies:  $\mathbf{M}(\mathbf{t}) = \mathbf{M}(\mathbf{q}(\mathbf{t}))$  is a  $n \times n$  symmetric and positive definite

inertia matrix, and  $\mathbf{N}(\mathbf{t})$  is a  $n \times 1$  vector contains other components of the manipulator dynamics. The simplicity costs that model parameters  $\mathbf{M}(\mathbf{t})$  and  $\mathbf{N}(\mathbf{t})$  are considered time-varying.

### 2.2 Controller Design

The design problem can be as follows: given the reference trajectories  $\mathbf{q}_{\text{ref}}(t)$ ,  $\dot{\mathbf{q}}_{\text{ref}}(t)$  and  $\ddot{\mathbf{q}}_{\text{ref}}(t)$  of position, velocity and acceleration respectively, without knowing the robot model, derive an adaptive control law for the actuator torques, and a real time estimation scheme for the adaptive components, such that the manipulator joint position  $\mathbf{q}(t)$  precisely tracks  $\mathbf{q}_{\text{ref}}(t)$  after an initial adaptation process. In order to design such a controller, we propose the following theorem.

*Theorem 1.* The controller

$$\tau = \mathbf{M}(\mathbf{t})(\ddot{\mathbf{q}}_{\text{ref}} - \lambda_d \dot{\mathbf{e}} - \lambda_p \mathbf{e} - \lambda_i \int_0^T \mathbf{e}) + \mathbf{N}(\mathbf{t}), \quad (3)$$

with proper PID gains  $\lambda_p$ ,  $\lambda_i$  and  $\lambda_d$ , is sufficient to asymptotically stabilize the tracking error  $\mathbf{e} = \mathbf{q} - \mathbf{q}_{\text{ref}}$  for the system modeled by (2).

**Proof.** At instant  $t$ , we can replace the computed motor torque  $\tau$  in equation (3) by robot model (2), and it deduces a state equation for tracking error  $\mathbf{e}$

$$\mathbf{M}(\mathbf{t})(\ddot{\mathbf{e}} + \lambda_d \dot{\mathbf{e}} + \lambda_p \mathbf{e} + \lambda_i \int_0^T \mathbf{e}) = 0. \quad (4)$$

Since  $\mathbf{M}(\mathbf{t})$  is always invertible (positive definite), the state equation becomes

$$\ddot{\mathbf{e}} + \lambda_d \dot{\mathbf{e}} + \lambda_p \mathbf{e} + \lambda_i \int_0^T \mathbf{e} = 0. \quad (5)$$

Derive equation (5) and we get a third order differential equation

$$\mathbf{e}^{(3)} + \lambda_d \ddot{\mathbf{e}} + \lambda_p \dot{\mathbf{e}} + \lambda_i \mathbf{e} = 0. \quad (6)$$

The asymptotically stability and convergence rate of tracking error  $\mathbf{e}$  can be ensured and are tunable by selecting the PID parameters  $\lambda_p$ ,  $\lambda_i$  and  $\lambda_d$ .

$\mathbf{M}(\mathbf{t})$  and  $\mathbf{N}(\mathbf{t})$  should be accurately estimated as  $\bar{\mathbf{M}}$  and  $\bar{\mathbf{N}}$  within small time interval, which will be discussed in the next section. This control scheme is easy to implement for robot manipulators without knowing the dynamic model of the robot, and simplicity in the model contributes to realize real time estimation. In return real time estimation offers quick response to variation of system dynamics as well as the initial conditions. Figure 1 shows the structure of the adaptive controller.

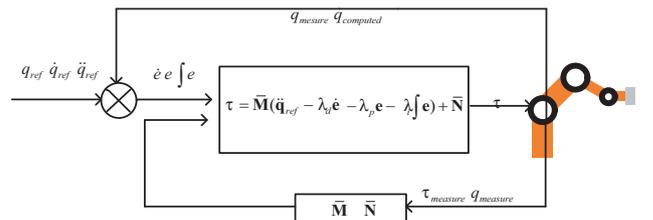


Fig. 1. Structure of adaptive controller

### 3. REAL TIME ESTIMATION

According to the model description (2), the conventional estimation approach needs the joint torques  $\tau$  and accelerations  $\ddot{\mathbf{q}}$ . Usually  $\tau$  are calculated from the current reference of the amplifier current loop and the gain of each joint drive chain. And  $\ddot{\mathbf{q}}$  are computed from discrete joint position measurement via robot sensor, see Gautier (1996). But reconstruction of high order derivatives from noisy data is long standing problem because noise component will be enlarged exponentially with increasing order during the numerical computation. Various numerical methods have been developed to obtain stable algorithms robust to additive noise, for example finite difference methods in Khan and Ohba (2000), wavelet differentiation methods in Shao and Ma (2003), Fourier transform methods in Fu et al. (2010), algebraic methods in Mboup et al. (2009), Liu et al. (2011) and so on. As robust differentiator, these approaches are complex to implement and time-consuming. Moreover most of them are applicable only for off-line cases, and the on-line differentiators usually induce a shift delay, which calculate the derivatives at the past time. These drawbacks make them less applicable for real time estimation.

Above all, we propose a modulating functions based structure regarding to the simplified model, which decreases the input order of derivatives in the estimation process. And their frequency domain properties are investigated so that they can be implemented as filtering tools.

#### 3.1 Modulating Functions

Let  $k \in \mathbb{N}^+$ ,  $T \in \mathbb{R}^+$ , and  $g$  be a function satisfying the following properties:  $g \in \mathcal{C}^k([0, T])$ ,  $g^{(i)}(0) = g^{(i)}(T) = 0$ , for  $i = 0, 1, \dots, k-1$ , where  $\mathcal{C}^k([0, T])$  refers to the set of functions being  $k$ -times continuously differentiable on  $[0, T]$  with  $k \in \mathbb{N}^+$ . Then  $g$  is called  $k^{\text{th}}$  **order modulating function** on  $[0, T]$ .

Modulating functions transform a differential expression into a sequence of algebraic equations using noisy data signals. Their filtering property makes this method interesting in several real processes. For years many authors have focused on the choice of different modulating functions types such as sinusoid modulating functions, polynomial modulating functions, Hermite functions in Jordan and Paterson (1986), Fourier modulating functions in Pearson and Lee (1985), Hartley modulating functions in Unbehauen and Rao (1997), Fedele and Coluccio (2010) and spline-type functions in Fedele et al. (2009).

#### 3.2 Estimation Model

Recall the simplified model (2). To update the parameters we need to discretize them. Regarding to small time window, we assume that  $\mathbf{M}(\mathbf{t})$  and  $\mathbf{N}(\mathbf{t})$  are approximated as a constant  $\bar{\mathbf{M}}$  and a linear relation  $\bar{\mathbf{N}} = \bar{\mathbf{N}}_0 + \bar{\mathbf{N}}_1 \mathbf{t}$ . More precisely, the inertia matrix  $\mathbf{M}(\mathbf{t})$  is a function of  $\mathbf{q}$  which can be considered constant in short time interval; while the vector  $\mathbf{N}(\mathbf{t})$  is a quadratic functions of  $\mathbf{q}$  and  $\dot{\mathbf{q}}$  whose variation cannot be ignored even in small time interval, in this case it can be treated as linear component. Thus the model rewrites as:

$$\tau = \bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{N}}_0 + \bar{\mathbf{N}}_1 \mathbf{t}. \quad (7)$$

In this case these parameters can be update in real time to reconstruct system dynamics. The number of parameters is greatly reduced so that for a  $n$ -link manipulator, the simplified model contains  $\frac{n^2+5n}{2}$  parameters, while in the general dynamic model case the number of parameters after regrouping rule is around  $6n$ . Not until link number exceeds 7, the simplified model has fewer parameters than dynamic model.

Then apply modulating functions to decrease the order of observed value. Let  $g$  be a  $k^{\text{th}}$  order modulating function on  $[0, T]$  where  $k \geq 2$ . Multiply  $g$  with acceleration  $\ddot{\mathbf{q}}$  and integrate on  $[0, T]$ . By partial integration, input  $\ddot{\mathbf{q}}$  decrease its order to position input  $\mathbf{q}$  and modulating function  $g$  increase to  $\dot{g}$  which is analytically known.

$$\int_0^T g\ddot{\mathbf{q}} = \int_0^T \dot{g}\dot{\mathbf{q}} = \int_0^T \ddot{g}\mathbf{q}. \quad (8)$$

Multiply equation (7) by modulating function  $g$  and integrate on  $[0, T]$ , using equation (8) we formulate the estimation model as:

$$\int_0^T g\tau = \bar{\mathbf{M}} \int_0^T \ddot{g}\mathbf{q} + \bar{\mathbf{N}}_0 \int_0^T g + \bar{\mathbf{N}}_1 \int_0^T g\mathbf{t} \quad (9)$$

Notice that equation (9) contains  $n$  equations. To solve the unknowns it need additional data from multi-equations whose number must not be smaller than that of the unknowns. This can be realized by adding a variable  $\ell$  to modulating function  $g$  where  $\ell \in \mathbb{R}$ , and a combination of different group of modulating functions. With enough sequence of  $\ell$ , the estimator forms an over-determined observation matrix and it can be solved by least square techniques. Finally we get the linear estimation model

$$\int_0^T g_\ell \tau = \left[ \int_0^T \ddot{g}_\ell \mathbf{q} \int_0^T g_\ell \int_0^T g_\ell \mathbf{t} \right] [\bar{\mathbf{M}} \bar{\mathbf{N}}_0 \bar{\mathbf{N}}_1]. \quad (10)$$

These scalar equations give an overdetermined system which is linear with respect to unknown parameters  $\mathbf{X}_s = [\bar{\mathbf{M}} \bar{\mathbf{N}}_0 \bar{\mathbf{N}}_1]$ , or can be expressed as  $\mathbf{B} = \mathbf{A}\mathbf{X}_s$ . This kind of problem can be solved by minimizing the Euclidian length of the residual vector  $\min_{\mathbf{X}_s} \|\mathbf{A}\mathbf{X}_s - \mathbf{B}\|$ , which gives a unique

optimal  $\hat{\mathbf{X}}_s$  as solution. There exists a lot of least square (LS) techniques such Moore-Penrose pseudoinverse, SVD decomposition, QR factorization, weighted LS, iterative LS and so on, which can be applied on this case.

#### 3.3 Frequency Analysis

The approach to investigate the frequency domain property of a modulating function is to consider the effect of integration as a filtering process. In Collado et al. (2009), the authors also analyze the differentiator frequency domain property. In real computation, the numerical integration  $\int gx$  is actually a discrete operation with integration time interval  $T_s$ , which calculates the sum of discrete points of a signal  $x$  associated with the modulating function  $g$ . In discrete version it writes as  $\sum_{i=1}^N g[i]x[i]$  with interval  $T_s$ . In this way the modulating functions can be discretized as a list of weighting coefficients. Moreover these weighting coefficients can be regarded as coefficients of a finite

impulse response (FIR) filter with respect to a discrete system with sampling time  $Ts$ . By studying the frequency domain behavior of the FIR filter, we can extend the results to integration effect with modulating functions. In the following part we look into several modulating functions and discuss their filtering property in order to give a standard to choose for applications.

The proposed functions  $g_\ell(t)$  are  $K$  order modulating functions on interval  $[0, T]$ , with  $K$  the order desired to decrease. They satisfy the two-point boundary conditions,  $g_\ell^{(i)}(t) = 0$ , for  $t = 0$  and  $t = T$ ,  $i = 0, 1, \dots, K - 1$ .

(1) Sinusoid based modulating functions (SMF): the sinusoid function value reaches 0 per half period, according to this property, propose  $g_\ell(t) = \sin^\ell(\frac{\pi}{T}t)$ , with  $\ell \in \mathbb{R}$ .

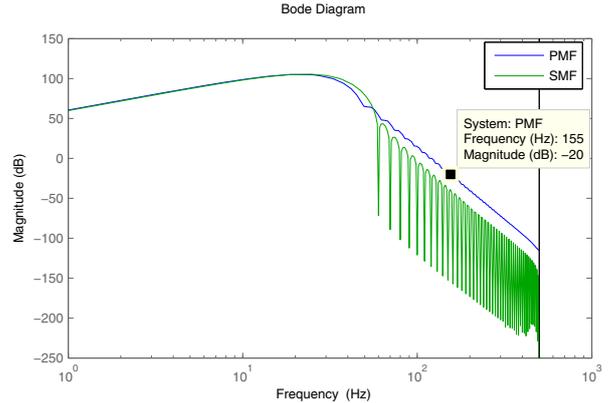
(2) Polynomial modulating functions (PMF): this group of functions are a combination of polynomials which equal to 0 at each end of interval. Remember the order of each polynomial is larger than  $K - 1$  and propose  $g_\ell(t) = t^{\ell_1}(t - T)^{\ell_2}$ , with  $\ell_1, \ell_2 \in \mathbb{R}$  and  $\ell_1, \ell_2 > K - 1$ .

(3) Fourier modulating functions (FMF): as known complex exponential function  $e^{ix} = \cos x + i \sin x$  is a period function which reaches 1 per period. Based on this, the fourier modulating functions can be written as  $g_\ell(t) = e^{-i\alpha\ell}(e^{-i\frac{2\pi}{T}t} - 1)^K$ , where  $\alpha$  is a tuning parameter and  $\ell \in \mathbb{R}$ .

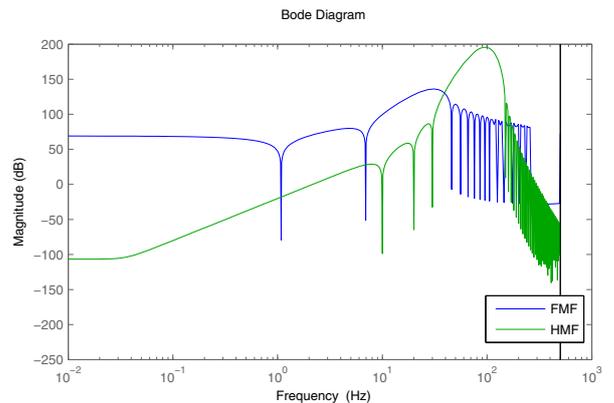
(4) Harley modulating functions (HMF): based on Shinbrot's method of moment functionals and Pearson Fourier modulating functions, this group of modulating functions are given:  $g_\ell(t) = \sum_{j=0}^n (-1)^j \binom{n}{j} \text{cas}((n + \ell - j)\omega_0 t)$ , where  $\ell = 0, \pm 1, \pm 2, \dots$  is integer,  $\omega_0 = \frac{2\pi}{T}$  is resolving frequency,  $\text{cas}(x) = \cos x + \sin x$ .

The integration effect with modulating functions can be a FIR filtering process. Suppose the system sampling time is  $Ts$  and extract modulating function value  $g_\ell(i)$  every  $Ts$  second as the coefficient of FIR filter. Then use bode plot to get the frequency contribution to magnitude of the modulating function. For example, take into account the second order derivatives of modulating functions, with system sampling time 1 millisecond, and draw their bode plots.

From figure 2, the frequency-magnitude response shows that for the groups of PMF and SMF, the filtering property of modulating functions are similar as low pass FIR filter, because the high frequency component of the signal contributes in a attenuation way to output and frequency higher than 150 Hz is considered to be cut off. When noise occurs at high frequency part of the signal, computation of integration using these modulating functions is robust to noise. While for the groups of FMF and PMF, it turns out that they enlarge the high frequency contribution to integration. Especially PMF can be regarded as a high pass FIR filter because it attenuates greatly the low frequency contribution to magnitude. This property makes these two groups of modulating functions not suitable in normal applications. In the next section, the estimation process considers only the groups of PMF and SMF modulating functions.



(a) PMF and SMF



(b) FMF and PMF

Fig. 2. Bode plot of second order derivatives of modulating functions when  $\ell = 10$

In conclusion, integration with modulating functions is an effective approach to decrease the order of input model. As well it has certain filtering property. Compared to filter techniques, it is causal and it has no phase shift because it calculates a scalar. The integration coefficients can be computed off-line so that it can be implemented easily and instantly for on-line applications. These advantages make modulating functions method interesting, but still it has drawback such as it has less excitation in the identifiability compared to the method treating each points of the interval as an independent equation, because the modulating function approach combines all to one scalar.

#### 4. SIMULATION RESULTS

The simulation part utilizes a two revolute joints planar robot model which moves in a horizontal plane and has no gravity effect. According to Gautier and Khalil (1990), the dynamic model depends on eight minimal dynamic parameters  $\mathbf{X} = [ZZ_{1R} \ ZZ_2 \ MX_2 \ MY_2 \ FV_1 \ FC_1 \ FV_2 \ FC_2]$ , with the regrouped parameter  $ZZ_{1R} = ZZ_1 + M_2L^2$ , where  $L$  is the length of first link,  $ZZ_1$  and  $ZZ_2$  are drive side moment of inertial of link 1 and 2 respectively,  $MX_2$ ,  $MY_2$  are first moment of link 2,  $FV_j$ ,  $FC_j$ , are the viscous and Coulomb friction coefficients of joint  $j$ . The simulation tests are running with value  $\mathbf{X}$  which is all in SI Units:  $\mathbf{X} = [3.9 \ 0.25 \ 0.45 \ 0.1 \ 0.3 \ 0.4 \ 0.15 \ 0.25]$ . Recall the robot dynamic model (1) and each component is:

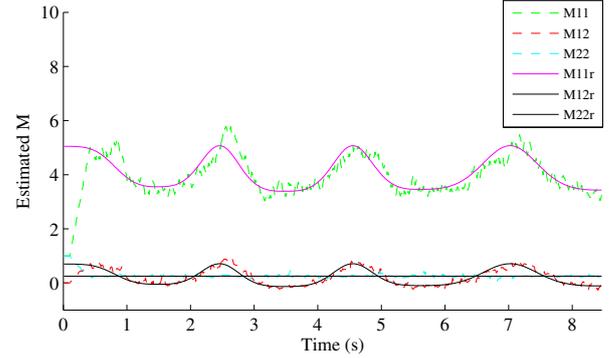
$$\begin{aligned}
\mathbf{H}(1,1) &= ZZ_{1R} + ZZ_2 + 2L(C2MX_2 - S2MY_2), \\
\mathbf{H}(1,2) &= ZZ_2 + L(C2MX_2 - S2MY_2), \\
\mathbf{H}(2,2) &= ZZ_2, \\
\mathbf{C}(1,1) &= -L\dot{q}_2(C2MY_2 + S2MX_2), \\
\mathbf{C}(1,2) &= -L(\dot{q}_1 + \dot{q}_2)(C2MY_2 + S2MX_2), \\
\mathbf{C}(2,1) &= L\dot{q}_1(C2MY_2 + S2MX_2), \\
\mathbf{C}(2,2) &= 0, \mathbf{Q} = \mathbf{0}, \\
\tau_{\mathbf{F}}(1) &= F_{V1}\dot{q}_1 + F_{C1}\text{sign}(\dot{q}_1), \\
\tau_{\mathbf{F}}(2) &= F_{V2}\dot{q}_2 + F_{C2}\text{sign}(\dot{q}_2), \\
\text{with } C1 &= \cos(q_1) \text{ and } C2 = \cos(q_2).
\end{aligned}$$

Consider the simplified robot model (2), we have  $\mathbf{M}(\mathbf{t}) = \mathbf{H}(\mathbf{q})$  and  $\mathbf{N}(\mathbf{t}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) + \tau_{\mathbf{F}}$ . And  $\bar{M}$  and  $\bar{N}$  are to be estimated at instant  $t$  which is approximately equal to the value of  $\mathbf{M}(\mathbf{t})$  and  $\mathbf{N}(\mathbf{t})$ . The simulation task is to track the desired trajectories using the proposed adaptive control and real time estimation associated with PMF and SMF modulating functions. The trajectories are defined from point to point and between each two points the joint position, velocity, and acceleration trajectories are planned as high order polynomials, which can offer good excitation. The simulation sampling time is 1 millisecond. The measured joint position and joint torques are with both high frequency sinusoid noise and high frequency random noise, the signal to noise ratio is 30dB. In the estimation process we bound the estimation increase step in order to attenuate the influence the wrong estimation at some points due to the ill excitation. We use QR factorization method to solve the least square problem and the estimation interval is 0.1s. For the initial 0.1s period, the parameters are set to be zero except the diagonal of  $\bar{M}$  is set to be 1.

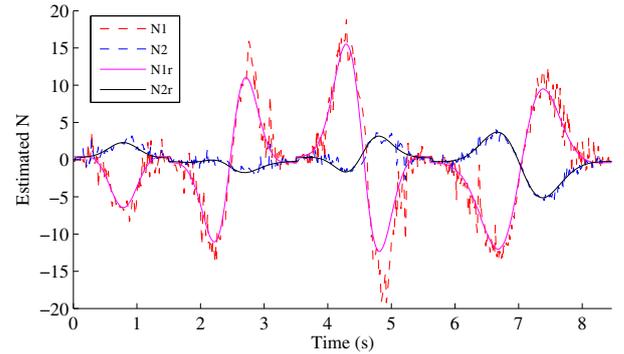
Using the configuration above, the simulation result is quit good with tracking error less than 0.004. And estimation value of  $\bar{M}$  and  $\bar{N}$  are quite fit to real value. The estimation time is 0.1 second so that it is real time estimation. From the figure 3 it can be found that  $M$  is quasi constant but  $N$  is varying fast with respect to estimation window. The estimated  $\bar{N}$  are reconstructed from two estimation  $\bar{M}_0$  and  $\bar{M}_1$ . The proposed simplified model which consider  $\bar{M}$  is constant and  $\bar{N} = \bar{N}_0 + \bar{N}_1 t$  is a linear component is reasonable from this result and it can be extended to common manipulator applications because most of their trajectories dynamic property is similar as this case.

#### 4.1 Results Robust To Variation Payload

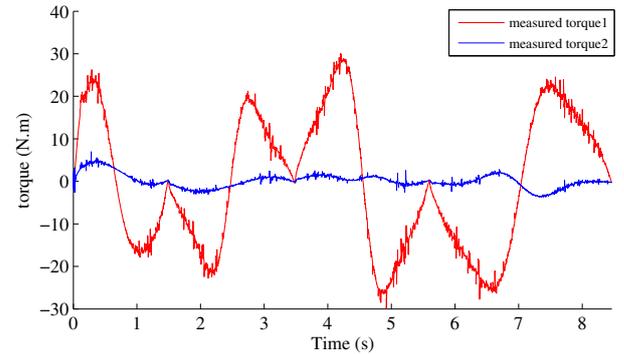
In real applications, sometimes the payload changes during the manipulator operation. To adjust them on-line is necessary for robust control. The adaptive control is a solution to variation of system dynamics, and in our case the real time estimation ensures the quick response to corrupt change. In simulation, at instant  $t = 2s$ ,  $ZZ_{1R}$  changes from 3.8 to 8, and at instant  $t = 5s$ ,  $ZZ_{1R}$  changes from 8 to 5. This can simulate the corrupt change of payload. Apply the adaptive control, and result is good with tracking error less than 0.003. The estimation of  $\bar{M}$  can be found in figure 4. Notice that there is a delay of



(a) Estimated  $\bar{M}$



(b) Estimated  $\bar{N}$



(c) Measured torque

Fig. 3. Estimated parameters and real parameters in normal tracking task

about 0.3s before getting the correct estimation of  $ZZ_{1R}$ . This delay is caused by the bounded estimation increase step and the estimation window 0.1s, as well it needs some time to recover from variation of system dynamics to re-estimate the changed parameter. During this transition period, the estimated parameters are varying smoothly to the correct value.

#### 4.2 Results Robust To Initial Condition

Let the reference trajectory starts from [1.5338; 2.0522], the robot manipulator starts from the origin [0; 0]. The tracking error is shown in figure 5, which proves the control structure is robust with initial condition and has a fast converge rate.

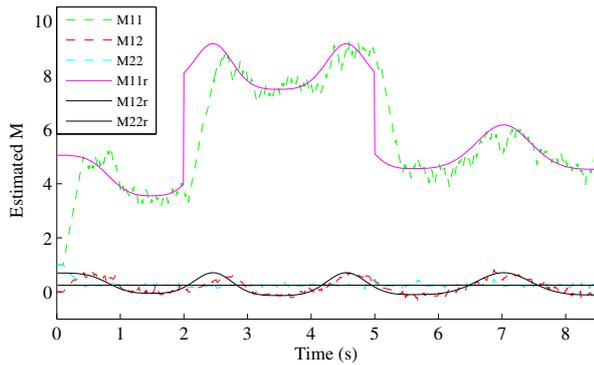


Fig. 4. Estimation of  $\bar{M}$  when  $ZZ_{1R}$  varies

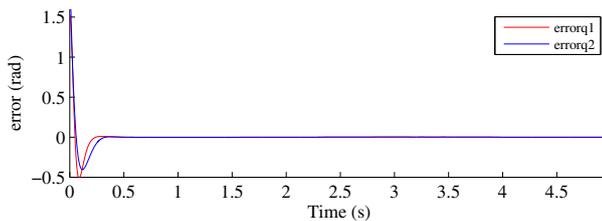


Fig. 5. Tracking error with initial condition

## 5. CONCLUSION

This paper propose an adaptive control structure associated with a robust real time estimation module for robot manipulator tracking task. A simplified differential robot model is proposed to decrease largely the number of estimated parameters and decreases the complexity of estimation process. With fewer parameters, the estimation time is sharply reduced and it responds faster to variation of system dynamics. The contribution of this paper is to apply the modulating function approach in estimation process, which allows to decrease the order of model input via integration. This can avoid the noisy numerical computation of high order derivatives of measured signal. Another contribution is to investigate the frequency domain response of different modulating functions. The selected modulating functions have a low pass filtering property. This gives simplicity to estimation module because it is not necessary to pre-process the signal to filter the noise component. And compared to to common filter, the modulating function approach needs only the causal data and calculates a scalar without considering phase shift. For future, experimental work should be carried out and test should be applied on robot with more links.

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