

POINTWISE CIRCUMSCRIPTION
is equivalent to
PREDICATE COMPLETION
(sometimes)

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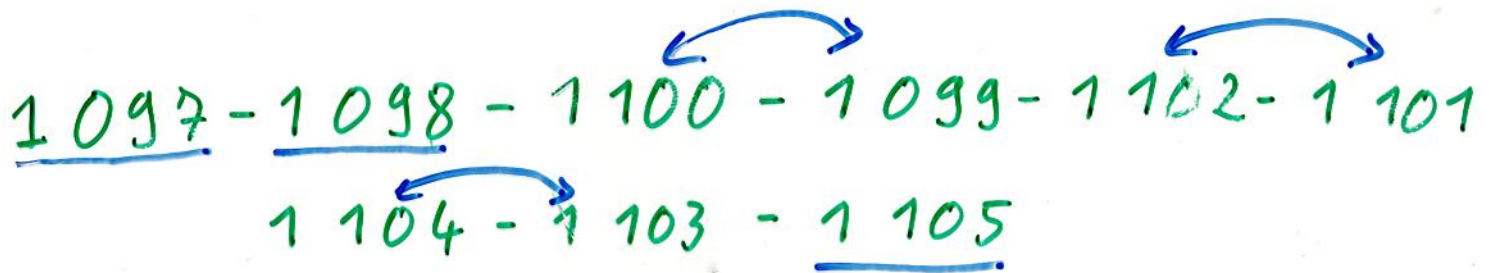
IRISA RENNES FRANCE

- 1) Minimizing positive information**
- 2) Predicate completion**
- 3) Circumscription**
- 4) Pointwise circumscription**
- 5) Pointwise circumscription and
Predicate completion**
- 6) Conclusion**

This

How to read this paper?

1097 - 1098 - 1100 - 1099 - 1102 - 1101
1104 - 1103 - 1105



except for the 1st and the last sheet,

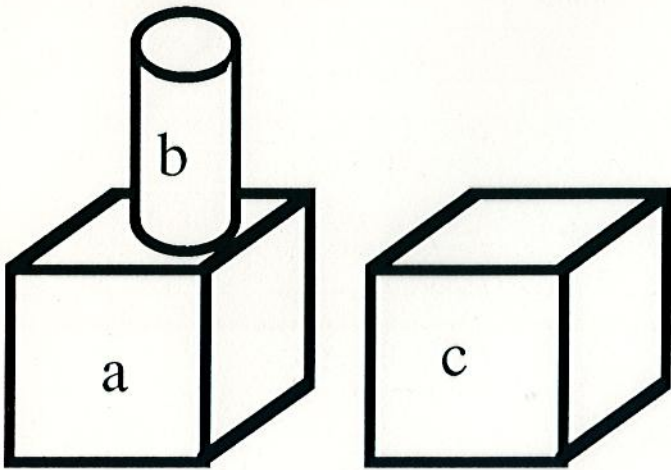
reverse order of the pages

on each sheet

of the paper.

MINIMIZING POSITIVE INFORMATION

Example 1:



T1 {
 CUB(a)
 CUB(c)
 CYL(b)
 ON(a,b)

\neg CUB(b) ?

\neg ON(a,c)?

CLOSED WORLD ASSUMPTION

(Reiter)

If a ground term $Q(t)$ cannot be proved,
then infer $\neg Q(t)$.

In example 1: $\neg \text{CUB}(b), \neg \text{ON}(a,c)$.

Example 2: T2: $P(a) \Leftrightarrow P(b)$.

CWA: $\neg P(a), \neg P(b)$

Example 3: T3: $P(a) \vee P(b)$

CWA: $\neg P(a), \neg P(b)$



PREDICATE COMPLETION

T = set of (definite) Horn clauses, clauses *in* P being:

$$T: \begin{cases} C_1(y_1) \Rightarrow P(t_1(y_1)) \\ \dots \dots \\ C_n(y_n) \Rightarrow P(t_n(y_n)) \end{cases}$$

General form of P in T :

$$\forall x \left(\begin{array}{l} (\exists y_1 \ x=t_1(y_1) \wedge C_1(y_1)) \\ \dots \vee \dots \\ (\exists y_n \ x=t_n(y_n) \wedge C_n(y_n)) \end{array} \right) \Rightarrow P(x)$$

\uparrow

The completion of P in T : \Leftrightarrow

(+ equality axioms + Clark's axioms)

In example 1:

 $(x=a \vee x=c) \Leftrightarrow CUB(x)$

Clark's axioms $(a \neq b, a \neq c, b \neq c)$, give: $\neg CUB(b)$.

(first order)

CIRCUMSCRIPTION

(McCarthy)

$T=T[P]$: set of formulas, P : predicate symbol.

Circumscription of P in T, $\text{Circ}(T:P)$:

$$T + \{ (T[p] \wedge \forall x (p[x] \Rightarrow P(x))) \Rightarrow \forall x (P(x) \Rightarrow p[x]) \}$$

for every formula p .

In example 1: $\text{Circ}(T1:CUB)$

The instance of the circumscription schema defined

by $p[x] \equiv x=a \vee x=c$ is enough to give the result:

$$T1 \wedge \forall x (CUB(x) \Rightarrow (x=a \vee x=c)) \text{ (id. completion)}$$

POINTWISE CIRCUMSCRIPTION (Lifschitz)

$T=T[P]$: set of formulas, P : predicate symbol.

$P_{/y}(x) \equiv P(x) \wedge x \neq y$.

Pointwise circumscription of P in T :

$C_{ppp}(T:P): T + \{ \forall y (T[P_{/y}] \Rightarrow \neg P(y)) \}$.

(one instance of circumscription schema)

- ◆ $\text{Circ}(T:P) \vdash C_{ppp}(T:P)$
- ◆ Converse true if T is positive in P (Lifschitz).

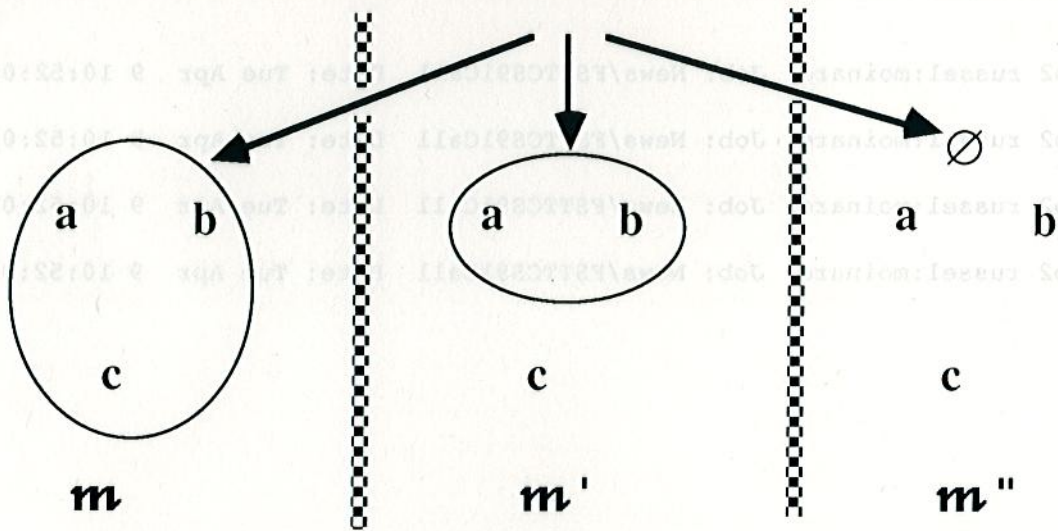
Example 2: T2: $P(a) \Rightarrow P(b), P(b) \Rightarrow P(a)$.

Completion of P in T2: $\forall x(P(x) \Leftrightarrow (P(a) \wedge (x=a \vee x=b)))$

$\text{Circ}(T2:P) \equiv \forall x(P(x) \Rightarrow \text{FALSE}) \equiv \forall x \neg P(x)$.

$\text{Cppp}(T2:P) \equiv \forall x(P(x) \Leftrightarrow (P(a) \wedge a \neq b \wedge (x=a \vee x=b)))$.

Extension of P:



$\neg T2$

$\neg T2$

$\neg T2$

**Model
of :**

\neg -completion
of P

\neg -completion
of P

\neg -Cppp(T2:P)

\neg -Cppp(T2:P)

\neg -Circ(T2:P)

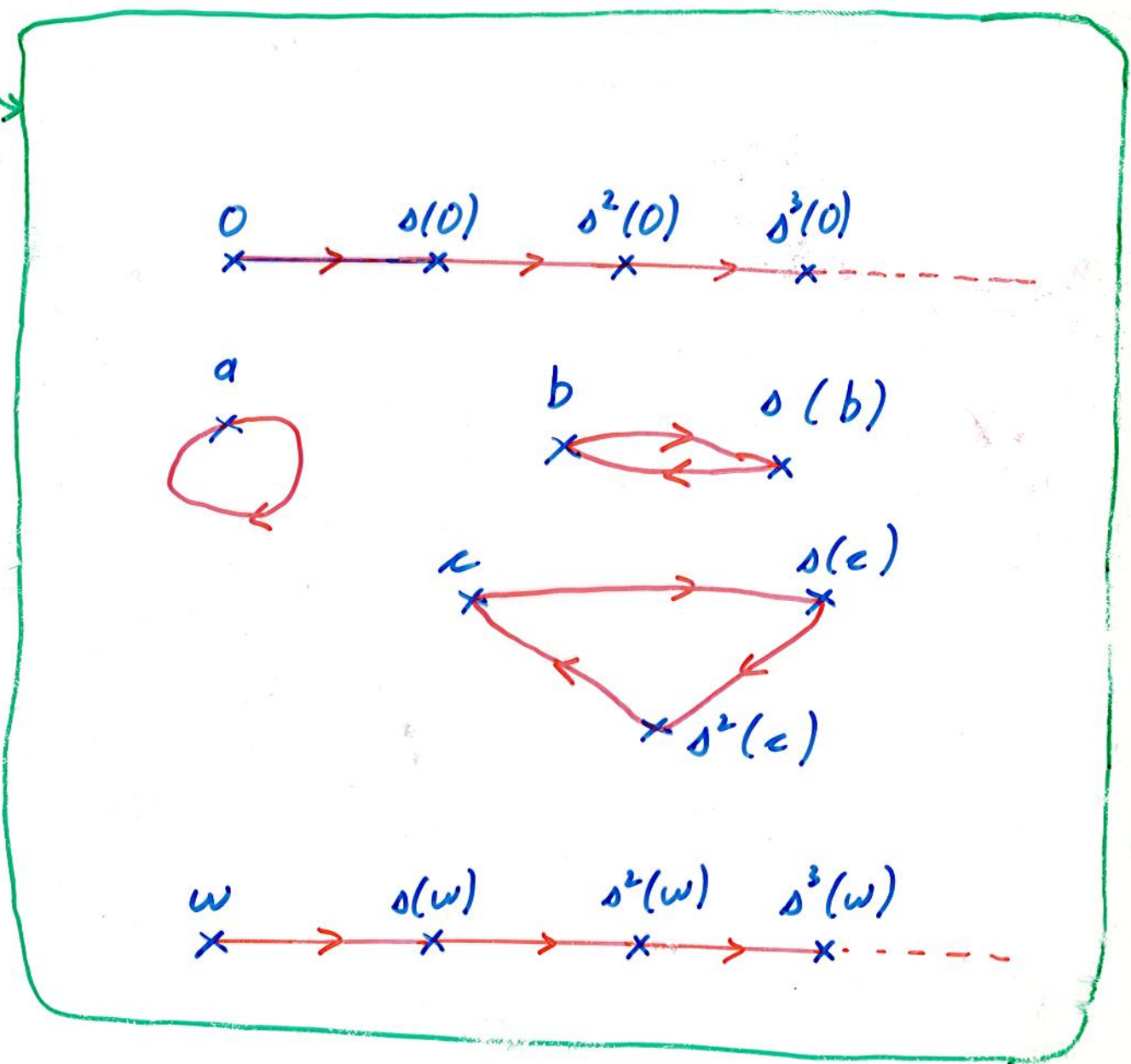
Circ / C_{PPP}

6 bis

Another Example:

$$T \begin{cases} P(0) \\ P(x) \Rightarrow P(\delta(x)) \end{cases} \quad \xrightarrow{\delta}$$

$|P|_m$



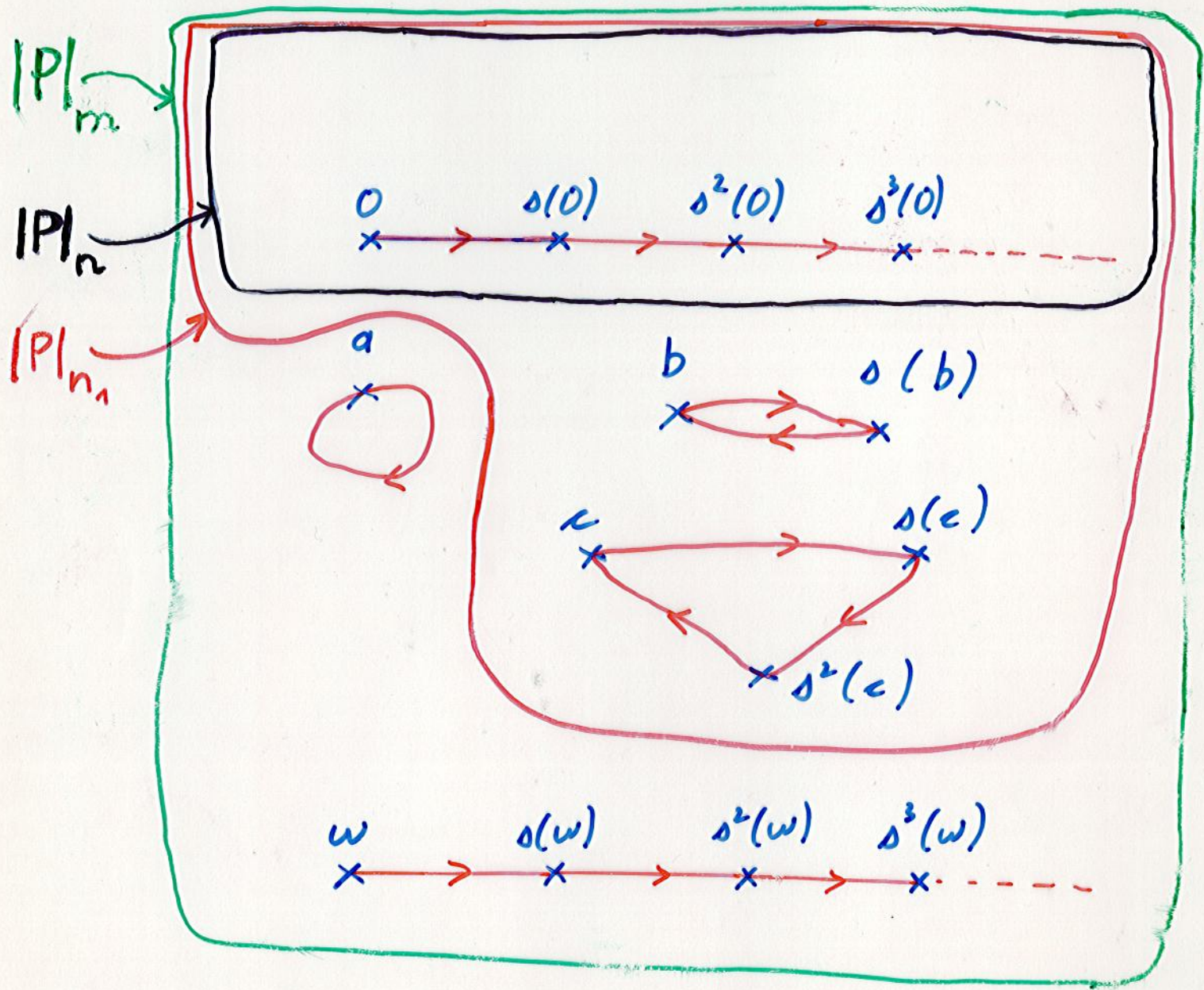
m : model of T

Circ⁶ / C PPP

6 bis

Another Example:

$$T \begin{cases} P(0) \\ P(x) \Rightarrow P(\Delta(x)) \end{cases} \xrightarrow{\delta}$$



- m : model of T
- n : $\text{Circ}(T:P)$
- n_1 : $\text{C PPP}(T:P)$

$$T \quad \left[\begin{array}{l} P(0) \\ P(x) \Rightarrow P(\sigma(x)) \end{array} \right. \quad \text{6 quart}$$

$$\models_{PPP} (T:P) \equiv \forall y \left\{ P(y) \Leftrightarrow \left(y=0 \vee \exists x (P(x) \wedge x \neq \sigma(x) \wedge y = \sigma(x)) \right) \right\}$$

$$\models_{irc} (T:P) \vdash \quad \text{and also:}$$

$$\models_{irc} (T:P) \vdash \forall y \left\{ P(y) \Rightarrow y \neq \sigma^n(y) \right\} \quad (n=1,2,\dots)$$

$$T' \equiv T + \forall x (\sigma(x) \neq 0)$$

$$\forall x \forall y \left(\sigma(x) = \sigma(y) \Rightarrow x = y \right) \quad \left. \begin{array}{l} \forall x \sigma^n(x) \neq x \\ n=1,2,\dots \end{array} \right\} \text{Clark's axioms}$$

$$\models_{irc} (T':P) \equiv \models_{PPP} (T':P)$$

Circ ^{6 quint} CPPP

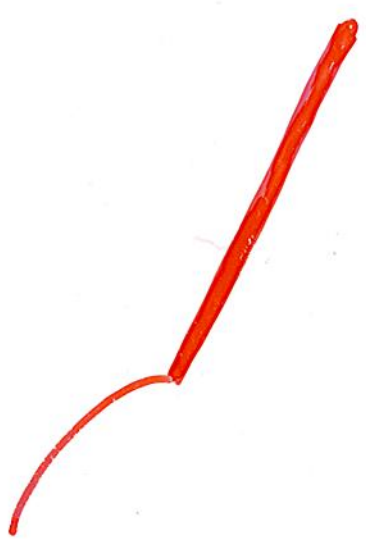
Circ



(Standard)
Circumscription

(as big as we need,
almost)

CPPP



pointwise
circumscription

(removes 1 particule
at a time,
steadily)

COMPLETION and SYNTAX

$$\begin{aligned}
 \text{Example 1': } T'1 = T1 \quad & (\text{CUB}(a) \\
 & \text{CUB}(c) \\
 & \text{CYL}(b) \\
 & \text{ON}(a,b)) \\
 & + \\
 & \text{CUB}(x) \Rightarrow \text{CUB}(x)
 \end{aligned}$$

The completion of CUB in T'1 gives:

$$\forall x \text{ (CUB}(x) \Leftrightarrow \text{CUB}(x) \vee x=a \vee x=c)} \\
 \equiv$$

$$\forall x \text{ (CUB}(x) \Leftarrow x=a \vee x=c) \quad \text{i.e.}$$

the old T1 before completion.

$$\text{Circ}(T'1:\text{CUB}) \equiv \text{Circ}(T1:\text{CUB})$$

$$\text{Cppp}(T'1:\text{CUB}) \equiv \text{Cppp}(T1:\text{CUB})$$

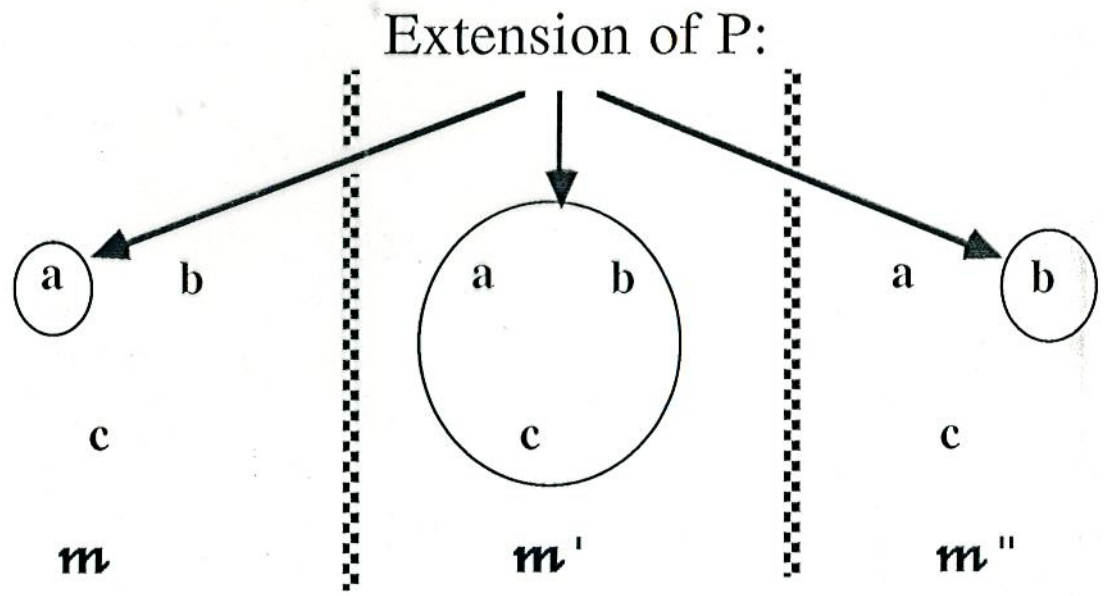
$$\equiv \text{Circ}(T1:\text{CUB})$$

7 bis
8 7'

Example 3: T3: $P(a) \vee P(b)$

$\text{Circ}(T3:P) \equiv \forall x (P(x) \Leftrightarrow x=a) \vee \forall x (P(x) \Leftrightarrow x=b)$

$\text{Cppp}(T3:P) \equiv \text{Circ}(T3:P)$



<p>Model of: $\neg T3$ $\neg \text{Circ}(T3:P)$ $\neg \text{Cppp}(T3:P)$</p>	<p>$\neg T3$</p>	<p>$\neg T3$ $\neg \text{Circ}(T3:P)$ $\neg \text{Cppp}(T3:P)$</p>
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Completion (extended definition: non Horn theory):

We must choose $\neg P(a) \Rightarrow P(b)$
or $\neg P(b) \Rightarrow P(a)$

HORN CLAUSES

Recursive clause: $(P(t(x)) \wedge \dots) \Rightarrow P(t'(x))$

Tautologous clause : $(P(t(x)) \wedge \dots) \Rightarrow P(t(x))$

↑ unifiable ↑

- ◆ **circumscription \vdash completion** (Reiter).
- ◆ **Converse true if no recursive clause** (Lifschitz).
- ◆ **pointwise circumscription \vdash completion.**
- ◆ **Converse true if:**
 - Clark's axioms
 - no tautologous clause .

CONCLUSION

– **Pointwise** circumscription and **negation as failure**.

$C_{ppp}(T:P) : \{ \forall y (T[P/y] \Rightarrow \neg P(y)) \}$.

If **P** is not forced to be true in y , then we conclude $\neg P(y)$

– **Computation:**

◆ Completion < Pointwise circumscription << circumscription
(Horn) (every theory)

– **Dependance vis-à-vis syntax:**

◆ strong for completion;

◆ weak for pointwise circumscription

which is only vocabulary dependant,

and applies to any theory.