



HAL
open science

Quantification in Frame Semantics with Hybrid Logic

Laura Kallmeyer, Timm Lichte, Rainer Osswald, Sylvain Pogodalla, Christian Wurm

► **To cite this version:**

Laura Kallmeyer, Timm Lichte, Rainer Osswald, Sylvain Pogodalla, Christian Wurm. Quantification in Frame Semantics with Hybrid Logic. 2015. hal-01151641v1

HAL Id: hal-01151641

<https://inria.hal.science/hal-01151641v1>

Preprint submitted on 13 May 2015 (v1), last revised 4 Sep 2015 (v4)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Quantification in Frame Semantics with Hybrid Logic

Laura Kallmeyer¹, Timm Lichte¹, Rainer Osswald¹, Sylvain Pogodalla^{1,2}, and Christian Wurm¹

¹Heinrich Heine Universität, Düsseldorf, Allemagne

²INRIA, Villers-lès-Nancy, F-54600, France, Université de Lorraine, LORIA, UMR 7503, Vandœuvre-lès-Nancy, F-54500, France, CNRS, LORIA, UMR 7503, Vandœuvre-lès-Nancy, F-54500, France

This paper aims at integrating logical operators into frame-based semantics. Frames are semantic graphs that allow to capture lexical meaning in a fine-grained way but that do not come with a natural way to integrate logical operators such as quantifiers. The approach we propose starts from the observation that modal logic is a powerful tool for describing relational structures, hence frames. We use its hybrid logic extension in order to incorporate quantification and thereby allow for inference and reasoning. We develop a type theoretic compositional semantics using this approach, formulated within Abstract Categorical Grammar.

1 Frames and Lexical Semantics

Frames emerged as a representation format of conceptual and lexical knowledge (Fillmore 1977; Barsalou 1992; Löbner 2014). They are commonly presented as semantic graphs with labelled nodes and edges, such as the one in Figure 1, where nodes correspond to entities (individuals, events, ...) and edges correspond to (functional or non-functional) relations between these entities. In Figure 1 all relations except *part-of* are meant to be functional.

Frames can be formalized as extended typed feature structures (Petersen 2007; Kallmeyer and Osswald 2013; Lichte et al. to appear), but a reformulation in first order logic is also straightforward (Kallmeyer and Osswald 2013). This conception of frames is therefore not to be confused with the somewhat simpler FrameNet frames (see Osswald et al. 2014).

Recent work has addressed the composition of lexical frames on the sentential level by means of an explicit syntax-semantics interface (Kallmeyer and Osswald 2013). However, the integration of logical operators remains a desideratum. While Kallmeyer and Richter (2014) experiment with a seamless intergration of “quantifier frames”, Muskens (2013) suggests to keep frames and logical operators separate. We follow the general approach of Muskens in this paper.

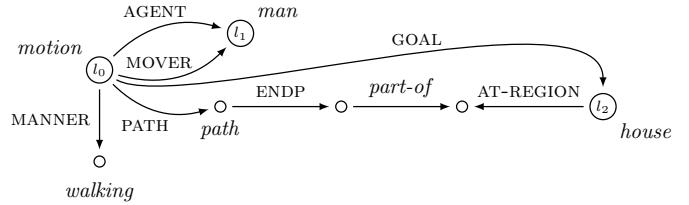


Figure 1: Frame for the meaning of *the man walked to the house* (adapted from Kallmeyer and Osswald 2013)

2 Hybrid Logic and Semantic Frames

2.1 Hybrid Logic. We use the notations of (Areces and ten Cate 2007) that uses *Rel* a set of relational symbols, *Prop* a set of propositional variables, *Nom* a set of nominals, and *Svar* a set of state variables ($\text{Stat} = \text{Nom} \cup \text{Svar}$). The language of formulas is $\text{Forms} ::= \top \mid p \mid s \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \langle R \rangle \phi \mid \exists\phi \mid @_s\phi \mid \downarrow x.\phi \mid \exists x.\phi$ where $p \in \text{Prop}$, $s \in \text{Stat}$, $R \in \text{Rel}$ and $\phi, \phi_1, \phi_2 \in \text{Forms}$. A *model* \mathcal{M} is a triple $\langle M, (R^M)_{R \in \text{Rel}}, V \rangle$ such that M is a non-empty set, each R^M is a binary relation on M , and the valuation $V : \text{Prop} \cup \text{Nom} \rightarrow \wp(M)$ is such that if $i \in \text{Nom}$ then $V(i)$ is a singleton. An assignment g is a mapping $g : \text{Svar} \rightarrow M$. For an assignment g , g_m^x is an assignment that differs from g at most on x and $g_m^x(x) = m$. For $s \in \text{Stat}$, we also define $[s]^{\mathcal{M}, g}$ to be the only m such that $V(s) = \{m\}$ if $s \in \text{Nom}$ and $[s]^{\mathcal{M}, g} = g(s)$ if $s \in \text{Svar}$.

Let \mathcal{M} be a model, $w \in M$, and g an assignment for \mathcal{M} . The *satisfaction relation* is defined as follows:

$\mathcal{M}, g, w \models \top$		$\mathcal{M}, g, w \models p$	iff $w \in V(p)$ for $p \in \text{Prop}$
$\mathcal{M}, g, w \models s$	iff $w = [s]^{\mathcal{M}, g}$ for $s \in \text{Stat}$	$\mathcal{M}, g, w \models @_s\phi$	iff $\mathcal{M}, g, [s]^{\mathcal{M}, g} \models \phi$ for $s \in \text{Stat}$
$\mathcal{M}, g, w \models \neg\phi$	iff $\mathcal{M}, g, w \not\models \phi$	$\mathcal{M}, g, w \models \downarrow x.\phi$	iff $\mathcal{M}, g_w^x, w \models \phi$
$\mathcal{M}, g, w \models \phi_1 \wedge \phi_2$	iff $\mathcal{M}, g, w \models \phi_1$ and $\mathcal{M}, g, w \models \phi_2$	$\mathcal{M}, g, w \models \exists x.\phi$	iff $\exists w' \mathcal{M}, g_w^x, w' \models \phi$
$\mathcal{M}, g, w \models \langle R \rangle \phi$	iff $\exists w' R^{\mathcal{M}}(w, w')$ and $\mathcal{M}, g, w' \models \phi$	$\mathcal{M}, g, w \models \exists\phi$	iff $\exists w' \mathcal{M}, g, w' \models \phi$

We also define $\mathbf{V}\phi \equiv \neg\exists(\neg\phi)$ ($\mathcal{M}, g, w \models \mathbf{V}\phi$ iff $\forall w' \mathcal{M}, g, w' \models \phi$)¹ and $\phi \implies \psi \equiv (\neg\phi) \vee \psi$. A formula ϕ is:

- *satisfiable* if there is a model \mathcal{M} , and assignment g on \mathcal{M} , and a state $w \in M$ such that $\mathcal{M}, g, w \models \phi$
- *globally true* in a model \mathcal{M} under an assignment g , i.e., $\mathcal{M}, g, w \models \phi$ for all $w \in M$. We write $\mathcal{M}, g \models \phi$

2.2 Feature Structures. In Kallmeyer and Osswald (2013), semantic frames are formally introduced as *base-labelled feature structure with types and relations*. This definition extends the standard definition of feature structures in two respects: In addition to features, proper relations between nodes can be expressed. Moreover, it is not required that every node is accessible from a single root node via a feature path; instead, it is required that every node is accessible from one of the base-labelled nodes. Semantic frames defined in this way can be seen as finite first-order structures which conform to a signature consisting of unary relation symbols $\text{Label} \cup \text{Type}$ and binary relation symbols $\text{Feat} \cup \text{Rel}$ subject to the constraints that the members of Label denote singletons, the members of Feat denote *functional* relations, and that the above accessibility condition holds. In the example frame of Figure 1, symbols inside nodes ($l_0, l_1 \dots$) indicate base labels, symbols attached to nodes ($man, motion \dots$) belong to Type , members of Feat are marked by small caps ($\text{AGENT}, \text{ENDP} \dots$), and *part-of* is the only member of Rel occurring in the frame.

Structures of this kind can easily be turned into Kripke structures by treating the interpretation of $\text{Label} \cup \text{Type}$ as a separate valuation function. Semantic frames, or feature structures, provide thus a natural application domain for modal languages and, in particular, for hybrid extensions because of the need to cope with node labels and feature path re-entrancies (Blackburn 1993). Under the formal set-up of Section 2.1, Type corresponds to Prop , Label corresponds to Nom , and Feat is subsumed under Rel . (The functionality of the members of Feat must be enforced separately.) The semantic frame of Figure 1 is a model that satisfies the formula (1) at the element named by l_0 .

$$(1) \quad l_0 \wedge motion \wedge \langle \text{AGENT} \rangle (l_1 \wedge man) \wedge \langle \text{MOVER} \rangle l_1 \wedge \langle \text{GOAL} \rangle (l_2 \wedge house) \wedge \langle \text{MANNER} \rangle walking \wedge \\ (\exists v w. \langle \text{PATH} \rangle (path \wedge \langle \text{ENDP} \rangle v) \wedge @_{l_2} (\langle \text{AT-REGION} \rangle w) \wedge @_v (\langle \text{part-of} \rangle w))$$

The logical framework of Kallmeyer and Osswald (2013) does not provide means for explicit quantification. As a consequence, the referential entities of the domain of discourse are implicitly treated as definite, which is reflected by the crucial role of nominals in (1). In the following, we will show how this limitation can be overcome by employing hybrid languages.

3 Type-Theoretic Semantics with Frames

We now provide the type-theoretic syntax-semantics interface allowing for a compositional building of the meanings. We describe it using the ACG (de Groote 2001) framework. As we are concerned in this article with semantic modeling and quantification rather than with parsing, we use higher-order types for quantified noun-phrases.

The models we are considering are *semantic frames* instead of arbitrary first-order models. So we first present some models in which we consider the sentences (2a–4a). When the model is the frame of Figure 2(a), we expect (2a) to be true as there is a *loving* event with AGENT and THEME attributes linking to persons named (linked with the NAMED attribute) *John* and *Mary* resp. (3a) is expected to be false, as well as (4a) with the object wide scope reading as there is a person named *Paul* (resp. named *Peter*) who is AGENT of a single *loving* event whose THEME is a person named *Sue* (resp. *Mary*). On the other hand, the subject wide scope reading of (4a) is expected to be true. (5) shows how state storing with the \downarrow operator correctly interacts with the $@$ operator used in specifying node sharing (for instance that the state linked with a GOAL relation in the verb semantic recipe has to be specified by the *PP*). This sentence is expected to be true (both readings) in the model given by the frame of Figure 2(b).

- (2) a. *John loves Mary*
b. $t_{2b} = \exists(\downarrow i. loving \wedge \langle \text{AGENT} \rangle (person \wedge \langle \text{NAMED} \rangle John) \wedge \langle \text{THEME} \rangle (person \wedge \langle \text{NAMED} \rangle Mary))$
- (3) a. *Every man loves Mary*
b. $t_{3b} = \mathbf{V}(\downarrow i. man \implies \exists(\downarrow i'. loving \wedge \langle \text{AGENT} \rangle i \wedge \langle \text{THEME} \rangle (person \wedge \langle \text{NAMED} \rangle Mary)))$
- (4) a. *Every man loves some woman*
b. $t_{4b} = \mathbf{V}(\downarrow i. man \implies \exists(\downarrow i'. woman \wedge \exists(\downarrow i''. loving \wedge \langle \text{AGENT} \rangle i \wedge \langle \text{THEME} \rangle i')))$
c. $t_{4c} = \exists(\downarrow i. woman \wedge \mathbf{V}(\downarrow i'. man \implies \exists(\downarrow i''. loving \wedge \langle \text{AGENT} \rangle i' \wedge \langle \text{THEME} \rangle i)))$
- (5) *Every man walked to some house*
a. t_{5a}
b. t_{5b}

¹According to the satisfaction relation, \downarrow and \exists bind state variables without changing the current evaluation state. (Blackburn and Seligman 1995) shows that they define a distinct hierarchy from the one we get using \exists (or some other binder Σ). It also shows that the fragment using operators from the two hierarchies is as expressive as the most expressive fragment.

with

$$\begin{aligned}
t_{5b} &= \exists(\downarrow i. \text{house} \wedge (\forall(\downarrow i'. \text{man} \implies \\
&\quad (\exists a g. \exists(\text{motion} \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g \wedge \langle \text{PATH} \rangle \text{path} \wedge \langle \text{MANNER} \rangle \text{walking} \wedge @_a i' \wedge \\
&\quad (\exists r v w. \text{event} \wedge \langle \text{PATH} \rangle (\text{path} \wedge \langle \text{ENDP} \rangle v) \wedge @_r(\langle \text{AT-REGION} \rangle w) \wedge @_v(\langle \text{part-of} \rangle w) \wedge @_r(g \wedge i)))))) \\
t_{5a} &= \forall(\downarrow i. \text{man} \implies (\exists(\downarrow i'. \text{house} \wedge \\
&\quad (\exists a g. \exists(\text{motion} \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g \wedge \langle \text{PATH} \rangle \text{path} \wedge \langle \text{MANNER} \rangle \text{walking} \wedge @_a i \wedge \\
&\quad (\exists r v w. \text{event} \wedge \langle \text{PATH} \rangle (\text{path} \wedge \langle \text{ENDP} \rangle v) \wedge @_r(\langle \text{AT-REGION} \rangle w) \wedge @_v(\langle \text{part-of} \rangle w) \wedge @_r(g \wedge i'))))))))
\end{aligned}$$

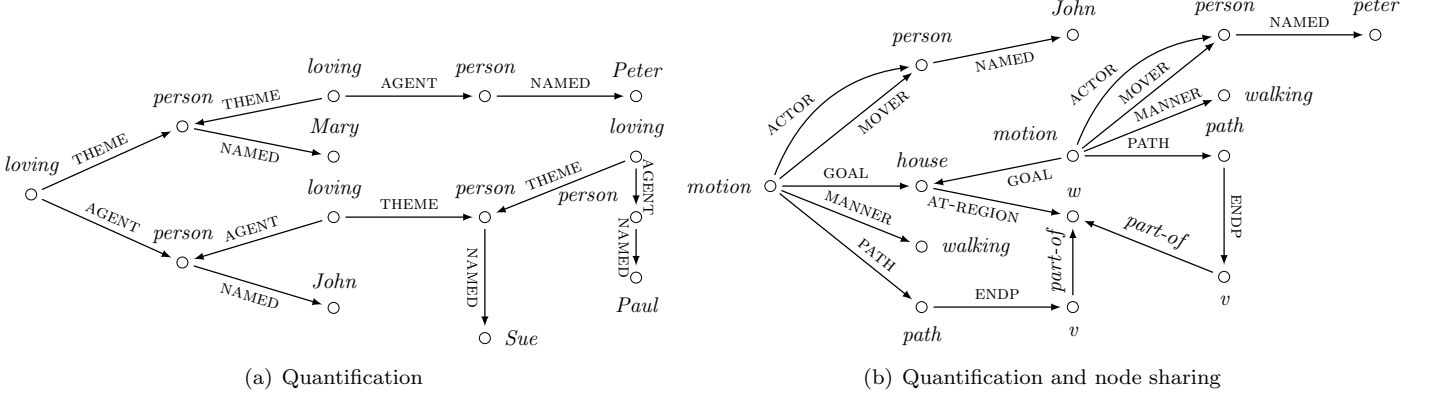


Figure 2: Frame samples

As in (Kallmeyer and Osswald 2013), the syntax-semantics interface we propose builds a *frame description* out of a sentence in natural language. This frame description is a logical formula that is checked against the possible models, and the sentence is true wrt. a model \mathcal{M} in case this model satisfies the logical formula. More precisely, given a sentence s and its semantic representation $\llbracket s \rrbracket$, we say that s is true iff for all assignments g , $\mathcal{M}, g \models \llbracket s \rrbracket$ (i.e., $\llbracket s \rrbracket$ is globally true in \mathcal{M} under any assignment).

We use the following syntactic types: NP , S , N , and PP and the following syntactic type assignments

John, Mary	: NP	loves	: $NP \rightarrow NP \rightarrow S$	to, into	: $NP \rightarrow PP$
man, woman, house	: N	every, some	: $N \rightarrow (NP \rightarrow S) \rightarrow S$	walk	: $PP \rightarrow NP \rightarrow S$

Table 1 shows the semantic constants we use, including logical operators and quantifiers. We follow (Kallmeyer and Osswald 2013) in the semantics and meaning decomposition of locomotion verbs.

Then we can use the semantic interpretation given in Table 2. For sake of conciseness and explanatory purposes, we use a single type t to denote modal formulas. This is not completely satisfactory as we can build terms that are not in **Forms** (in principle, any proposition could specify the $@ : t \rightarrow t \rightarrow t$ operator. But in our lexicon example, we of course restrict the first parameter to state variables). A more faithful encoding could use the standard parametrization of the propositions with a s type for states, or use a dedicated hybrid type-theory (Areces, Blackburn, et al. 2014). Then the following equalities hold:

$$\begin{aligned}
(6) \quad & \llbracket \text{loves Mary John} \rrbracket = t_{2b} \\
(7) \quad & \llbracket (\text{every man}) (\lambda x. \text{loves Mary } x) \rrbracket = t_{3b} \\
(8) \quad & \llbracket (\text{every man}) (\lambda x. (\text{some woman}) (\lambda y. \text{loves } y x)) \rrbracket = t_{4b} \\
(9) \quad & \llbracket (\text{some woman}) (\lambda y. (\text{every man}) (\lambda x. \text{loves } y x)) \rrbracket = t_{4c}
\end{aligned}$$

$event, loving, motion, person, John, Mary \dots$: t	$@$: $t \rightarrow t \rightarrow t$
\wedge	: $t \rightarrow t \rightarrow t$	\implies	: $t \rightarrow t \rightarrow t$
\exists, \forall	: $t \rightarrow t$	$\downarrow \exists$: $(t \rightarrow t) \rightarrow t$

Table 1: Constant terms of the semantic language

In (10) and (11), we have storing for quantification and path equalities compositionally issuing from the verb and the preposition interact. In the verb semantics, the path equalities specify that the **MOVER** and the **AGENT** attributes of the event are the same, and that the information provided by the pp argument should hold for the **GOAL** g . In its

	$S, NP, N := t$	$PP := t \rightarrow t$
John := <i>John</i>	Mary := <i>Mary</i>	
man := <i>man</i>	woman := <i>woman</i>	house := <i>house</i>
some := $\lambda P Q. \exists (\downarrow i. P \wedge (Q i))$	every := $\lambda P Q. \forall (\downarrow i. P \implies (Q i))$	
loves := $\lambda o s. \exists (\downarrow i. loving \wedge \langle \text{AGENT} \rangle s \wedge \langle \text{THEME} \rangle o)$		
walks :=	$\lambda pp s. \exists a g. \exists (motion \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g$	
	$\wedge \langle \text{PATH} \rangle path \wedge \langle \text{MANNER} \rangle walking \wedge @_a s \wedge (pp g))$	
to := $\lambda n g. \exists r v w. event \wedge \langle \text{PATH} \rangle (path \wedge \langle \text{ENDP} \rangle v) \wedge @_r \langle \text{AT REGION} \rangle w \wedge @_v \langle \text{part-of} \rangle w \wedge @_r (g \wedge n)$		
into := $\lambda n g. \exists r v w. event \wedge \langle \text{PATH} \rangle (path \wedge \langle \text{ENDP} \rangle v) \wedge @_r \langle \text{IN REGION} \rangle w \wedge @_v \langle \text{part-of} \rangle w \wedge @_r (g \wedge n)$		

Table 2: Semantic interpretation of constants

semantics, the preposition contributes on the one hand to the main event (as the *event* proposition is evaluated at the current state) and on the other hand by specifying that the g state (meant to be the target node of the verb that the proposition modifies, here the target of the GOAL attribute) should be identified to the n argument (the noun phrase which is argument of the preposition).

$$(10) \quad \llbracket (\text{every man}) (\lambda x. (\text{some house}) (\lambda y. \text{walked} (\text{to } y) x)) \rrbracket = t_{5a}$$

$$(11) \quad \llbracket (\text{some house}) (\lambda y. (\text{every man}) (\lambda x. \text{walked} (\text{to } y) x)) \rrbracket = t_{5b}$$

4 Conclusion and Perspectives

We used hybrid logic as a means to integrate logical operators with frame semantics. A type theoretic semantics was presented that shows how to compositionally derive different quantifier scope readings. This approach has much in common with (Muskins 2013), which combines data semantics with frame semantics. The exact relation between the two approaches needs to be spelled out in future work. We see applications of our approach of using hybrid logic for frame semantics in the context of various formalisms; we plan in particular to pursue this approach in the framework of Lexicalized Tree Adjoining Grammars (LTAG) (Kallmeyer and Osswald 2013).

We also think that the compositional account we presented allows us to consider an embedding within a under-specified representation language. The object language (in the sense of (Bos 1995)) would be the hybrid logic language instead of the usual first-order logic language, following a standard modeling of scope ambiguity in LTAG.

Finally, we plan to investigate the computational properties of the framework we propose with respect to the hybrid inferential systems (Blackburn and Marx 2002) and the specific properties induced by the frame models we consider, typically the functionality of the attribute relations (Schneider 2007).

References

- Arecas, Carlos, Patrick Blackburn, et al. (2014). “Completeness in Hybrid Type Theory”. In: *Journal of Philosophical Logic* 43.2-3, pp. 209–238. DOI: [10.1007/s10992-012-9260-4](https://doi.org/10.1007/s10992-012-9260-4).
- Arecas, Carlos and Balder ten Cate (2007). “Hybrid logics”. In: *Handbook of Modal Logic*. Ed. by Patrick Blackburn, Johan Van Benthem, and Frank Wolter. Vol. 3. Studies in Logic and Practical Reasoning. Elsevier. Chap. 14, pp. 821–868. DOI: [10.1016/S1570-2464\(07\)80017-6](https://doi.org/10.1016/S1570-2464(07)80017-6).
- Barsalou, Lawrence (1992). “Frames, concepts, and conceptual fields”. In: *Frames, fields, and contrasts: New essays in semantic and lexical organization*. Ed. by Adrienne Lehrer and Eva Feder Kittay. Hillsdale: Lawrence Erlbaum Associates, pp. 21–74.
- Blackburn, Patrick (1993). “Modal Logic and Attribute Value Structures”. In: *Diamonds and Defaults*. Ed. by Maarten de Rijke. Dordrecht: Kluwer, pp. 19–65.
- Blackburn, Patrick and Maarten Marx (2002). “Tableaux for Quantified Hybrid Logic”. In: *Automated Reasoning with Analytic Tableaux and Related Methods*. Ed. by Uwe Egly and Christian G. Fermüller. Vol. 2381. Lecture Notes in Computer Science. Springer Berlin Heidelberg, pp. 38–52. DOI: [10.1007/3-540-45616-3_4](https://doi.org/10.1007/3-540-45616-3_4).
- Blackburn, Patrick and Jerry Seligman (1995). “Hybrid languages”. In: *Journal of Logic, Language and Information* 4.3, pp. 251–272. DOI: [10.1007/BF01049415](https://doi.org/10.1007/BF01049415).
- Bos, Johan (1995). “Predicate Logic Unplugged”. In: *Proceedings of the Tenth Amsterdam Colloquium*. URL: <http://www.let.rug.nl/bos/pubs/Bos1996AmCo.pdf>.
- de Groote, Philippe (2001). “Towards Abstract Categorical Grammars”. In: *Association for Computational Linguistics, 39th Annual Meeting and 10th Conference of the European Chapter, Proceedings of the Conference*, pp. 148–155. ACL anthology: [P01-1033](https://doi.org/10.1007/978-3-319-01541-5_2).
- Fillmore, Charles J. (1977). “The case for case reopened”. In: *Grammatical Relations*. Ed. by Peter Cole and Jerrold M. Sadock. Vol. 8. Syntax and Semantics. New York: Academic Press, pp. 59–81.
- Kallmeyer, Laura and Rainer Osswald (2013). “Syntax-Driven Semantic Frame Composition in Lexicalized Tree Adjoining Grammars”. In: *Journal of Language Modelling* 1.2, pp. 267–330. DOI: [10.15398/jlm.v1i2.61](https://doi.org/10.15398/jlm.v1i2.61).
- Kallmeyer, Laura and Frank Richter (2014). “Quantifiers in Frame Semantics”. In: *Formal Grammar. 19th International Conference, FG 2014*. Ed. by Glyn Morrill et al. Lecture Notes in Computer Science 8612. Springer, pp. 69–85.
- Lichte, Timm and Simon Petitjean (to appear). “Implementing semantic frames as typed feature structures with XMG”. In: *Journal of Language Modelling*.
- Löbner, Sebastian (2014). “Evidence for frames from human language”. In: *Frames and Concept Types. Applications in Language and Philosophy*. Ed. by Thomas Gamerschlag et al. Vol. 94. Studies in Linguistics and Philosophy 94. Dordrecht: Springer, pp. 23–67. DOI: [10.1007/978-3-319-01541-5_2](https://doi.org/10.1007/978-3-319-01541-5_2).
- Muskens, Rheinard (2013). “Data Semantics and Linguistic Semantics. A festschrift for Jeroen Groenendijk, Martin Stokhof, and Frank Veltman”. In: *The dynamic, inquisitive, and visionary life of ϕ , $?\phi$, and $\diamond\phi$* . Ed. by Maria Aloni, Michael Franke, and Floris Roelofsen. Pumbo.nl. Chap. 24, pp. 175–183. URL: http://www.illc.uva.nl/Festschrift-JMF/papers/23_Muskens.pdf.

- Osswald, Rainer and Robert D. Van Valin Jr. (2014). "FrameNet, Frame Structure, and the Syntax-Semantics Interface". In: *Frames and Concept Types*. Ed. by Thomas Gamerschlag et al. Vol. 94. Studies in Linguistics and Philosophy. Springer. Chap. 6, pp. 125–156. DOI: [10.1007/978-3-319-01541-5_6](https://doi.org/10.1007/978-3-319-01541-5_6).
- Petersen, Wiebke (2007). "Representation of Concepts as Frames". In: *The Baltic International Yearbook of Cognition, Logic and Communication* 2, pp. 151–170.
- Schneider, Thomas (2007). "The Complexity of Hybrid Logics over Restricted Classes of Frames". PhD thesis. University of Jena, Germany. URL: http://www.cs.man.ac.uk/~schneidt/publ/sch07_phd.pdf.