Investigation on the sensitivity of subspace based damage detection technique to damage and noise levels

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INVESTIGATION ON THE SENSITIVITY OF SUBSPACE BASED DAMAGE DETECTION TECHNIQUE TO DAMAGE AND NOISE LEVELS

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ABSTRACT

Damage detection techniques are one of the main tools in health monitoring of structures. This paper addresses the effect of noise in the measured data on a robust damage detection method, namely statistical subspace-based damage detection technique. In this method the need of evaluating the modal parameters of the structure is circumvented which makes this method capable in real-time monitoring of structures. Moreover, this method identifies the changes in the eigen-structure of the model which makes it a robust approach to function with high amount of noise in the input data. In order to investigate the effect of noise on this method, a bridge structure located in Reibersdorf, Austria, is considered. This structure is modeled and calibrated to the real test data; subsequently the damage is modeled in one of the elements for different damage ratios. With using white noise excitation, ambient vibration test data is simulated and different noise ratios are applied to the data. A reference state of the structure is evaluated using this technique. A subspace-based residual between the reference and possibly damaged states is defined independently from the input excitations employing a χ^2 test and then is compared to a threshold corresponding to the reference state. Subsequently, the effect of noise ratios in the reference state and the measured data and their difference is investigated. It was concluded that the SSDD technique is capable of estimating the damage in almost all damage ratios and even for high noise ratios in the data. Moreover, it was observed that the noise ratio difference in the reference state and measured data may be interpreted as damage, since it is reflected in the computed residual. An optimum range of the noise in the data is also assessed and proposed.

Keywords: Damage detection, Noise, Subspace, Health monitoring, Real-time monitoring

1. INTRODUCTION

Monitoring the condition of infrastructures is of great importance to the researchers, due to their direct influence on the health and economy of the society. Existing civil structures deteriorate by aging and under different loading conditions imposed from natural phenomena such as earthquakes, typhoons, flood and etc. Therefore, it is imperative to investigate the safety of continuing using these structures, especially after occurring major demands on the structure from these phenomena. Numerous researches can be found in the literature and different approaches are proposed to deal with this problem.

Inspecting the safety condition of these structures is performed in several ways that can be categorized into two groups [1], i.e. local and global techniques. Local techniques usually need to have access to all parts of the structure in order to assess a specific part. This may lead to interference in the operation of the structure and is not suitable for major structures. However, in global techniques the condition of the structure can be assessed without the need of accessing to all parts of it and by monitoring the response of structure in specific locations. Therefore these techniques can be performed effectively without any need to interrupt the functionality of the structure.

The global techniques can be also categorized into two groups based on their approach to the problem. In the first category, the structural properties are identified and employed to assess the condition of the structure. The structural properties identified from these approaches include stiffness, damping, mass, load paths and boundary conditions (supports, connections and etc). In the second category the eigen-structure of the problem is employed to evaluate the safety condition of the structure. In these methods, modal properties such as natural frequencies, modal damping values and mode shapes are used to identify any changes in the structure. Any change in the structural properties leads to a change in the modal parameters of the structure. However generally, identifying the modal parameters in a structure is more practical and accurate than the structural properties.

The process of evaluating the modal parameters of a structure is also time consuming and it usually cannot be employed in real-time monitoring. Evaluation of these dynamic characteristics can be avoided by using statistical approaches, e.g. statistical subspace-based damage detection technique (SSDD) [2-5]. This technique evaluates the global condition of structure by identifying changes in the eigen-structure of the problem. The damage can be detected by comparing a statistical model from the possibly damaged structure to the one obtained from a reference state. In other words a subspace based residual function between these states is defined and compared using a $\chi 2$ test. In this way there is no need to estimate the natural frequencies and mode shapes, making this approach capable of being used in real-time monitoring of structures. In [4-6] it is investigated that this approach can also perform robustly under ambient excitations with changing statistics.

A bridge structure, i.e. S101, located at Reibersdorf, Austria, is investigated and simulated for this purpose. This structure was damaged artificially in a progressive manner and it was continuously measured during each damage level [7]. A finite element model of this structure is created and calibrated using the available measured data. In our previous paper [8], the performance of the SSDD technique was assessed for different damage types and ratios. It was shown that this technique can efficiently identify the damage occurred in the structure. However, the effect of noise in the data was not investigated in our previous study and it motivates the present paper.

Existence of noise in experimental data is inevitable. There are different sources of the noise in measuring a structure [4] such as the change in excitation sources [6], noise of measuring instruments and human error. Moreover, the data quality (noise ratio) can affect significantly on the damage detection output (e.g. [9]). Therefore, investigating the effect of this inherent characteristic of the measurements on the SSDD technique is an important factor in assessing its functionality.

In this paper the functionality and performance of the SSDD technique is investigated for different noise ratios in the data and for different damage ratios simulated in a specific member. Noise-to-data ratio is one of the major parameters defining the effect of the noise on the data. This ratio is used herein to simulate and add the noise to the simulated measurements. It is worth mentioning that the simulated measurements are generated by measuring the acceleration time histories of the nodes typically measured in a bridge structure. The noise ratio is then applied to the measured data and it is consequently processed by ARTeMIS software [10].

2. STATISTICAL SUBSPACE-BASED DAMAGE DETECTION

Statistical subspace-based damage detection (SSDD) technique detects the damage in a structure by using a χ^2 test on a residual function [2-5]. Therefore in this method, there is no need to compute and compare modal parameters of the reference and possibly damaged states of the system. In other words, this residual function represents the changes occurred to the model which can be caused by a damage in structure. This changes are basically identified in the eigen-structure of the problem.

2.1. Models and parameters

The dynamic system of the model can be considered as a discrete time state space model of

$$\begin{cases} X_{k+1} = FX_k + \varepsilon_k \\ Y_k = HX_k + \vartheta_k \end{cases} \tag{1}$$

where, the state is represented by $X \in \mathbb{R}^n$ and the measured output is $Y \in \mathbb{R}^r$. F also represents the state transition matrix and H shows the observation matrix with dimensions n×n and r×n, respectively. The state noise, ε_k and measurement noise ϑ_k are assumed to be Gaussian unmeasured white noise with zero mean. The covariance of output measurements Y_k can be computed from the state space model (1) by

$$R_i = \mathbf{E}(Y_{k+i}Y_k^T) \tag{2}$$

in which operator **E** is the expectation function. With choosing parameters q and p such as $q \geq p+1$, the Hankel matrix **H** can be written as

$$\mathbf{H} = \begin{pmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & & R_{q+1} \\ \vdots & & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \dots & R_{p+q} \end{pmatrix} \tag{3}.$$

As mentioned earlier the measurements are performed in a reference state and a possibly damaged state. The Hankel matrix of the measurements in reference state, $\mathbf{H}_0$, can then be computed from (2) and (3). This matrix is then decomposed using singular value decomposition in order to compute the left null space **S**. Defining **H** for the possibly damaged state of the system, the left null space matrix **S** in the reference state is characterized by $\mathbf{S}^T\mathbf{H} = \mathbf{0}$ ([2] and [3]). Therefore the residual vector $\boldsymbol{\zeta}_n$ can be written as

$$\boldsymbol{\zeta}_n = \sqrt{n}\,\mathrm{vec}(\mathbf{S}^T\mathbf{H}) \tag{4}$$

in which, n represents the number of samples measured for computing **H**. This residual can now be used in order to check if any change is made in the model due to damage. The residual vector $\boldsymbol{\zeta}_n$ is asymptotically Gaussian with zero mean in reference state; significant changes in its mean value indicates the structure is moved from its reference state. In order to check this change from the residual vector mean, the χ^2 test can be performed as following [2-5].

$$\chi^2 = \zeta_n^T \Sigma^{-1} \zeta_n \tag{5}$$

Herein, Σ represents the covariance matrix of the residual in the reference state, and can be shown as

$$\Sigma = \mathbf{E}\left[\zeta_n \zeta_n^T\right] \tag{6}$$

It is worth mentioning that the covariance of the input noise ε_k is assumed to not change between the reference state and the possibly damaged state. when using the residual defined in (4). In [4,5] it was shown that the modified residual

$$\tilde{\zeta}_n = \sqrt{n}\,\mathrm{vec}(\mathbf{S}^T \mathbf{U}_1) \tag{7}$$

is robust to changes in the covariance of the input noise ε_k in the same framework, where $\mathbf{U}_1$ is the matrix of the principal left singular vectors of $\mathbf{H}$.

By monitoring the value of χ^2 and comparing it to a threshold value, the state of the damage of the system can be estimated. This threshold can be simply evaluated using several data sets measured from the structure in its reference state. Subsequently, some other data sets measured from the reference state are used to check the threshold. Then the χ^2 value is computed for the possibly damaged structure. If the χ^2 value is computed to be higher than the threshold it can be inferred that the structure may be damaged. In other words, the amount of effect of damage on statistics of the measured data has a straight relation with the amount of the χ^2 value.

3. DAMAGE AND DATA SIMULATION

Simulating the damage in a structure and subsequently generating the ambient vibration test data can be a straightforward approach to evaluate damage detection techniques. This data can be an acceptable benchmark to evaluate the functionality of these techniques by providing control on the test conditions, e.g. structural properties and damage effects. In order to investigate the effect of noise on these techniques, a predefined amount of white noise is superposed to the simulated data, which will be described in the next section.

In order to evaluate the functionality of the subspace-based damage detection technique, the ambient vibration test data can be simulated for different damage ratios. In order to simulate this data, a finite element model of the structure is created and then calibrated to the real structure. It should be mentioned that calibration of the structure does not have a straight effect on the damage detection technique. In other words, the damage detection technique should be able to detect the damage in any structural model including the uncalibrated one as long as the base of comparison is identical. However in this study, calibration to a real structure is performed to obtain a realistic model.

The damage can be modeled in different locations by reducing the dimensions of one or some short elements corresponding to it. The amount of the damage can be presented by the ratio of this reduction. For each damage and noise ratio separate analysis model is created.

Several points of the structure are excited using white noise excitation in all three directions. Different excitations are imposed on the structure in order to excite the structure as randomly as possible. This excitation can be done by acceleration or load forces in different points of the structure. Subsequently, the simulated data can be obtained by measuring acceleration time histories of the nodes typically measured and instrumented in a bridge.

The simulated data can then be analyzed in order to compute the natural frequencies and their corresponding mode shapes. These can be used to check which mode shapes can be captured by the simulated white noise excitation. Based on the positioning of the sensors and or insufficient excitation of the structure, some mode shapes may not be captured. For the latter, the excitation must be modified to impose an excitation to the structure close to the white noise in different points of the structure.

4. NOISE APPLICATION

The imposed noise on the data is created using a random generation algorithm. The simulated test data in each point and each direction is defined as a measurement channel. The probability distribution of the random generator is evenly distributed and its magnitude is chosen as a ratio, i.e. noise ratio, of the maximum value of each channel. Therefore, the maximum value, m_i, in each measurement channel, i, is evaluated and then by multiplying it to the noise ratio (N_r) the interval of the random numbers is defined. The random vector $\mathbf{R}_i$ can be evaluated from

$$\mathbf{R}_i = \text{random}(N_r m_i, \text{even}) \tag{8}$$

In the next step, the random vector $\mathbf{R}_i$ is added to the measured data $\mathbf{D}_i$ for the corresponding channel. Therefore the modified measured data $\mathbf{ND}_i$ can be evaluated as

$$\mathbf{ND}_i = \mathbf{D}_i + \mathbf{R}_i \tag{9}$$

It should be noted that the mean of the generated noise vector, $\mathbf{R}_i$, is zero. Different noise ratios are investigated in this study which will be described in section 5.2 and 5.3. As an example, a Gaussian wave packet function is modified with 10% of noise ratio. The original data and the generated white noise with amplitude of 10% of the maximum value of the data are added together to create the modified data with noise.

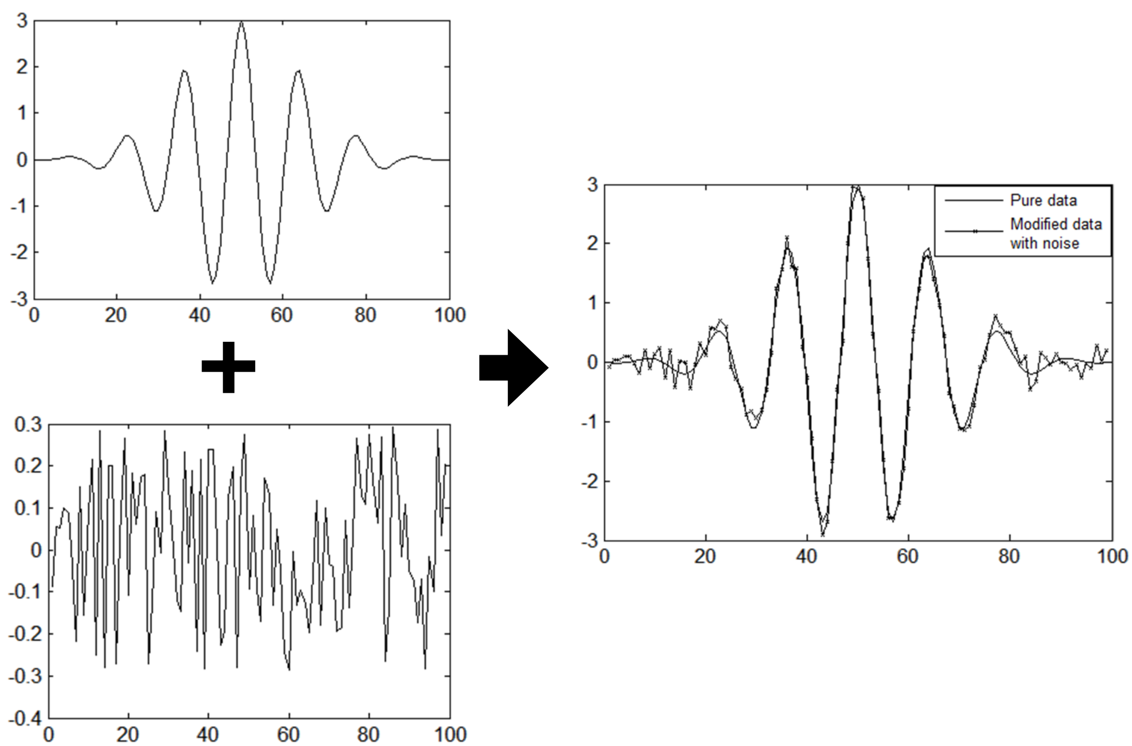


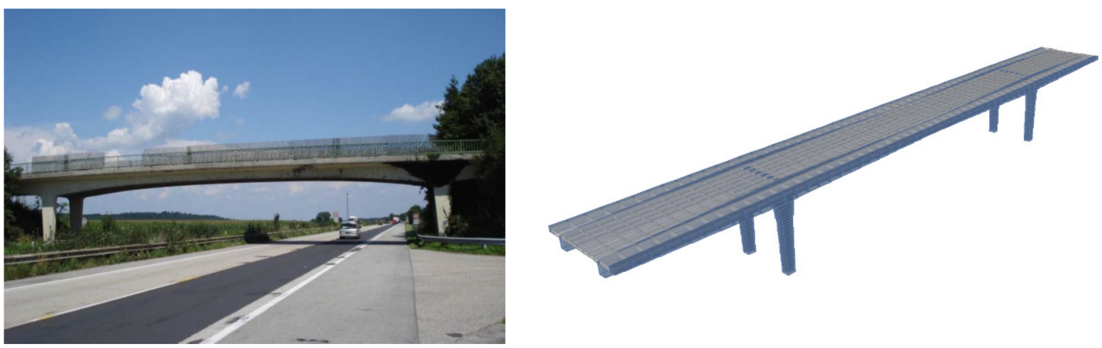
Figure 1. The original and modified data with noise ratio of 10%; the noise signal is a white noise (at bottom left) and the data is a Gaussian wave packet function (at top left)

It can be inferred from Figure 1 that the imposed noise affects the low amplitude parts of the measurement more than the higher amplitude parts of the excitation. This is due to the constant amplitude of the noise during the time and the fact that the amplitude of the noise is chosen as a ratio of the maximum peak of the data.

In this paper, the simulated data in reference state and damaged conditions are modified for each level of noise. Then the effect of noise in reference state along with the effect of noise in measured conditions are investigated. Six cases are chosen for the reference state, from which four are used in ARTeMIS for evaluating a threshold and two are used to check that threshold.

5. CASE STUDY AND DISCUSSION

Herein, the case study is the model of a bridge structure, namely S101, located in Reibersdorf, Austria. In [7], this structure was progressively damaged and the ambient vibration data was recorded continuously to evaluate the SSDD method. In this study, the finite element model of this structure is used to simulate the damage in a specific location of the bridge, i.e. center of one of the main girders, with various extents. The finite element model is calibrated using the measured data from the bridge to have a precise estimation of the behaviour of structure. The bridge structure and its finite element model are shown in Figure 2. The natural frequencies of the analytical model and the bridge structure are also compared at Table 1.



(a) **(b)**

Figure 2. a S101 bridge structure, Austria, and **b** its calibrated finite element model

Table 1. Natural frequencies of the bridge structure in undamaged condition obtained from the measured data and finite element model

	Measured data (Hz)	Finite element model (Hz)
First bending mode	4.05	4.04
First torsional mode	6.30	6.08
Second bending mode	9.69	10.72
Second torsional mode	13.29	12.85
Third bending mode	15.93	19.58

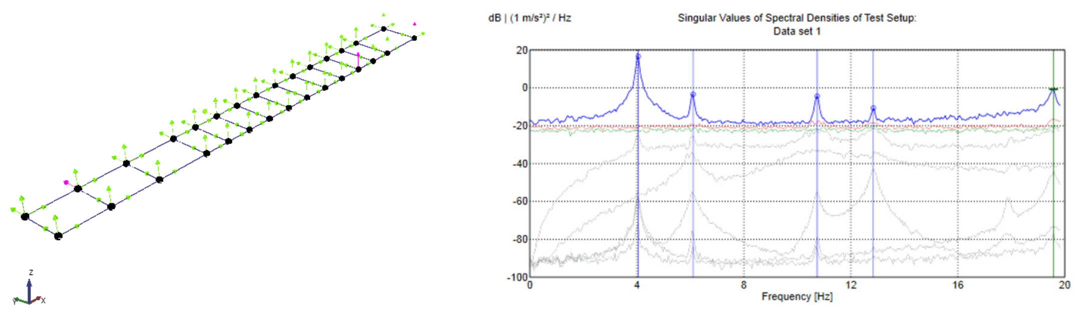
It can be inferred from Table 1 that the finite element model of the structure can be a good representative of the dynamic properties of the bridge. As mentioned in previous section, the purpose of this calibration is to have a realistic model of a bridge and it does not affect the assessment of the functionality of the damage detection technique. The effect of bearings in simulating the damage in other elements of the bridge is neglected.

5.1. Damage and data simulation

As a demonstrative example, the damage is modeled only in the center of one of the main girders of the bridge. The reason of choosing the main girder for the damage location is because of its significant effect on the functionality of the bridge and investigating the sensitivity of the damage detection technique to such a damage. The effect of damage location in different element types was investigated in [8], in which the data was assumed as pure and without noise. It is assumed that the noise ratio and damage location should not have any interaction that affects the functionality of the damage detection technique. However, this interaction is intended to be studied in future research papers.

In the girder, the damage is simulated by reducing a ratio, namely damage ratio, of its section dimension around the strong axis. The damage ratio varies among 20% (mild damage), 40% (medium damage) and 80% (severe damage).

The finite element model of the structure is excited with a white noise excitation as an acceleration time history in three directions. Moreover, the structure is vibrated by different white noise loads in various locations. The measured points to record acceleration time histories are illustrated in Figure 3a. Spectral densities of the simulated data obtained from undamaged reference case are shown in Figure 3b.



(a) **(b)**

Figure 3. a measuring-points corresponding to sensor locations; **b** Frequency domain decomposition of the simulated measurement data in undamaged structure

It can be seen in Figure 3b that the natural frequencies of the analytical model can be obtained from processing the simulated data accurately. Although, the structure is properly excited by white excitation, but some mode shapes cannot be captured. This stems from the location of the sensors and their resolution. As an example, the mode shapes associated to the longitudinal edges of the bridge cannot be captured by the sensors due to their small accelerations occurring in sensor locations.

5.2. Noise addition

In order to investigate the effect of noise on the SSDD technique, a white noise vector is created using a random number generator and is applied to the data as mentioned in section 4. There are 90 sensors (channels) modeled for this bridge and the noise for each channel is applied based on the maximum response of that channel. Therefore, for each noise ratio, 90 vectors of time history of noise is created and applied to the data. It should be noted that the noise ratio in the reference state is different from the noise in measured data; for different noise ratios in reference state, different noise ratios in the measured data is investigated. The noise ratios chosen for this study are 0%, 2%, 5%, 10%, 30%, 70%, 80% and 90% in the measured data. As an example, the pure and the modified data for 10% noise ratio in one of the channels are shown in Figure 4.

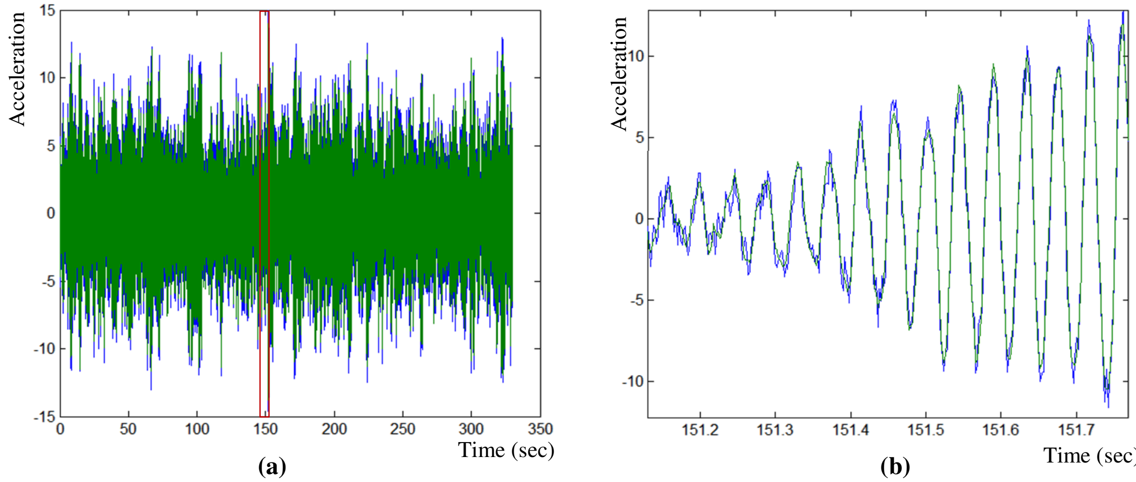


Figure 4. a The original and modified data with noise ratio of 10%, **b** a magnified part of the graph (blue: modified data; green: original data; red: the magnified box)

As mentioned in section 4 and demonstrated in Figure 4b, the effect of the noise in the data is more visible on low amplitude parts of the signal. It is worthwhile to illustrate how noise may affect on the eigen-structure of the measurements. Especially that the SSDD technique is sensible to the changes in the eigen-structure of the problem.

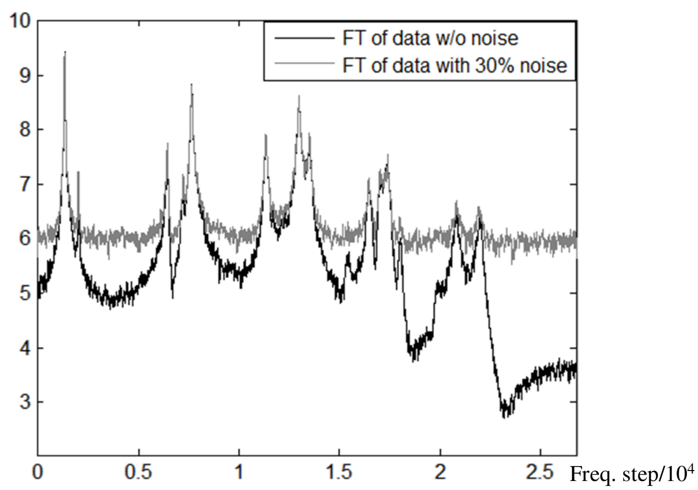
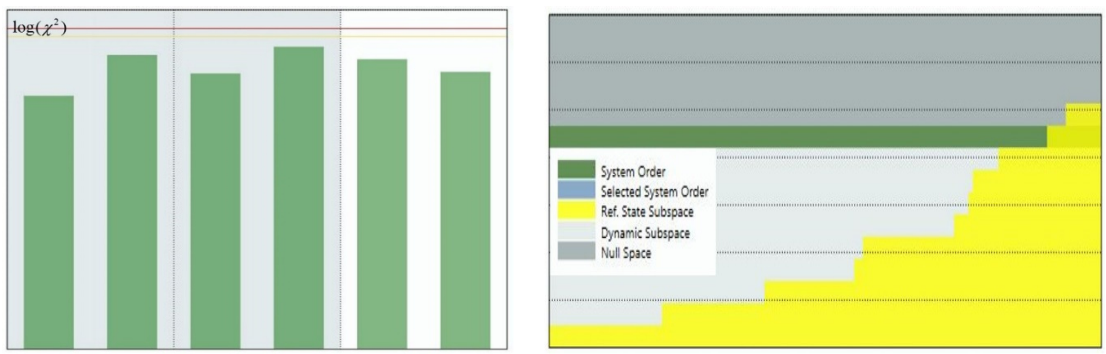


Figure 5. The Fourier transformation of the data without noise and the data with 30% noise ratio

In Figure 5 the measured data in one of the channels is shown in frequency domain. In this figure the Fourier transformation of the data without noise and with 30% noise ratio are compared. It can be inferred that the lower amplitude frequencies of the data are *drowning* in the noise. In other words, since the probability distribution of the random number generator is evenly distributed, the generated noise from it in the frequency domain is almost even too (about 6.0 in Figure 5). The higher the noise ratio becomes, the higher the level of the *drowning* amplitudes becomes. It should be expected that the higher noise ratios make the mode shapes and natural frequencies with lower amplitudes to become unidentifiable.

5.3. Damage detection results and discussion

The undamaged structure is excited for six cases from which four are used to create a threshold for the χ^2 value. The two remaining cases are then used to check the threshold. For each damage and noise ratio, the simulated data is created and the χ^2 test is performed. Subsequently, this value is compared to the computed threshold. As an example for the reference state data, the reference state for the data without noise is shown in Figure 6a. In order to validate the reference state, the null space of the Hankel matrix is illustrated in Figure 6b, which shows that only a small portion of the singular values are more than the system order. This suggests that the reference state in both cases are reliable.

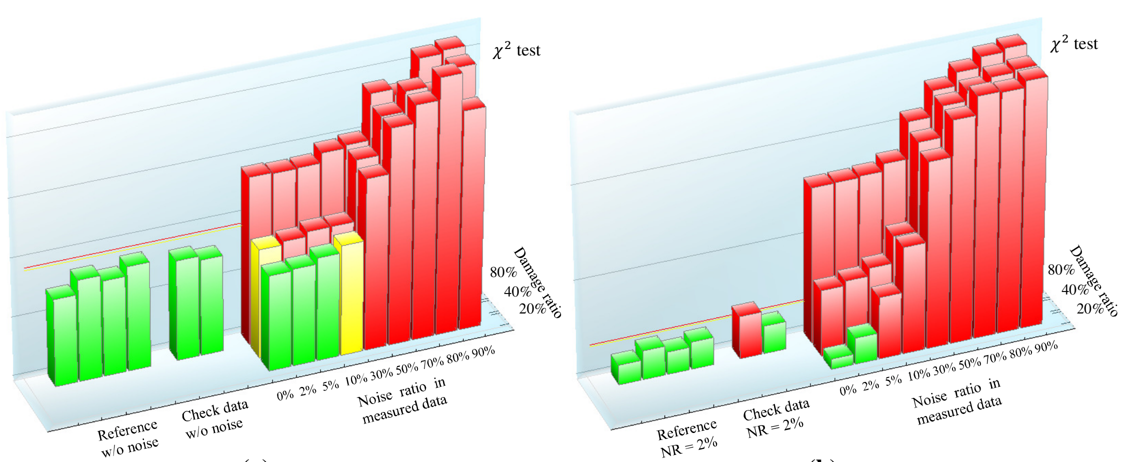


(a) **(b)**

Figure 6. a χ^2 values and thresholds of the reference state, and **b** validation of the reference state for data without noise

The threshold is computed for two significance values, namely critical zone for significance level 95% (shown with yellow line in Figure6-7) and unsafe zone for significance level 99% (shown with red line in Figure6-7). If the χ^2 test value computed from the structure becomes more than the yellow line, it suggests that the structure is in critical state. Similarly, if this value passes the red line, then the structure is estimated to be in unsafe conditions.

The χ^2 test values of the simulated data from different damage cases of the model are illustrated in Figure 7. For each damage ratio, the χ^2 value is computed and compared to the threshold acquired from the reference state of the structure.



(a) **(b)**

Figure 7. χ^2 test from SSDD technique for different damage and noise ratios in reference and checking data: **a** with no noise in reference data, **b** noise ratio of 2%, **c** noise ratio of 5%, **d** noise ratio of 10%, **e** noise ratio of 30%, **f** noise ratio of 50%, **g** noise ratio of 70% and **h** noise ratio of 90% (green: safe zone, yellow: critical zone and red: unsafe zone)

Considering Figure 7b, d, f and g, one of the checking data measurements is identified as critical state. This shows that when there is noise in the reference data, there should be more samples and measurements performed in order to acquire a reliable safety threshold.

It may be inferred from Figure 7 that the noise in the data may help the SSDD technique to identify the damage even for mild damage ratio. In fact, the case of no noise on the outputs in Figure 7a is a particular case that changes the kind of the linear model (1) that is used. Hence, the results without noise in 7a are not comparable to 7b-h from a theoretical viewpoint. It should be noted that the case of no noise on the outputs is purely theoretical and does not appear on real measurements.

One may interpret that the noise is misidentified as damage by this technique, since this noise difference leads to a higher variance in the residual in equation (4) for a fixed number of samples n. Thus, the information on the damage is drowned in the noise when the noise ratio increases in the each of the cases in Figures 7b-h, leading to unpredictable results. It should be noted that not only the higher noise in the measured data than the reference state can be interpreted as damage but also the lower noise in the measured data considering the reference state noise ratio can also affect the outcome of the damage detection technique. However, in most cases, results are stable for the region between 2% and 30% noise on the outputs.

Comparing Figures 7b-h with each other, it becomes clear that a higher noise ratio in the reference state leads to more uncertainty on the estimated null space S in equation (4) and on the residual covariance, thus leading to a lower damage detection resolution. Since the asymptotic properties of the damage detection residual for a large number of data samples n are in theory not affected by the output noise, this effect should be compensated by using longer datasets when noise ratios are high.

Moreover, in most of the cases the minimum of the χ^2 value is corresponding to the equivalent noise ratio in the measured and reference data, which may confirm the previous statement. Therefore, in order to have a comparison regardless of this effect, in Figure 8, the results from the χ^2 test of measured data with equal noise ratio in the reference state and measured data are illustrated. The χ^2 values are scaled for each noise ratio in order to make the thresholds identical.

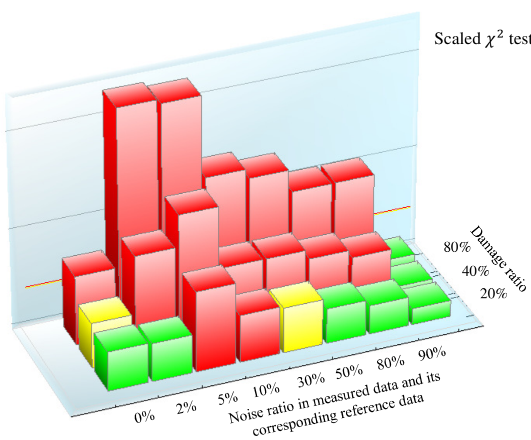


Figure 8. Scaled χ^2 test from SSDD technique for different damage ratios and different noise ratios in reference, checking data and measured data (green: safe zone, yellow: unsafe zone and red: critical zone)

It can be seen in Figure 8 that the SSDD technique identifies the damage in the data for most of the noise ratios. Moreover, expectedly the noise in the data can affect more on the outcome for lower damage ratios. It can be inferred from this image that existence of low ratios of noise in the data may help the damage detection technique. In other words the damage may not be identified from the data without noise or with high noise ratios. It can be seen that the optimum range of noise ratio in this set of data is from 2% to 30% in both reference and measured data with a peak about 5%.

It can be seen from Figure 7 and 8 that in most of the cases, even with very high noise ratios the damage can be identified using the SSDD technique. The reason for the detection power of such high noise ratios can stem from the fact that the subspace based damage detection technique evaluates the changes in the eigen-structure of the model. Therefore, since the noise implemented in the measurements is simply created by adding a white noise to the measured data, the eigen-structure can be affected only for high noise ratios, as shown in Figure 5.

6. CONCLUSION

In this paper the effect of noise on the input measurements of a damage detection technique, namely statistical subspace-based damage detection technique was investigated. A finite element model from a real bridge structure was created and calibrated. This model was employed as a realistic base model to simulate different damage-noise scenarios. In a single element the damage was modeled for different damage ratios. The damage ratio varied among mild (20%), medium (40%) and severe (80%) conditions. For each case, a time history analysis was performed with white noise excitation imposed on the structure. Different noise ratios were applied and added to the data to investigate the effect of the noise for different damage ratios.

It was concluded that this method can detect the damage in the structure for most of the cases even with high noise ratios in the data. The reason is that this method is monitoring the change in the Eigen-structure of the measurements in which the white noise cannot significantly affect the Eigen-structure of the data except for high noise ratios.

Moreover, the difference of noise ratios in the data used in reference state and the measured data can be also identified as damage, affecting the χ^2 test value from SSDD technique. Therefore it should be concluded that the noise in the data and reference state should be almost the same ratio to have a valid output result from this technique. In addition, if there is noise in the data, more samples are needed to be measured in order to acquire reliable safety thresholds. Especially, taking longer datasets in the reference state should compensate for the influence of the noise ratios on the outputs.

It was shown that the noise in the data can affect more on the identification of lower damage ratios comparing to higher damage ratios which can be identified almost in all of the cases. Furthermore, existence of low ratios of noise in the data may help the damage detection technique. In other words, no noise in the data cannot refer to better estimation of the damage compared to the data with low noise ratio. It was observed in the case study, that the optimum range of noise ratio is from 2% to 30% with a peak at 5%, in both reference and measured data.

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