RNS Modular Computations for Cryptographic Applications
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1. Elliptic Curve Cryptography (ECC)

Elliptic curve over $\mathbb{F}_p$: $y^2 = x^3 + ax + b$ with $P$ a $\ell$-bit prime

Security levels: $\ell \in (160, \ldots, 600)$ bits

Curve level operations:
- point addition (ADD): $Q + Q'$
- point doubling (DBL): $Q + Q$
- scalar multiplication: $|k|Q = Q + \ldots + Q$

Security (ECDLP): knowing $Q$ and $|k|Q$, $k$ cannot be recovered

ECMPL: Elliptic Curve Discrete Logarithm Problem

3. RNS Computation Flow in ECC Applications

RNS allows to perform some field level operations in parallel

5. New RNS Modular Inversion (MI) (CHES 2013)

State-of-the-art RNS MI methods:
- based on Fermat’s Little Theorem (FLT-MI): $X^{-1} = X^{p-2} \mod P$
  - i.e. a large exponentiation with a lot of modular reductions
  - which costs $O(\log_2 P \times n^2)$ EMUs
- very limited parallelisation due to internal data dependencies

Proposed method PM-MI:
- extended binary Euclidean algorithm (binary-ternary version)
- uses the plus-minus trick:
  - if $X$ and $Y$ are odd then $X + Y \equiv 0 \mod 4$ or $X - Y \equiv 0 \mod 4$
  - PM-MI works without BE and costs $O(\log_2 P \times n)$ EMUs

Example: # EMUs for $\ell = 192$ bits

<table>
<thead>
<tr>
<th>$n \times w$</th>
<th>PM-MI</th>
<th>PM-MI’</th>
<th>Gain Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 $\times$ 17</td>
<td>103140</td>
<td>5474</td>
<td>18</td>
</tr>
<tr>
<td>9 $\times$ 22</td>
<td>61884</td>
<td>4106</td>
<td>15</td>
</tr>
<tr>
<td>7 $\times$ 29</td>
<td>40110</td>
<td>5193</td>
<td>12</td>
</tr>
</tbody>
</table>

6. Fast Patterns for RNS Computations (ASAP 2014)

Cost of standard and modular multiplications in RNS:
- standard: $n$ EMUs
- modular: $2n^2 + O(n)$ EMUs

Proposed method:
- splits operands into 2 parts: $\overline{x} = (K_1) \times (M_3) + (R_s)$
- allows to replace $2n$ moduli by only $\frac{2n}{2}$
- allows for modular reduction
- requires an hypothesis on $P$: OK for ECC/DH, but not for RSA

Cost for some patterns (#EMUs):

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Cost</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB \mod P$</td>
<td>$2n^2 + 4n$</td>
<td>$2.5n^2 + 12.5n$</td>
</tr>
<tr>
<td>$A^2 \mod P$</td>
<td>$2n^2 + 4n$</td>
<td>$1.75n^2 + 10.5n$</td>
</tr>
<tr>
<td>$\ell A \mod P$</td>
<td>$2n^2 + 8n$</td>
<td>$1.75n^2 + 16.5n$</td>
</tr>
<tr>
<td>$\ell A^2 \mod P$</td>
<td>$2n^2 + 12n$</td>
<td>$2.5n^2 + 32.5n$</td>
</tr>
</tbody>
</table>

Usage for Diffie-Hellman or ElGamal:

In hardware:

1. Elliptic Curve Cryptography (ECC)
2. Residue Number System (RNS)
3. RNS Computation Flow in ECC Applications
4. State-of-the-Art Algorithms and Architectures
5. New RNS Modular Inversion (MI) (CHES 2013)
6. Fast Patterns for RNS Computations (ASAP 2014)

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