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The Modified Bidomain Model with Diffusive Inclusions

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The standard bidomain model

- Widely accepted model
- Very good for the healthy tissues

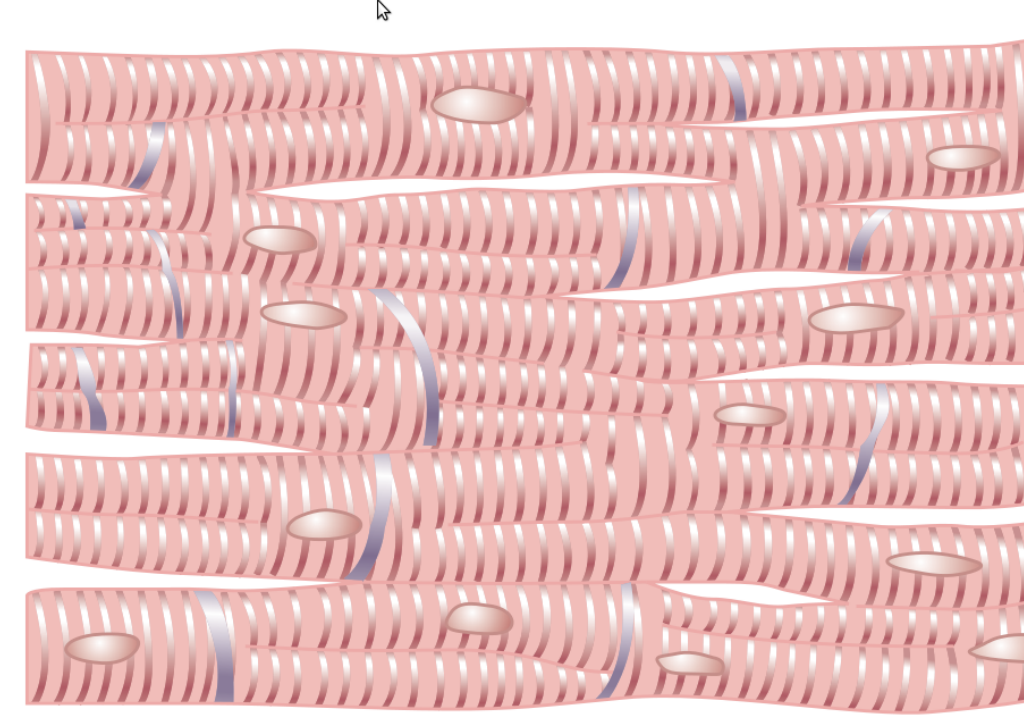


Figure: The patch of the cardiac tissue [4].

- Derived from the microscopic problem
- Assumptions used
 - periodicity
 - uniformly distributed ECM
 - anisotropic conductivities

Drawbacks - Examples

- There are non-small regions where myocytes are not present
- The **laminar structure**

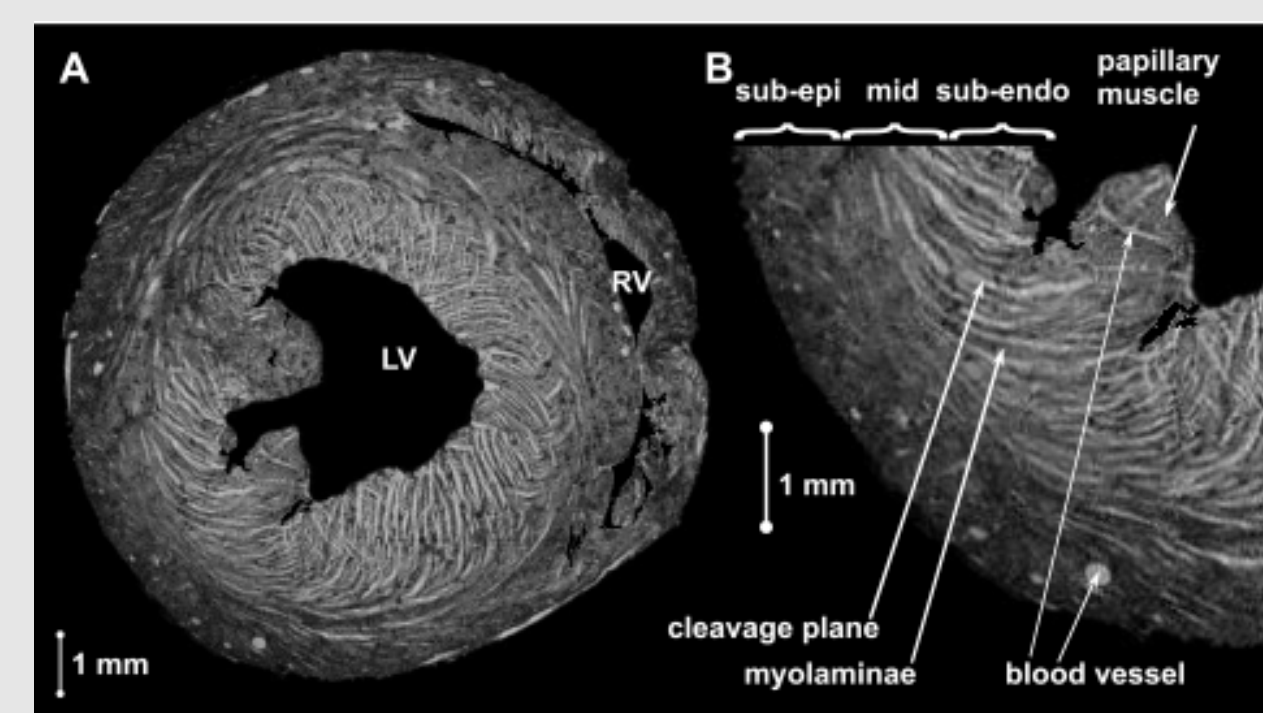


Figure: The laminar structure of myocardium. [2]

- The **infarct border zone**.

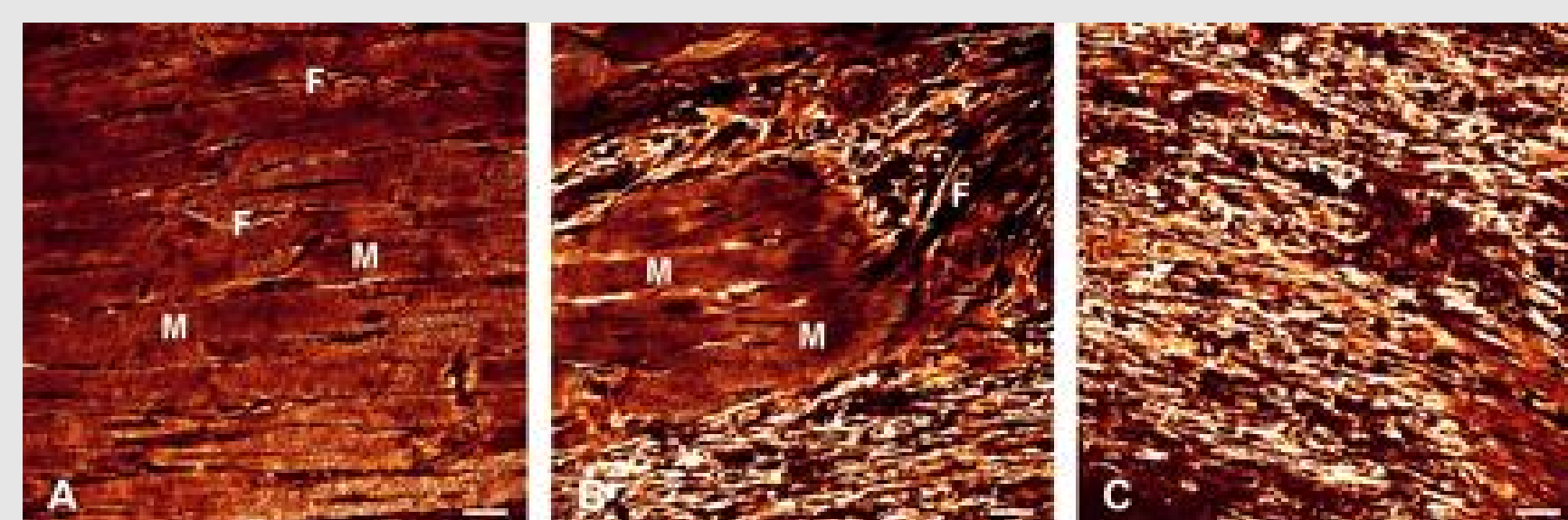


Figure: Fibroblast organization in sheep: normal ventricular myocardium (A), infarct border zone (B) and centre (C), 1 week after infarction. [3]

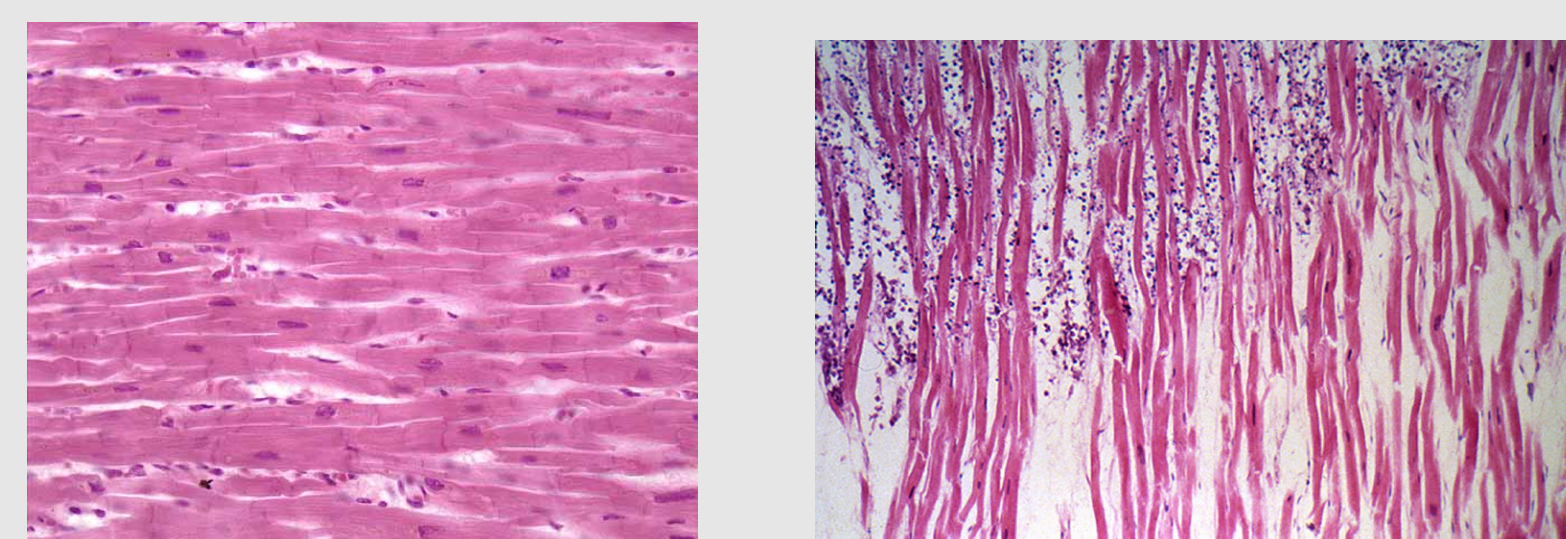
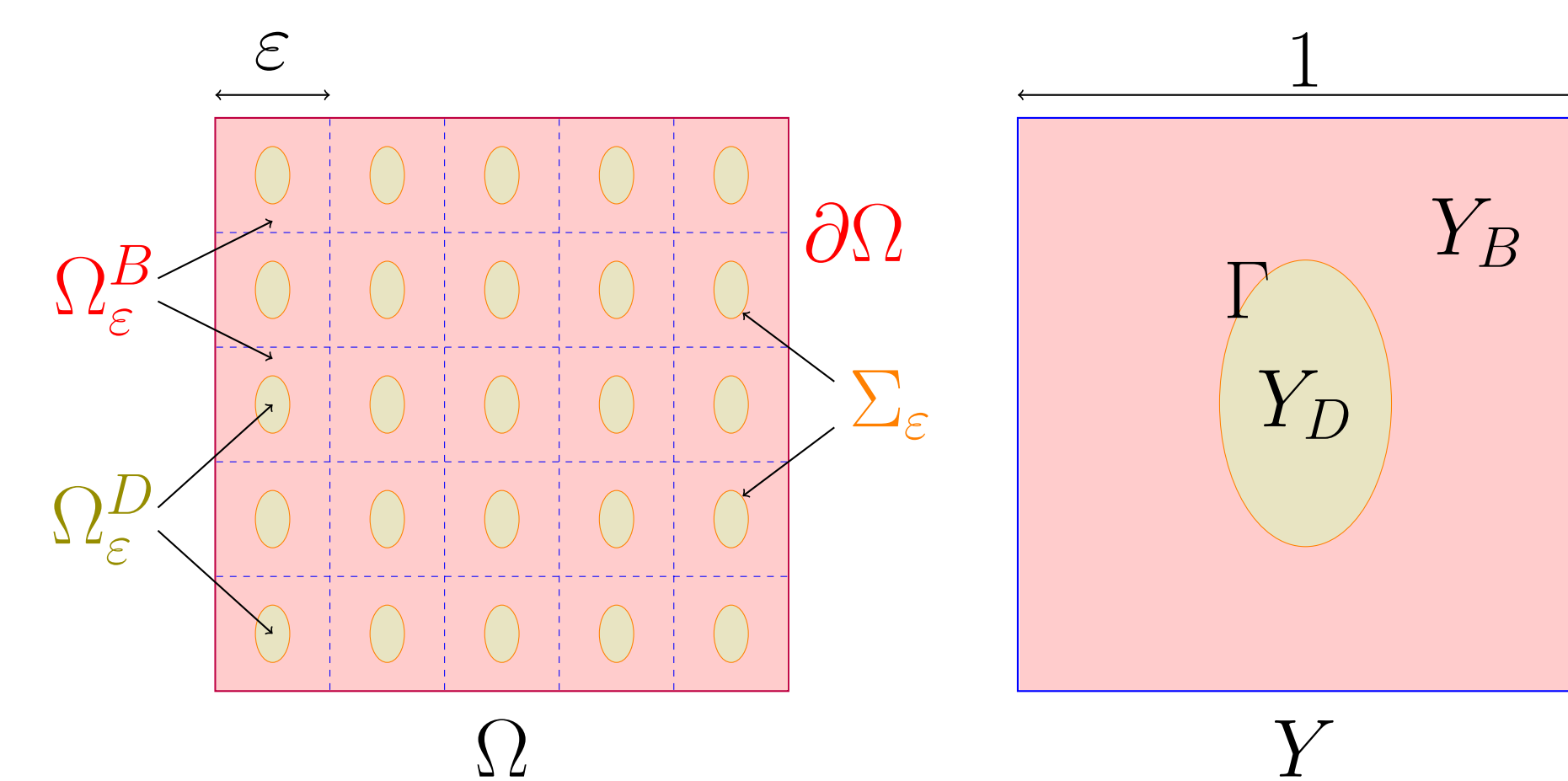


Figure: Normal tissue and Myocardial infarction in human. Tissue histology.

Modelling assumptions

- Periodic diffusive inclusions of size ε .
- Isotropic conductivity in the inclusions.
- No cells in the inclusions. Passive conductors.
- Standard transmission conditions.

Microscale model



- Bidomain model

$$\begin{aligned} \partial_t h_\varepsilon + g(v_\varepsilon, h_\varepsilon) &= 0, & \text{in } \Omega_\varepsilon^B, \\ \partial_t v_\varepsilon + I_{ion}(v_\varepsilon, h_\varepsilon) &= \nabla \cdot (\sigma_\varepsilon^i \nabla u_\varepsilon^i), & \text{in } \Omega_\varepsilon^B, \\ \partial_t v_\varepsilon + I_{ion}(v_\varepsilon, h_\varepsilon) &= -\nabla \cdot (\sigma_\varepsilon^e \nabla u_\varepsilon^e), & \text{in } \Omega_\varepsilon^B. \end{aligned}$$

- intra- and extra-cellular conductivities σ_ε^i and σ_ε^e
- unknown potentials u_ε^i and u_ε^e
- transmembrane potential $v_\varepsilon := u_\varepsilon^i - u_\varepsilon^e$
- $I_{ion}(v, h)$ and $g(v, h)$ define specific ionic model

- Diffusive inclusions

$$\nabla \cdot (\sigma_\varepsilon^d \nabla u_\varepsilon^d) = 0, \quad \text{in } \Omega_\varepsilon^D.$$

- conductivity σ_ε^d
- unknown potential u_ε^d

- Interface

$$\left. \begin{aligned} \sigma_\varepsilon^i \nabla u_\varepsilon^i \cdot \mathbf{n}_{\Sigma_\varepsilon} &= 0, \\ \sigma_\varepsilon^e \nabla u_\varepsilon^e \cdot \mathbf{n}_{\Sigma_\varepsilon} &= \sigma_\varepsilon^d \nabla u_\varepsilon^d \cdot \mathbf{n}_{\Sigma_\varepsilon}, \\ u_\varepsilon^e &= u_\varepsilon^d, \end{aligned} \right\} \quad \text{on } \Sigma_\varepsilon.$$

Method: Homogenisation

- Assume:

$$\begin{aligned} u_\varepsilon^i(t, x) &= u_0^i(t, x) + u_1^i(t, x, \frac{x}{\varepsilon}) + \dots, \\ u_\varepsilon^e(t, x) &= u_0^e(t, x) + u_1^e(t, x, \frac{x}{\varepsilon}) + \dots. \end{aligned}$$

- Let $\varepsilon \rightarrow 0$. Derive the homogenised problem and the cell problems.
- According to [1], *a priori* estimates ensure the two-scale convergence

Limit macroscale model

- The limit problem in whole Ω :

$$\begin{aligned} \partial_t h_0 + g(v_0, h_0) &= 0, \\ |Y_B|(\partial_t v_0 + I_{ion}(v_0, h_0)) &= \nabla \cdot (\tilde{\sigma}_i \nabla u_0^i), \\ |Y_B|(\partial_t v_0 + I_{ion}(v_0, h_0)) &= -\nabla \cdot (\tilde{\sigma}_e \nabla u_0). \end{aligned}$$

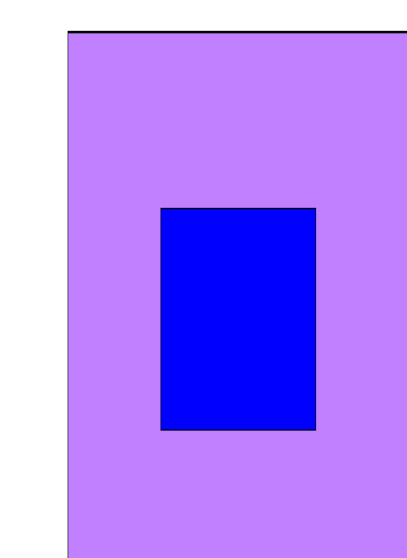
- New conductivities

$$\begin{aligned} \tilde{\sigma}_i &= \sigma^i |Y_B| + A^i(\sigma_i, w^i), \\ \tilde{\sigma}_e &= \sigma^e |Y_B| + \sigma^d |Y_D| + A(\sigma_e, \sigma_d, w) \end{aligned}$$

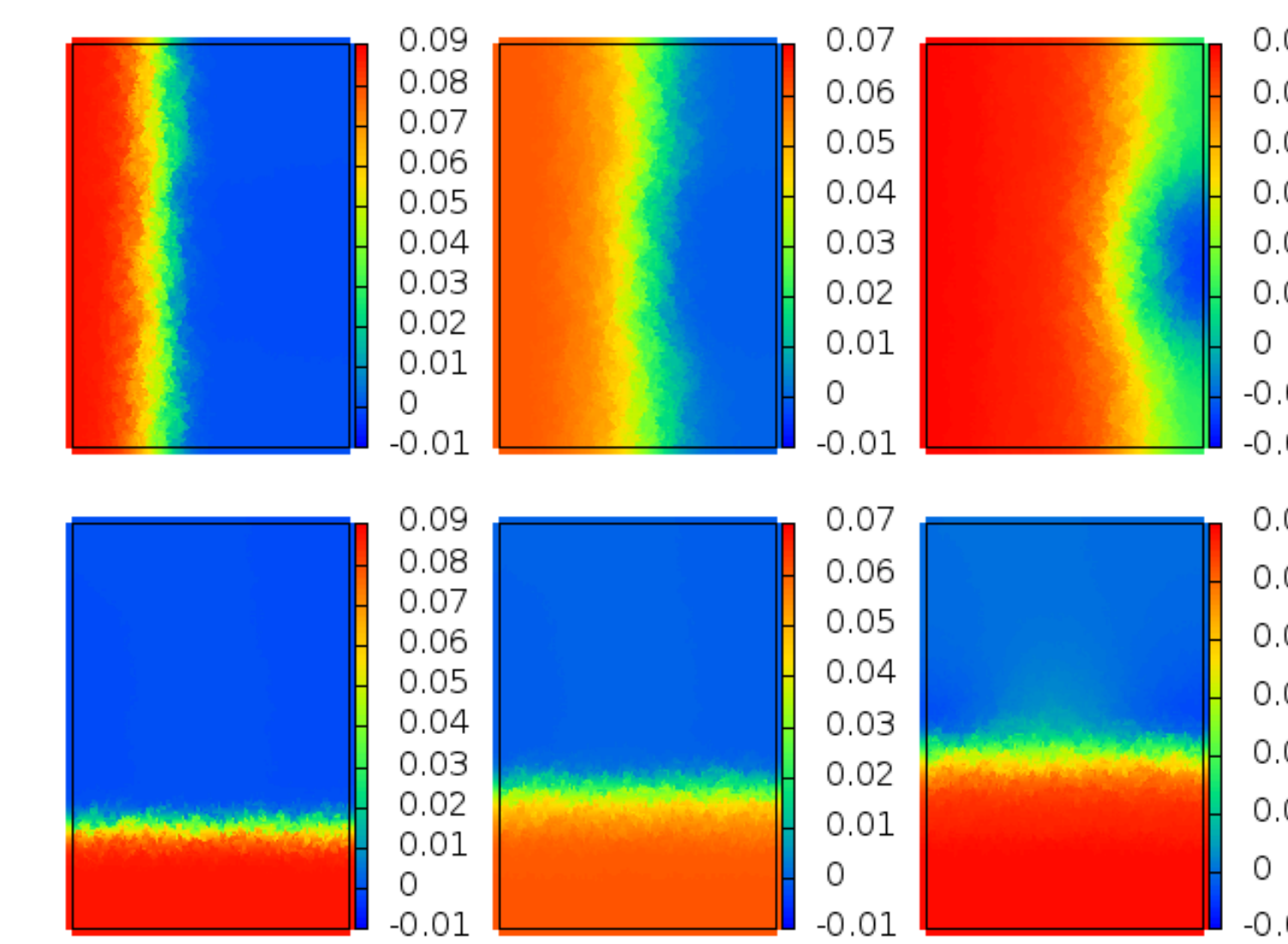
where A_i and A are matrices obtained by solving the cell problems on Y for unknown functions w^i and w .

Remarks:

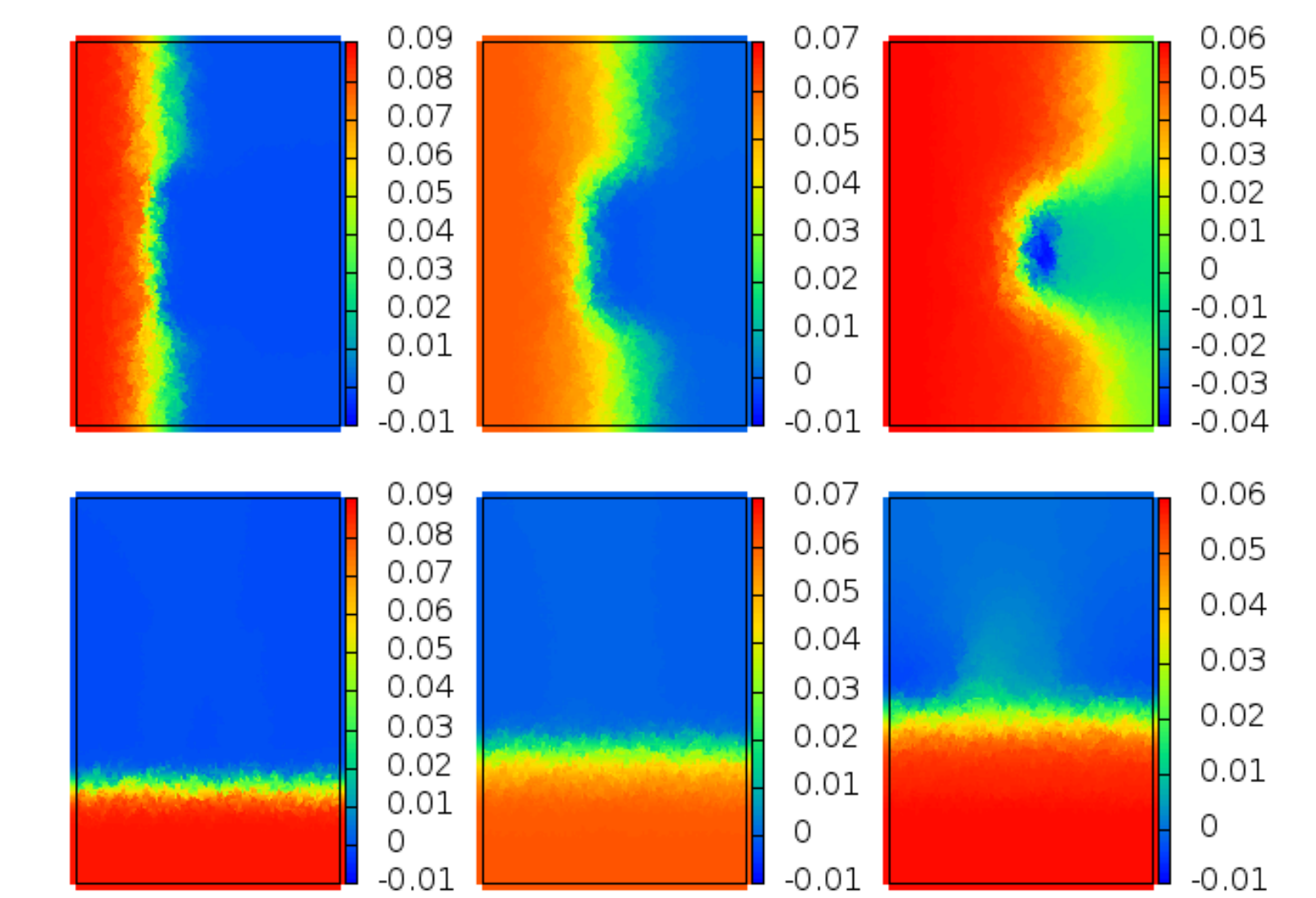
- The conductivities depend on the volume fraction and on the geometry of the diffusive part.
- If $\sigma_e = \sigma_d$ then on the volume fraction only.
- If $|Y_D| = 0$ it is the standard bidomain model.
- If $|Y_B| = 0$ it is only the diffusion model.



(a) Domain, purple: standard model, blue: modified.



(b) Circular inclusions.



(c) Elliptical inclusions. Axes length ratio 1:4.

Figure: Simulation of the standard bidomain with the "scar patch". $\sigma^d = 3$, volume fraction 0.18, comparing inclusions geometries.

Conclusions

- Inclusions change the speed of propagation and depending on geometry, the anisotropy ratio.
- Rigorous and practical way to link the structural disease to macroscopic conductivities.

Future work

- Use the late enhancement MRI for the volume fraction of the extracellular space.
- Cells in the additional ECM. Boundary conditions.

Numerical results

- FreeFem++* and *gnuplot*.
- Mitchell Schaeffer ionic model.
- 2D simulations on Ω rectangle, SBDF2 numerical scheme.
- Simple shapes of inclusions (circles and ellipses).

Table: The values of the new conductivities depending on the diffusion inclusions [10^{-1} S/m]. The first row: values for the standard bidomain model, without inclusions.

geometry	vol frac	σ^d	σ_{i11}^*	σ_{i22}^*	σ_{e11}^*	σ_{e22}^*
-	-	-	1.74	0.19	3.9	1.97
circle	0.18	3	1.29	0.18	3.28	2.45
ellipse (1:4)	0.18	3	0.31	0.19	0.75	2.61
circle	0.2	3	1.26	0.17	3.29	2.53
ellipse (1:1.5)	0.2	3	1.13	0.18	2.82	2.58
ellipse (1:2)	0.2	3	0.86	0.18	2.09	2.64
circle	0.4	3	1.07	0.15	3.42	3.53
ellipse (1:1.5)	0.4	3	0.81	0.16	2.53	3.70
circle	0.7	3	0.69	0.09	5.08	7.89

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