

Supplemental Material: Position-dependent Importance Sampling of Light Field Luminaires

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Abstract—This complementary document provides the reader with more details on the derivation of the different equations contained in the paper entitled "Position-dependent Importance Sampling of Light Field Luminaires".

Index Terms—Models of Light Sources, Light Field, Importance Sampling, Real-time Rendering,

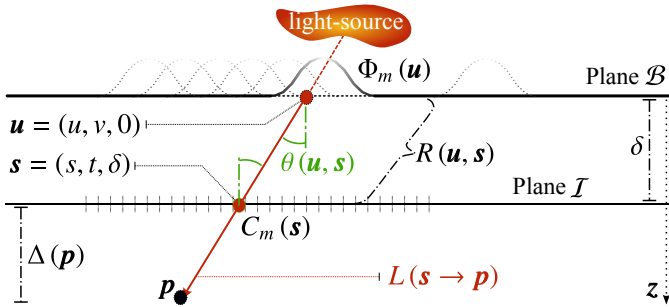


Fig. 1. Model parametrization of [1]. The 4D space of rays emitted from the light source is parametrized by a position \mathbf{u} on a plane \mathcal{U} that supports the reconstruction 2D basis functions Φ_m , and a position s on a plane \mathcal{S} , that supports the C_m images. $\Phi_m(\mathbf{u})$ is a short notation for $\Phi_m(u, v)$ and $C_m(s)$ for $C_m(s, t)$. δ is the inter-plane distance and $\Delta(\mathbf{p})$ is the distance between \mathbf{p} and \mathcal{S} . \mathbf{u} , s and \mathbf{p} are aligned.

1 FROM GOESELE'S MODEL TO EQUATION 1

In their paper, Goesele et al. describe their (cf. Equation 8 in [1]) light field luminaire by the following equation (using the notation introduced in Figure 1):

$$L(\mathbf{u} \rightarrow s) = \frac{R^2(\mathbf{u}, s)}{\cos^2 \theta(\mathbf{u}, s)} \sum_m C_m(s) \Phi_m(\mathbf{u})$$

with $L(\mathbf{u} \rightarrow s)$ representing the radiance transferred from \mathbf{u} to s . To simplify the notation, reader can note that the geometric

configuration leads to

$$R(\mathbf{u}, s) = |\mathbf{u} - s|$$

and to

$$\delta = |\mathbf{u} - s| \cos \theta(\mathbf{u}, s).$$

By combining these three equations together we obtain the equation used in our paper:

$$L(\mathbf{u} \rightarrow s) = \frac{|\mathbf{s} - \mathbf{u}|^4}{\delta^2} \sum_m C_m(s) \Phi_m(\mathbf{u}). \quad (1)$$

2 DERIVATION FOR $I_m(\mathbf{p})$

The irradiance $I(\mathbf{p})$ that can potentially reach \mathbf{p} from the light source is:

$$I(\mathbf{p}) = \int_{s \in \mathcal{I}} L(s \rightarrow \mathbf{p}) \frac{\cos \theta(\mathbf{u}, s)}{|\mathbf{s} - \mathbf{p}|^2} ds$$

According to the geometric configuration introduced in Figure 1, $\Delta(\mathbf{p}) = |\mathbf{s} - \mathbf{p}| \cos \theta(\mathbf{u}, s)$. Therefore, it can also be written as presented in our paper:

$$I(\mathbf{p}) = \int_{s \in \mathcal{I}} L(s \rightarrow \mathbf{p}) \frac{\Delta(\mathbf{p})}{|\mathbf{s} - \mathbf{p}|^3} ds \quad (2)$$

Combining Equation 1 with Equation 2 leads to

$$I(\mathbf{p}) = \sum_m I_m(\mathbf{p})$$

$$I_m(\mathbf{p}) = \int_{s \in \mathcal{I}} \frac{|\mathbf{s} - \mathbf{u}|^4}{\delta^2} \frac{\Delta(\mathbf{p})}{|\mathbf{s} - \mathbf{p}|^3} C_m(s) \Phi_m(\mathbf{u}) ds.$$

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We replace $\Delta(\mathbf{p})$ by $|s - \mathbf{p}| \cos \theta(\mathbf{u}, s)$ and δ by $|\mathbf{u} - s| \cos \theta(\mathbf{u}, s)$ to obtain

$$\begin{aligned}
 I_m(\mathbf{p}) &= \int_{s \in \mathcal{I}} \frac{|s - \mathbf{u}|^4 |s - \mathbf{p}| \cos \theta(\mathbf{u}, s)}{|\mathbf{u} - s|^2 \cos^2 \theta(\mathbf{u}, s) |s - \mathbf{p}|^3} C_m(s) \Phi_m(\mathbf{u}) ds \\
 \Leftrightarrow I_m(\mathbf{p}) &= \int_{s \in \mathcal{I}} \frac{|s - \mathbf{u}|^2 |s - \mathbf{p}| \cos \theta(\mathbf{u}, s)}{\cos^2 \theta(\mathbf{u}, s) |s - \mathbf{p}|^3} C_m(s) \Phi_m(\mathbf{u}) ds \\
 \Leftrightarrow I_m(\mathbf{p}) &= \int_{s \in \mathcal{I}} \frac{|s - \mathbf{u}|^2 |s - \mathbf{p}| \cos^2 \theta(\mathbf{u}, s)}{\cos^3 \theta(\mathbf{u}, s) |s - \mathbf{p}|^3} C_m(s) \Phi_m(\mathbf{u}) ds \\
 \Leftrightarrow I_m(\mathbf{p}) &= \int_{s \in \mathcal{I}} |s - \mathbf{p}| \frac{|s - \mathbf{u}|^2 \cos^2 \theta(\mathbf{u}, s)}{\cos^3 \theta(\mathbf{u}, s) |s - \mathbf{p}|^3} C_m(s) \Phi_m(\mathbf{u}) ds \\
 \Leftrightarrow I_m(\mathbf{p}) &= \int_{s \in \mathcal{I}} |s - \mathbf{p}| \frac{\delta^2}{\Delta^3(\mathbf{p})} C_m(s) \Phi_m(\mathbf{u}) ds
 \end{aligned}$$

Since $\Delta(\mathbf{p})$ and δ do not depend on s , we finally obtain the equations introduced in our paper:

$$I(\mathbf{p}) = \sum_m I_m(\mathbf{p}) \quad (3)$$

$$I_m(\mathbf{p}) = \frac{\delta^2}{\Delta^3(\mathbf{p})} \int_{s \in \mathcal{I}} |s - \mathbf{p}| C_m(s) \Phi_m(\mathbf{u}) ds. \quad (4)$$

REFERENCES

- [1] M. Goesele, X. Granier, W. Heidrich, and H.-P. Seidel, "Accurate light source acquisition and rendering," *ACM Trans. Graph.*, vol. 22, no. 3, pp. 621–630, 2003.