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# Parallel Kelvin-Helmholtz instability in edge plasma

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**Abstract.** In the scrape-off layer (SOL) of tokamaks, the flow acceleration due to the presence of limiter or divertor plates rises the plasma velocity in a sonic regime. These high velocities imply the presence of a strong shear between the SOL and the core of the plasma that can possibly trigger some parallel shear flow instability. The existence of these instabilities, denoted as parallel Kelvin-Helmholtz instability in some works [1, 2] have been investigated theoretically in [3] using a minimal model of electrostatic turbulence composed of a mass density and parallel velocity equations. This work showed that the edge plasma around limiters might indeed be unstable to this type of parallel shear flow instabilities. In this work, we perform 3D simulations of the same simple mathematical model to validate an original finite volume numerical method aimed to the numerical study of edge plasma. This method combines the use of triangular unstructured meshes in the poloidal section and structured meshes in the toroidal direction and is particularly suited to the representation of the real complex geometry of the vacuum chamber of a tokamak.

The numerical results confirm that in agreement with the theoretical expectations as well as with other numerical methods, the sheared flows in the SOL are subject to parallel Kelvin-Helmholtz instabilities. However, the growth rate of these instabilities is low and these computations require both a sufficient spatial resolution and a long simulation time. This makes the simulation of parallel Kelvin-Helmholtz instabilities a demanding benchmark.

## 1. Introduction

Poloidal up-down asymmetries of turbulence are known to exist in tokamaks as Tore-Supra. These asymmetries correlate with the edge geometry and the location of the limiters : Increased turbulence level being observed in the close vicinity of limiters. An explanation on these poloidal asymmetries relies on the triggering of parallel shear flow instabilities between the rotating core plasma and the accelerated parallel flow caused by the limiters : In the scrape-off layer (SOL), the flow acceleration due to the presence of limiter or divertor plates rises the plasma velocity in a sonic regime. These high velocities imply the presence of a strong shear between the SOL and the core of the plasma that can possibly trigger some unstable behavior denoted as parallel shear flow instability in [1, 2]. In this contribution, we investigate this possible explanation with the help of 3D numerical simulations and indeed show that instabilities can develop due to the presence of limiters. The outline of this paper is as follows : in section 2, we present

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the mathematical model used for this study. Section 3 is devoted to a brief description of the numerical method that we have used. We conclude by presenting our results in section 4.

## 2. Mathematical model

The possible presence of a parallel instability created by the strong shear between the SOL velocity and the core plasma is here investigated using the same minimal electrostatic model than in [3]. In this model, the electron are adiabatic and ions are assumed to be cold, moreover the temperature variation across the SOL is not taken into account. The magnetic field is considered steady and given in a toroidal coordinate system  $(r, \theta, \phi)$  by

$$\vec{B} = B_t \vec{e}_\phi + B_p \vec{e}_\theta \text{ with } q = \frac{r B_t}{R_0 B_p} = 6 \quad (1)$$

The electric field is electrostatic  $\Rightarrow \vec{E} = -\nabla \Phi$  and the velocity is assumed to have the following form

$$\vec{u} = u_{\parallel} \vec{b} + \vec{u}_{drift} \quad (2)$$

where the perpendicular velocity retains only the electric drift contribution  $\vec{u}_{drift} = \frac{\vec{B} \times \nabla \Phi}{B^2}$

Given an electron reference temperature  $T_e$  and length scale  $L_{ref}$ , the variables are made dimensionless as follows

$$\tilde{x} = \frac{x}{L_{ref}}, \quad \tilde{u}_{\parallel} = \frac{u_{\parallel}}{c_s}, \quad \tilde{t} = t \frac{c_s}{L_{ref}}, \quad (3)$$

$$\tilde{\Phi} = \Phi \frac{e}{T_e}, \quad \tilde{B} = \frac{B}{B_0} \quad (4)$$

where  $c_s = \sqrt{\frac{T_e}{m_i}}$  is the sound speed. With this normalization, the model takes the form

$$\left\{ \begin{array}{l} \frac{\partial n}{\partial t} + \nabla \cdot n \vec{u} = \nabla \cdot (\mu_n D \nabla n) \\ \frac{\partial n u_{\parallel}}{\partial t} + \nabla \cdot (n u_{\parallel} \vec{u}) + \nabla_{\parallel} p = -n \vec{u} \cdot \frac{D \vec{b}}{Dt} + \nabla \cdot (\mu_u D \nabla n u_{\parallel}) \\ \vec{u} = u_{\parallel} \vec{b} + \rho^* \frac{\vec{B} \times \nabla \Phi}{B^2}, \quad \Phi = \ln \left( \frac{n}{n_0} \right) \\ p = n, \quad D = \mathbf{I} - \vec{b} \otimes \vec{b}^T \end{array} \right. \quad (5)$$

and is completely defined by the following three parameters representing respectively the dimensionless mass diffusion, parallel momentum diffusion and normalized Larmor radius.

$$\tilde{\mu}_n = \frac{\mu_n}{L_{ref} c_s}, \quad \tilde{\mu}_u = \frac{\mu_u}{L_{ref} c_s}, \quad \rho^* = \frac{m_i c_s}{e B_0 L_{ref}} \quad (6)$$

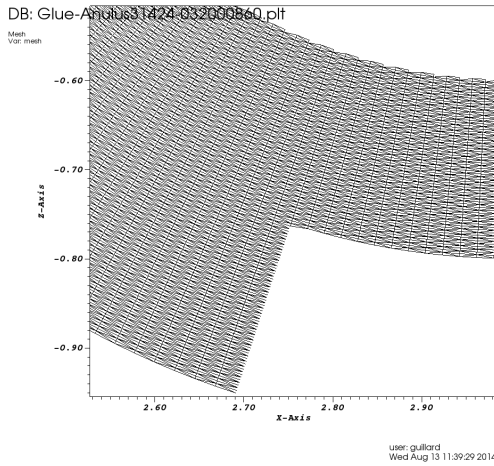
The computational domain is the full torus in  $\phi$  and an annulus of the poloidal cross section limited by a limiter of extend  $2\pi/10$  in  $\theta$  and  $0.25$  in  $r$ . The boundary conditions are given by

- Bohm boundary conditions on the limiter target plates :  $u_{\parallel}/c_s \geq 1$
- slip boundary condition on the exterior wall
- interior wall
  - Imposed parallel Mach number  $Ma_{\parallel} = Ma_{\parallel}(R_{min})$
  - Imposed mass flux  $(\mu_n D \nabla n) \cdot \vec{e}_r = given$

### 3. The numerical scheme

#### 3.1. Spatial discretization

The numerical scheme is described in details in [4]. It uses an unstructured finite element triangular mesh in the poloidal plane and a structured mesh in the toroidal direction. The use of Finite Element type mesh makes easy the representation of complex geometries as well as the use of possible adaptive mesh refinement. Although these possibilities are not exploited here, these features may be extremely useful as the need of more realism in plasma simulations will grow. The figure 1 displays an enlarged view of the mesh near the limiters target plates. The



**Figure 1.** Zoom on the poloidal mesh near the limiter,

discretization method uses a finite volume method using an exact Riemann solver to compute the convective fluxes at the interfaces between cells. This type of numerical method do not separate explicitly the parallel and perpendicular directions. This can be a shortcoming of the method in case of strongly anisotropic flow as this has to be compensated by an increased resolution and the use of large meshes. On the other hand, the present numerical method is genuinely conservative, is very robust with no addition of explicit artificial diffusivity and incorporates in an automatic way the curvature terms. Since, the system (5) is not written in cartesian coordinates, we briefly comment on the way to treat vector equations in our scheme : Assume we want to solve a (vector) conservative equation.

$$\frac{\partial \vec{u}}{\partial t} + \nabla \cdot F = \vec{0} \quad (7)$$

After integration on the cell  $i$  this equation becomes :

$$Vol_i \frac{d\vec{u}_i}{dt} + \sum_{j \in \kappa(i)} F_{ij} = \vec{0} \quad (8)$$

where  $F_{ij}$  are the numerical fluxes and  $\kappa(i)$  denotes the set of the neighbors of  $i$ . Note that the spatial integration is a mathematical operation that does not depend on the system of coordinate used. This step of the method is thus independent of this system of coordinates and can be performed in any convenient system (cartesian or not). However, in curvilinear system of coordinates, the basis functions depend on the space position and their projection does not commute with space integration. In our numerical scheme, we take this into account by projecting the sum of the fluxes on a representative basis vector of the cell  $i$  (that is thus different from cells to cells).

$$\left( Vol_i \frac{d\vec{u}_i}{dt} + \sum_{j \in \kappa(i)} F_{ij} \right) \cdot \vec{e}_i = \mathbf{0} \Rightarrow \quad (9)$$

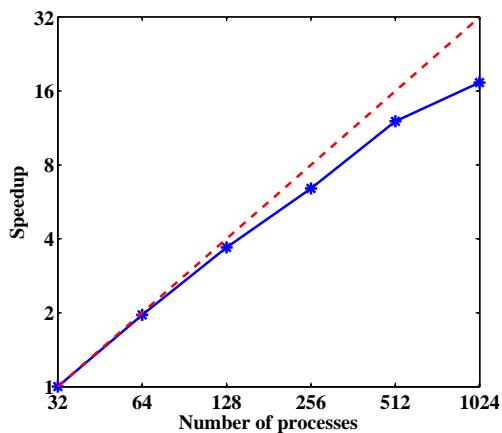
or with  $\mathbf{u}_i = \vec{u} \cdot \vec{e}_i$

$$Vol_i \frac{du_i}{dt} + \left( \sum F_{ij} \right) \cdot \vec{e}_i = \mathbf{0} \quad (10)$$

This procedure can be applied to any general curvilinear coordinate system. In this work, we have applied it with  $\vec{e}_i = \vec{b}$  to obtain the parallel velocity. Note that the curvature terms are implicitly discretized during this projection step. The method is made  $2^{nd}$ -order in space and time with the help of the MUSCL method in space and RK2 in time.

### 3.2. Parallel efficiency

The method uses domain partitioning for parallel computations with the MPI communication library to transmit information between processors. In the present study, we have used a mesh of  $31424 \times 128 = 4.02$  M of nodes. This corresponds to a spatial resolution of  $\sim 5mm$  in the radial and of  $1.2^\circ$  in the poloidal directions. This large number of nodes and in particular the large number of poloidal planes have been first used to ensure a sufficient resolution and accuracy of the numerical scheme but also to test the parallel implementation of the method and its performance when a large number of processors is used. Since, the method is explicit in time, we expect its scalability performances to be good. Figure 2 shows that this is indeed the case and that the speed-up obtained with up to 1024 processors scales linearly with the number of processors when the mesh is large enough.



**Figure 2.** Scalability on a mesh of 4.02 M nodes

#### 4. Results

The local linear stability theory developed in [3] predict instability growth if the background axisymmetric steady state verifies :

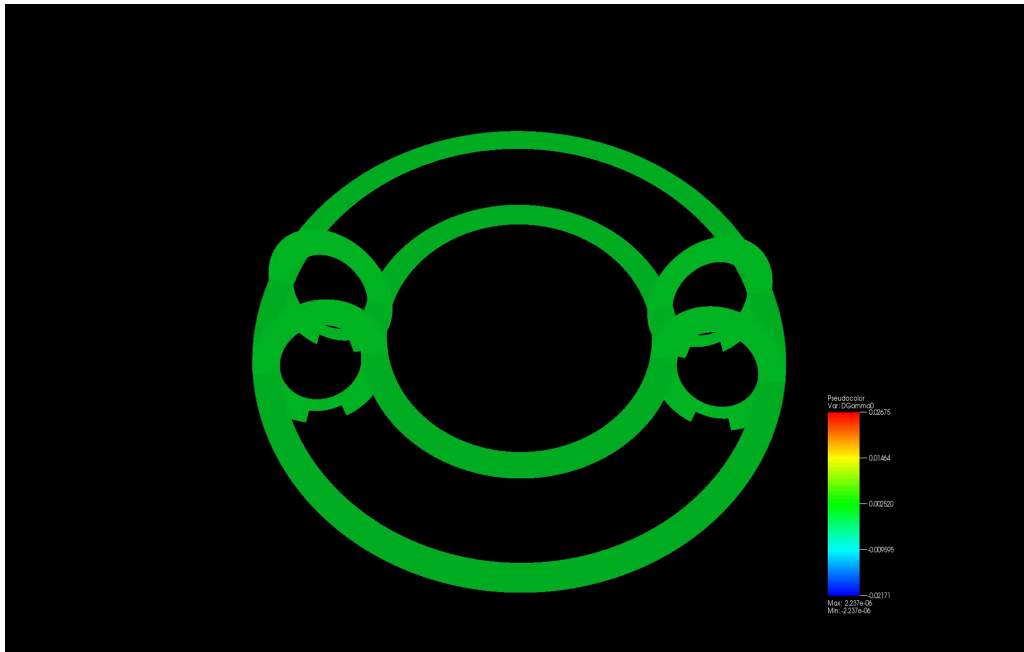
$$\left(\frac{dM_{//}}{dR}\right)^2 - \left(\frac{d \ln n}{dR}\right)^2 > 0 \quad (11)$$

For the present simulations, the values of the physical parameters defining the problems were:  $B_0 = 1.3T$ ,  $L_{ref} = 0.25m$ . An uniform temperature set to  $100eV$  corresponding to a sound speed of

$$c_s = \sqrt{\frac{T}{m_i}} = \sqrt{\frac{100 \cdot 1.6022 \cdot 10^{-19}}{1.6727 \cdot 10^{-27}}} \simeq 100000m/s \quad (12)$$

was used and the momentum diffusivity coefficient was  $\mu_u = 1m^2/s$  With this choice, the dimensionless parameters governing the flows are equal to :  $\tilde{\mu}_n \simeq 0.410^{-3}$   $\tilde{\mu}_u = 10^{-1}\tilde{\mu}_n$  and  $\rho^* = 3.10^{-3}$ . The core flow in the tokamak is defined by a velocity corresponding to  $Ma_i = 0.75$ .

For these values, the criterion (11) is satisfied on a large part of the domain upstream of the target plates. To check this theory, we have thus perturbed the 2D axisymmetric steady state displayed in figure 3

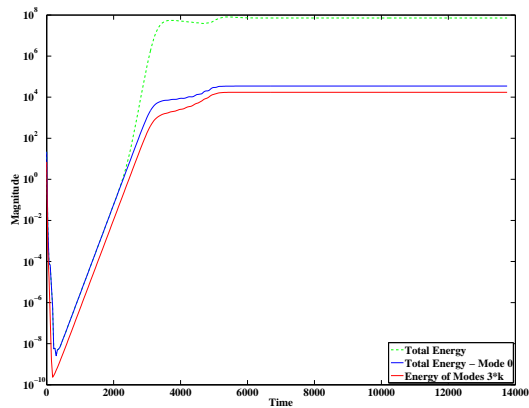


**Figure 3.** The figure displays the momentum perturbation i.e the difference between the parallel momentum and its corresponding value in the 2D axisymmetric steady state. At time  $t = 0$ , these perturbations are of order  $10^{-6}$  and cannot be observed on this figure.

by small parallel momentum fluctuations

$$\delta\Gamma = \Gamma_0(R, Z)(1 + \sum A_m \cos(m\phi)) \quad (13)$$

where the  $A_m$  are of order  $10^{-6}$ .



**Figure 4.** Growth of the perturbation

As shown in figure 4 after a phase where the perturbation decreases, the flow re-organizes and enters in a quasi-periodic behavior with regular creation/destruction of perturbations. The figure 5 displays this saturated quasi - steady state characterized by continuous creation/convection and destruction of perturbation.

## 5. Concluding remarks

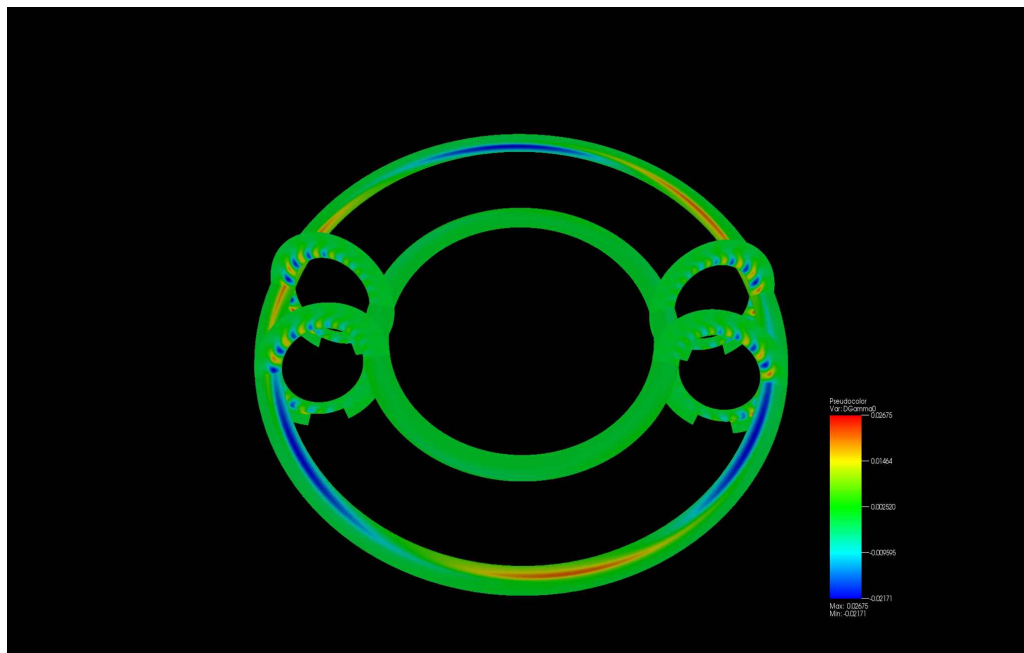
We have performed 3D simulations of a simple mathematical model to study the possible triggering of parallel shear flow instabilities in a tokamak in presence of limiters. These numerical results confirm that in agreement with the theoretical expectations, the sheared flows in the SOL can be subject to parallel Kelvin-Helmholtz instabilities. These results have also been confirmed using a different numerical method as reported in [5]. The growth rate of these instabilities is however low and it takes a very long time for an unstable mode to arise. Due to the large mesh used in this study, we believe that the numerical resolution is sufficient and that these simulation have been able to capture the parallel Kelvin-Helmholtz instability phenomenon. This must however be checked by performing simulations with increased resolution. In the future, it would be useful to consider also simpler benchmarks such as linear growth rates of drift or ballooning modes. This will be a next step in the development of this numerical method.

## Acknowledgments

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- [1] Garbet X, Fenzi C, Capes H, Devynck P and Antar G 1999 *Phys. Plasmas* **6** 3955-3965
- [2] Catto P, Rosenbluth M and Liu C 1973 *Phys. Fluids* **16** 1719
- [3] Schwander F, Chiavassa G, Ciralo G, Ghendrih P, Isoardi L, Paredes A, Sarazin Y, Serre E and Tamain P 2011 *Journal of Nuclear Materials* **415** S601 - S604 URL <http://www.sciencedirect.com/science/article/pii/S0022311510006677>
- [4] Bonnement A, Fajraoui T, Guillard H, Martin M, Mouton A, Nkonga B and Sangam A 2011 *ESAIM-Proc* **32** 163-176
- [5] Serre E, Bigot B, Colin C, Paredes A, Schwander F, Bufferand H, GCiralo, Ghendrih P, Tamain P, Bilanceri M, Guillard H and Nkonga B *15th European Fusion Theory Conference, 23-26 September 2013, Merton College, Oxford, UK*





**Figure 5.** Saturated quasi - steady state : The figure displays the difference between the parallel momentum and its corresponding value in the 2D axisymmetric steady state.