

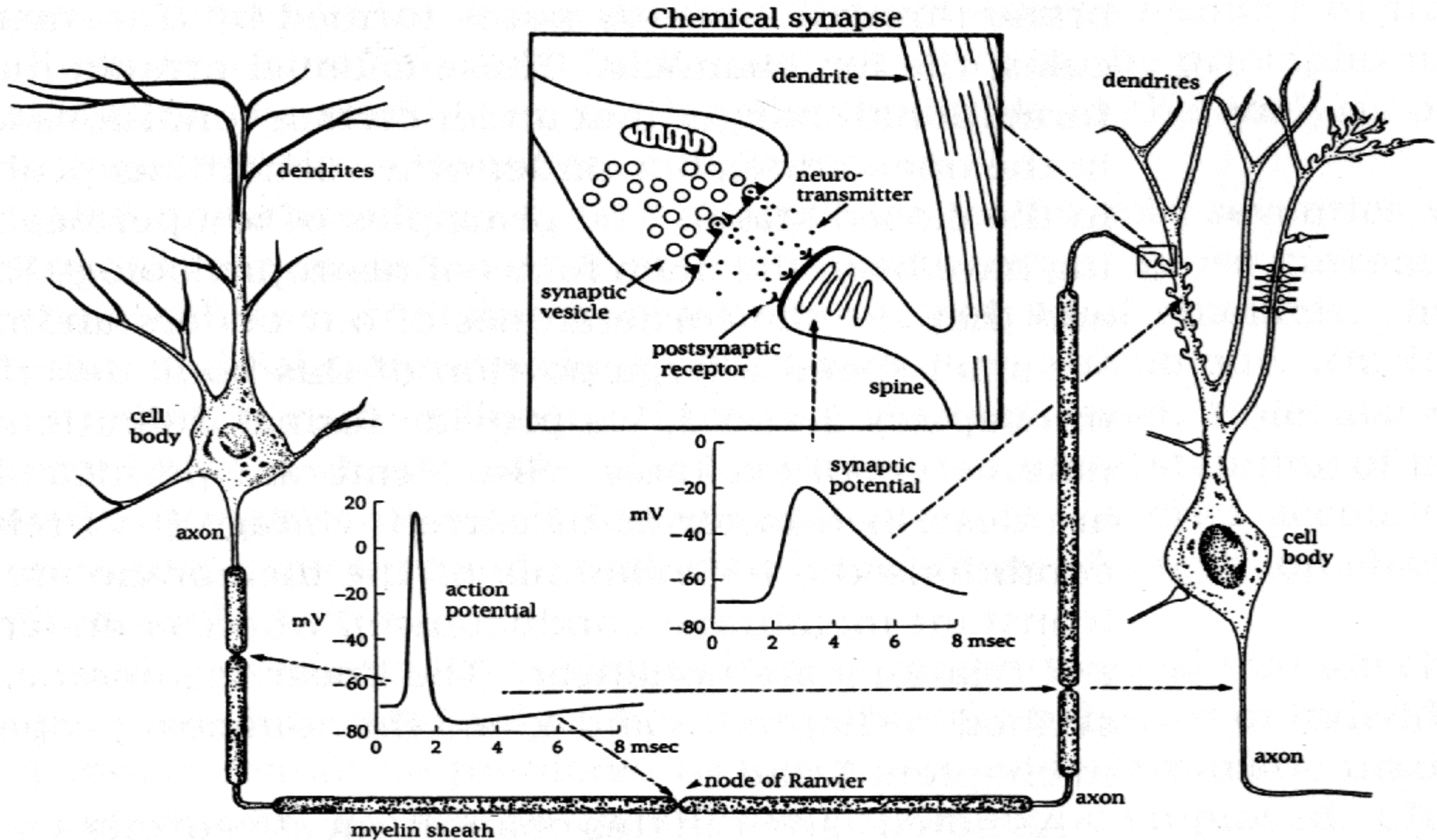
« Effects of Cellular Homeostatic Intrinsic
Plasticity on Dynamical and Computational
Properties of Biological Recurrent Neural
Networks »

Jeremie Naude, Bruno Cessac, Hugues
Berry, and Bruno Delord,

Goal of the talk

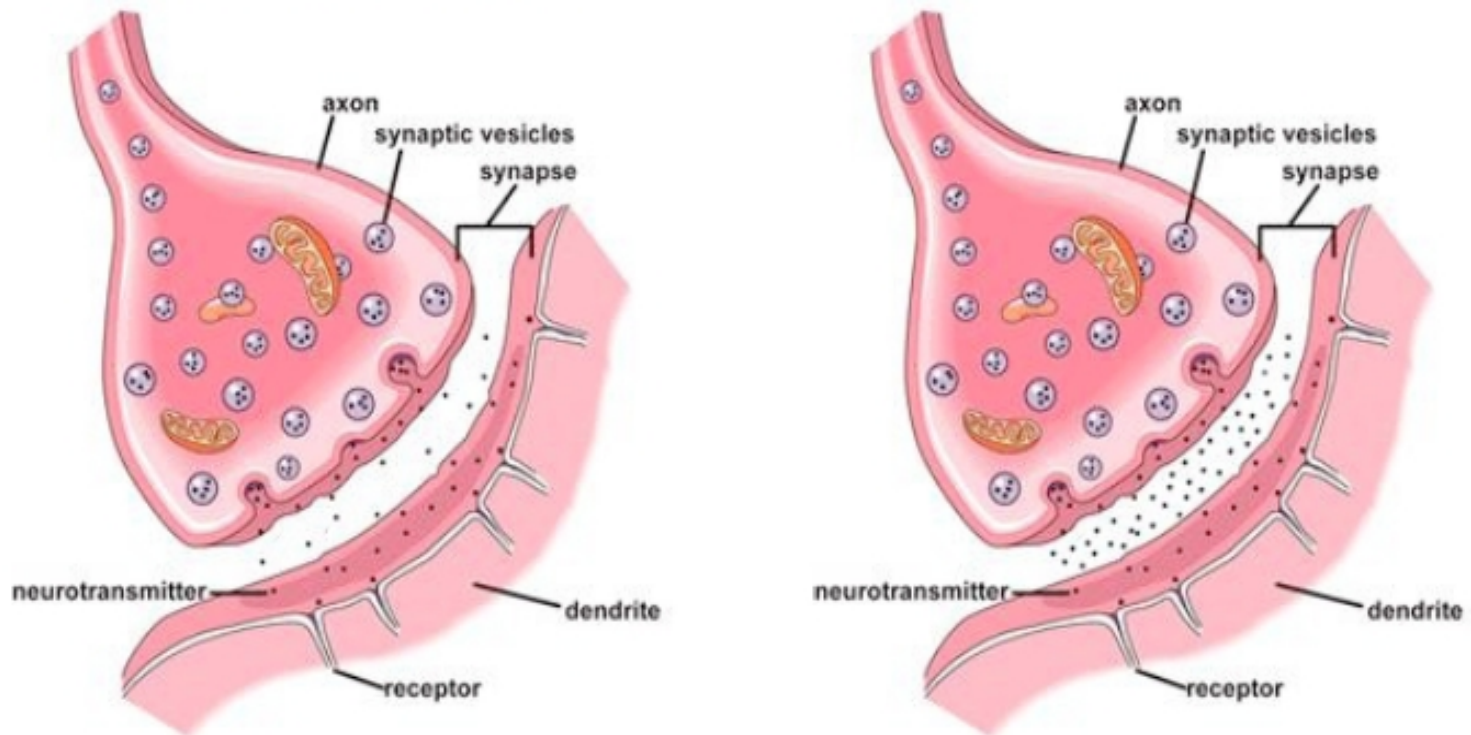
To study the joint effects of synaptic and intrinsic plasticity on a neural network model.

Neurons and synapses.



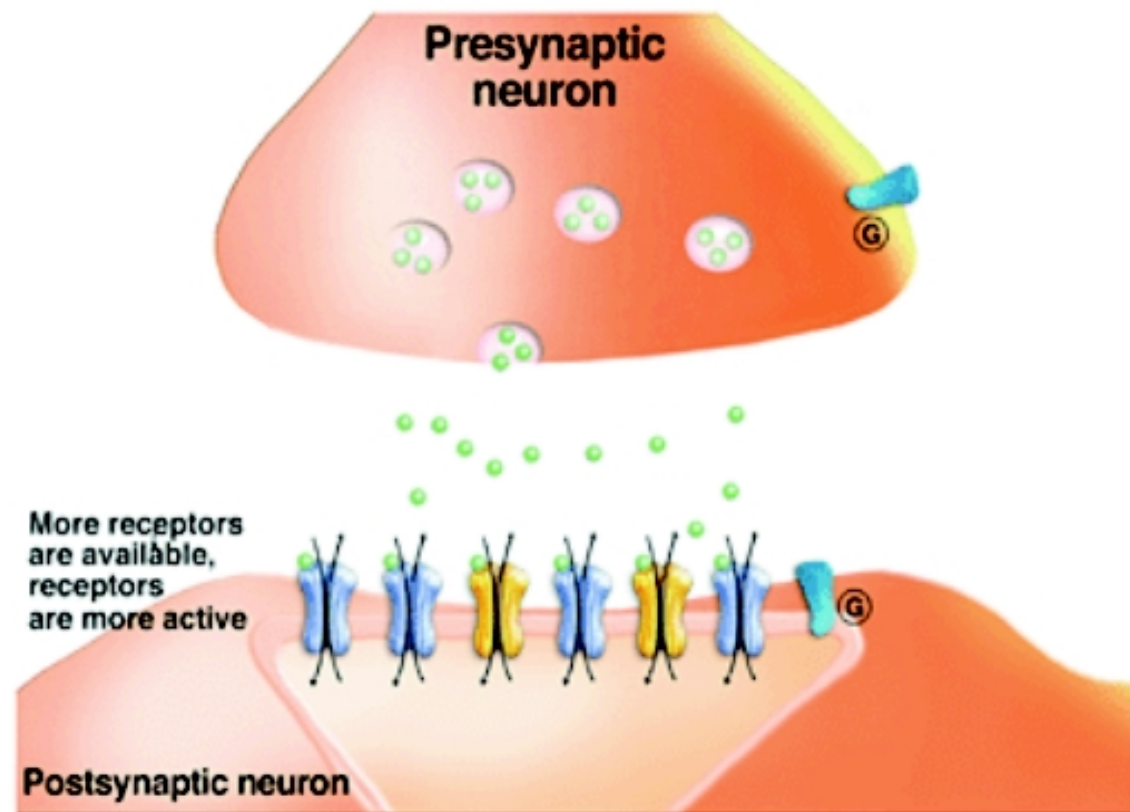
Synapses and Plasticity

- The synaptic weight (cellular and molecular factors)
 - As a function of the quantity of neurotransmitters released at a time



Synapses and Plasticity

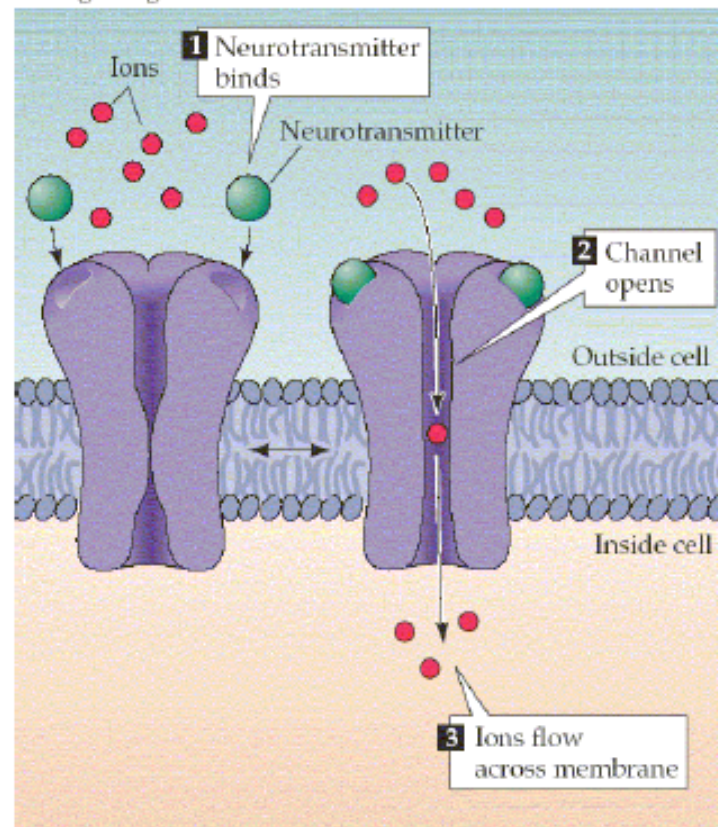
- The synaptic weight (cellular and molecular factors)
 - As a function of the quantity of receptors



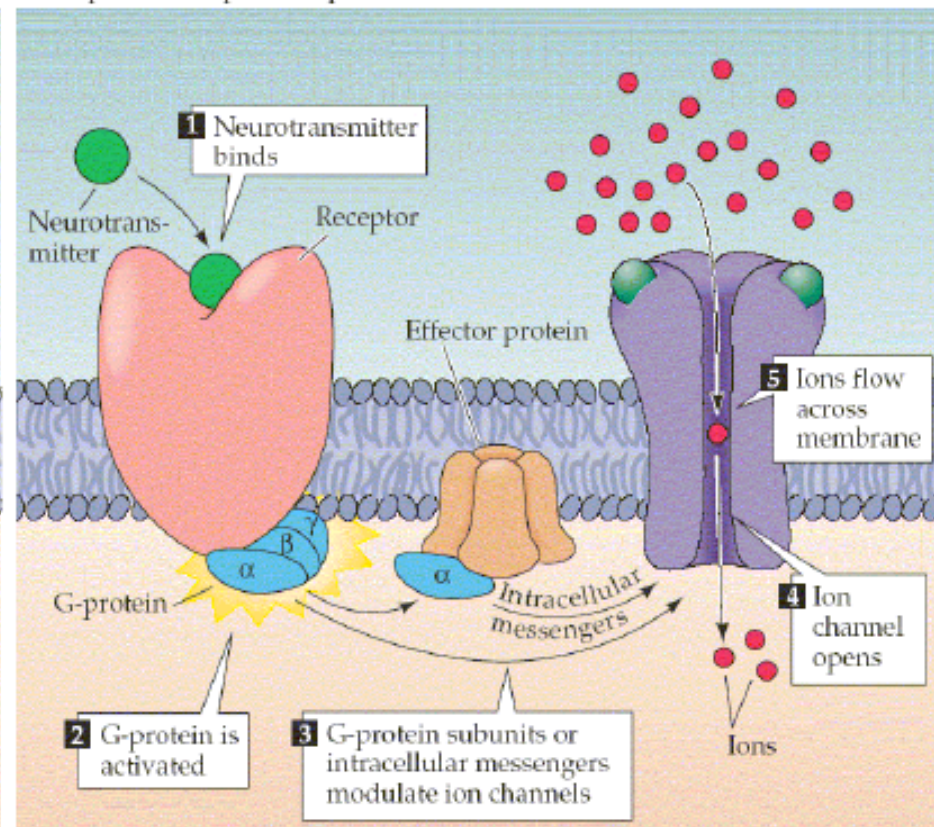
Synapses and Plasticity

- The synaptic weight (cellular and molecular factors)
 - As a function of the type of receptors

Ligand-gated ion channels

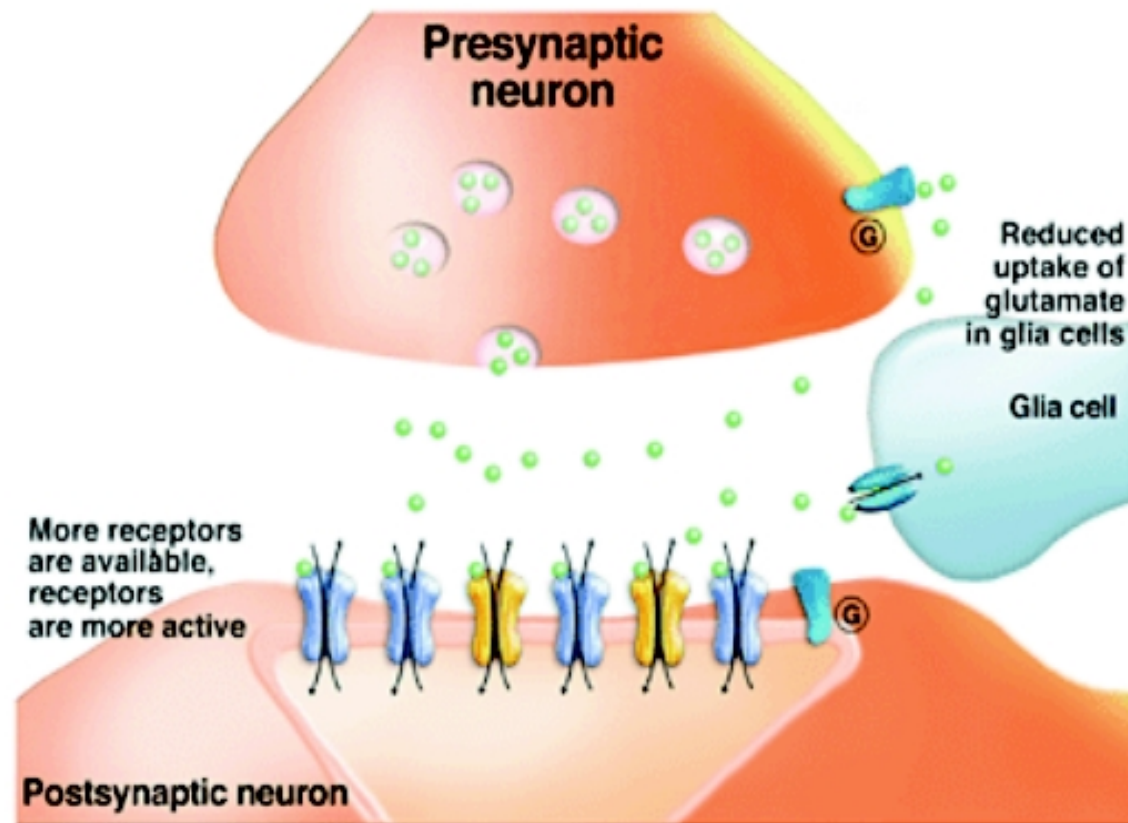


G-protein-coupled receptors



Synapses and Plasticity

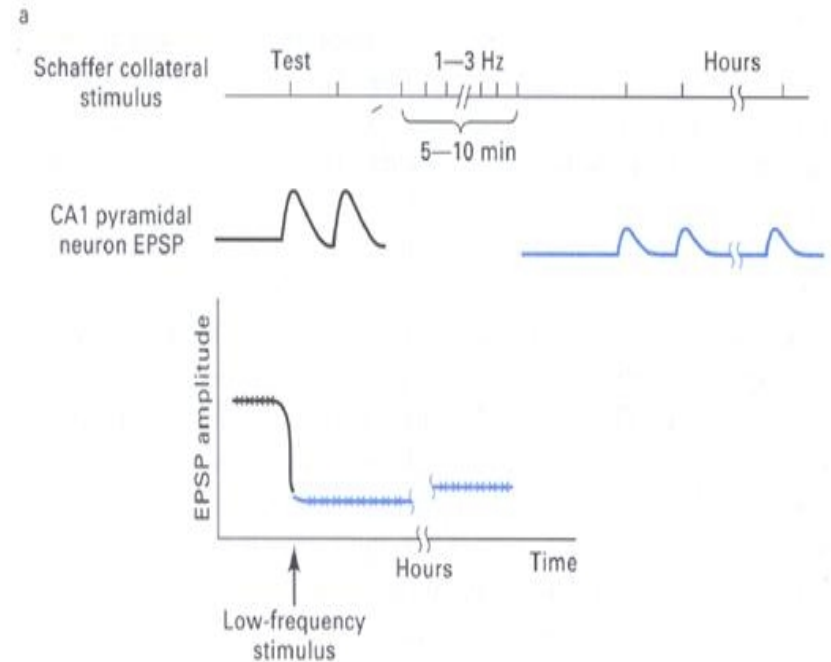
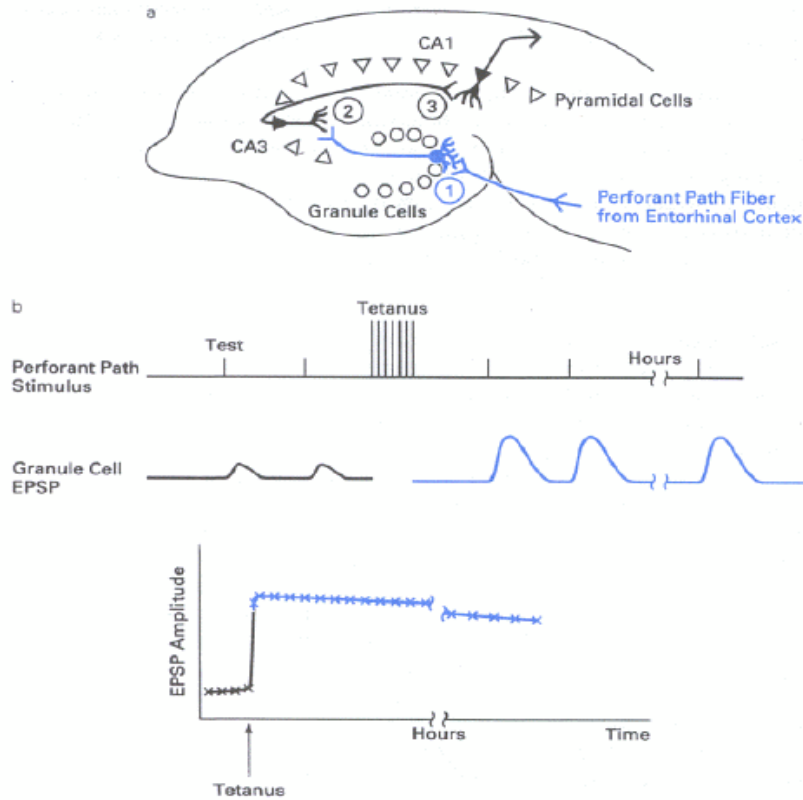
- The synaptic weight (cellular and molecular factors)
 - As a function of the uptake and the degradation



Synaptic plasticity.

Long-Term Potentiation: long lasting **enhancement** of synaptic transmission

Long-Term Depression: long lasting **reduction** in synaptic transmission



Hebbian theory

Let us assume that the persistence or repetition of a reverberatory activity (or "trace") tends to induce lasting cellular changes that add to its stability.... When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased

Hebb, D.O. (1961). "Distinctive features of learning in the higher animal". In J. F. Delafresnaye (Ed.). Brain Mechanisms and Learning. London: Oxford University Press.

Dynamical effects of Hebbian plasticity

Daucé & al, *Neural Networks*, 1998
Siri & al, *Neural Comput*, 2007

The SNT model again

$$u_i(t+1) = \gamma u_i(t) + \sum_{j=1}^N J_{ij} f(u_j(t)) - \theta_i + \sigma B_i(t)$$

$$u_i(t+1) = \sum_{j=1}^N W_{ij} f(u_j(t)) - \theta_i$$

$$x_i(t) = f(u_j(t))$$

$$x_i(t+1) = f\left(\sum_{j=1}^N W_{ij} x_j(t) - \theta_i\right)$$

Effects of Hebbian learning.

« Hebbian » learning.

$$W_{ij}^{(T+1)} = \lambda W_{ij}^{(T)} + \frac{\alpha}{N} \Gamma_{ij}^{(T)}$$

Decay rate (forget)
« passive LTD »

« Small » parameter.

Depends on
pre- and post-
synaptic neuron
« activity ».

Neurons dynamics

$$x_i^{(T)}(t+1) = f \left(\sum_{j=1}^N W_{ij}^{(T)} x_j^{(T)}(t) - \theta_i^{(T)} \right)$$

Example.

$$m_i^{(T)} = \frac{1}{\tau} \sum_{t=1}^{\tau} x_i^{(T)}(t) - d_i \quad \Gamma_{ij}^{(T)} = m_i^{(T)} m_j^{(T)} H \left(m_j^{(T)} \right)$$

Effects of Hebbian learning.

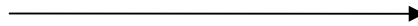
Coupled evolution of neurons and synapses.

Nonlinear neurons dynamics depends
on the synaptic graph structure.

Synaptic graph evolves according to
the neurons activity.

Generic effect of Hebbian learning.

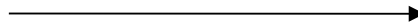
Changes in the network structure.



Increase in the number of positive circuits.

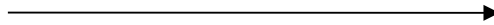
Generic effect of Hebbian learning.

Changes in the network structure.



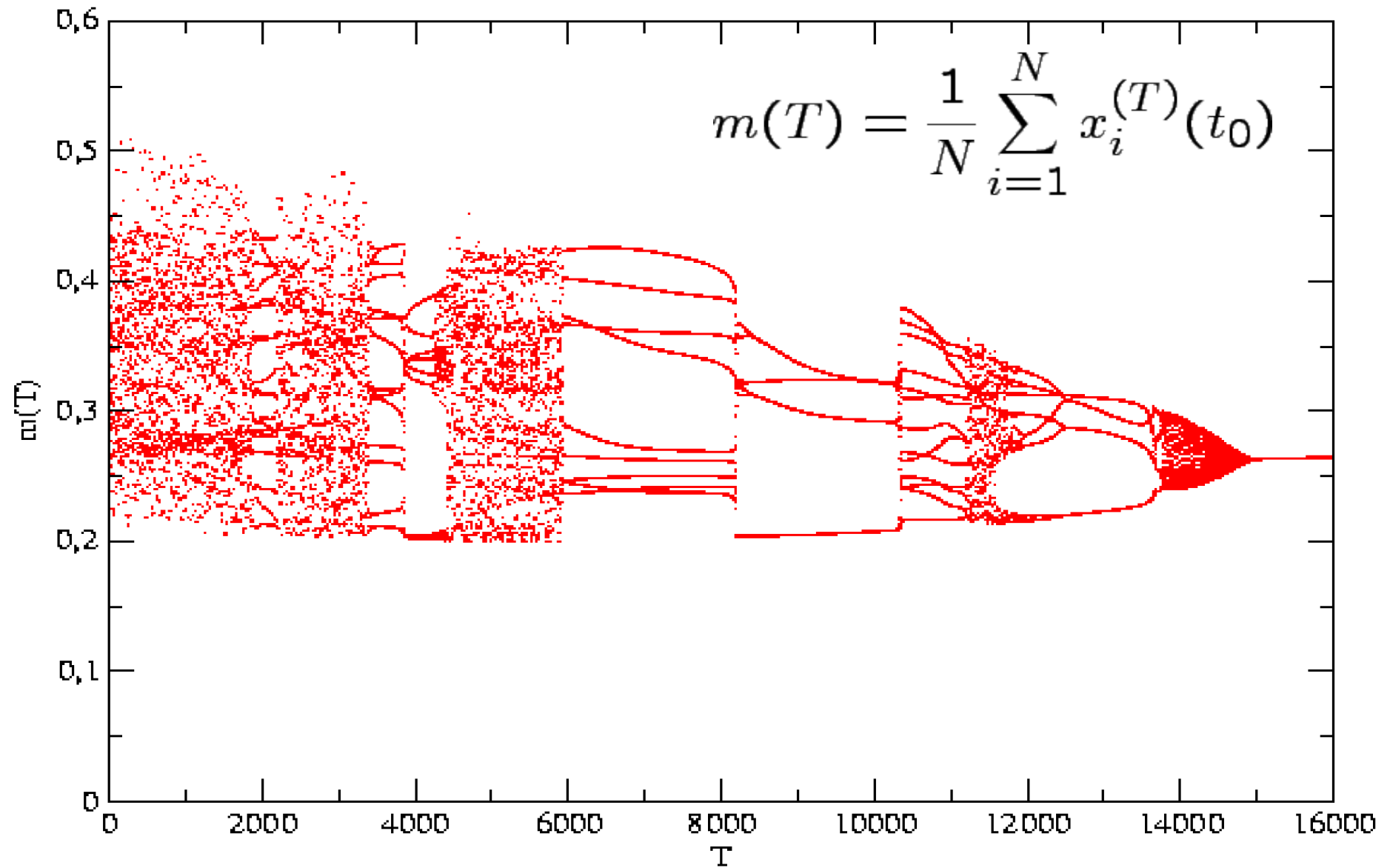
Increase in the number of positive circuits.

Dynamical effect.



Reduction of chaos.

Generic effect of Hebbian learning.

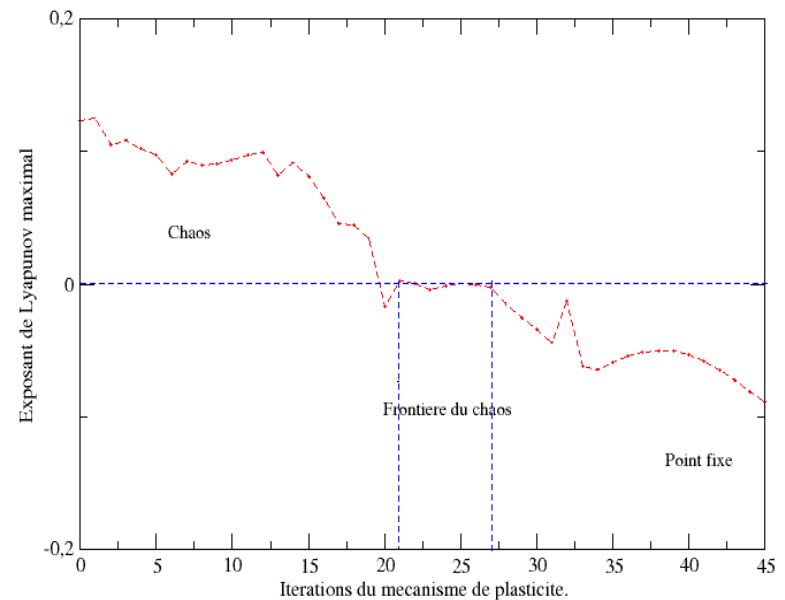
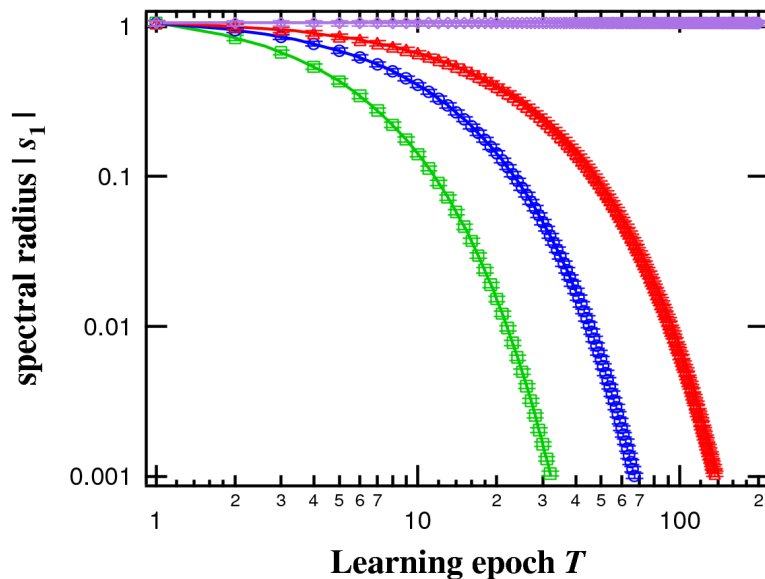


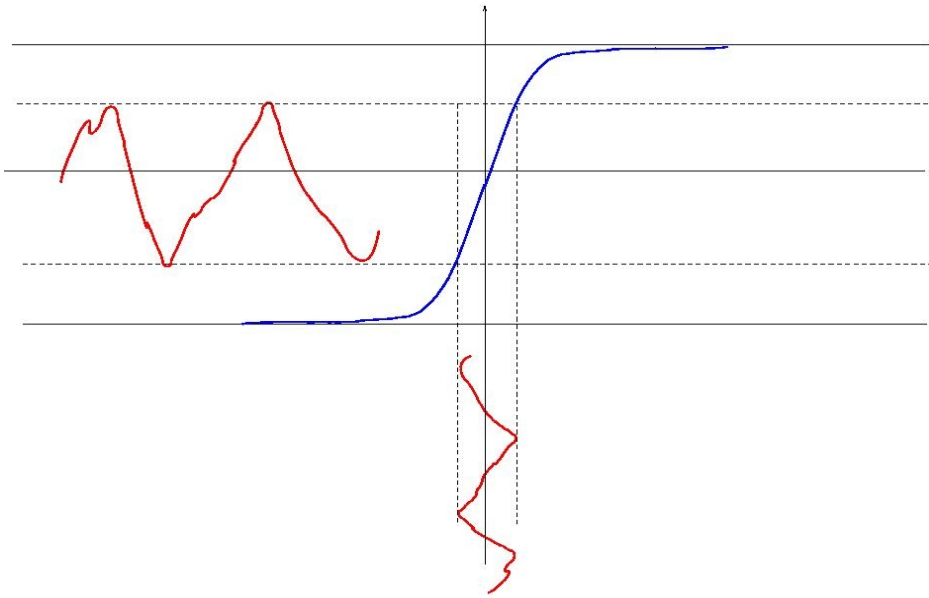
Generic effect of Hebbian learning.

Th. (Siri, Berry, Cessac, Delord, Quoy, 2007)

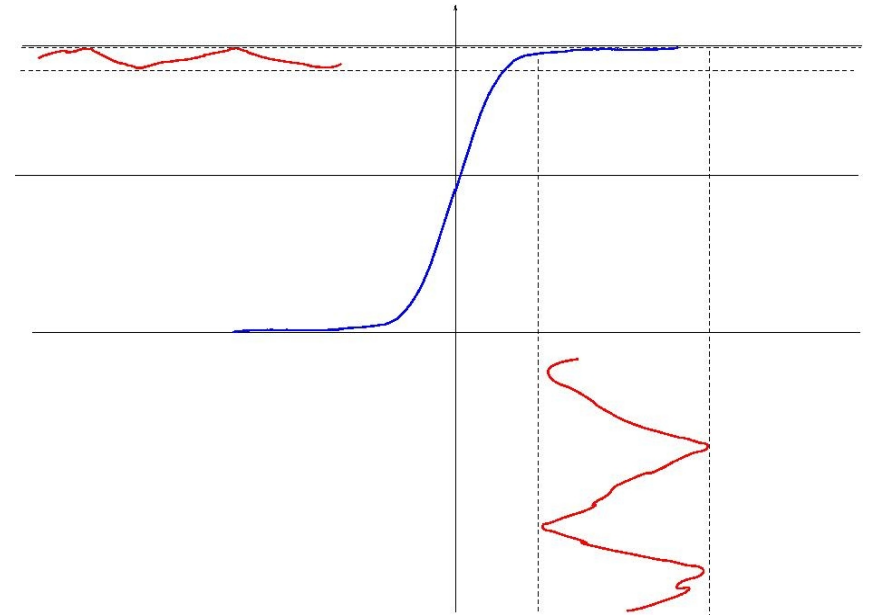
$$\|\mathcal{W}^{(T)}\| \leq \lambda^T \|\mathcal{W}^{(1)}\| + \frac{\alpha}{N} \sum_{n=1}^T \lambda^{T-n} \|\Gamma^{(n)}\|$$

$$\lambda_1^{(T)} \leq \log(\|\mathcal{W}^{(T)}\|) + \left\langle \log(\max_i f'(u_i)) \right\rangle^{(T)}$$



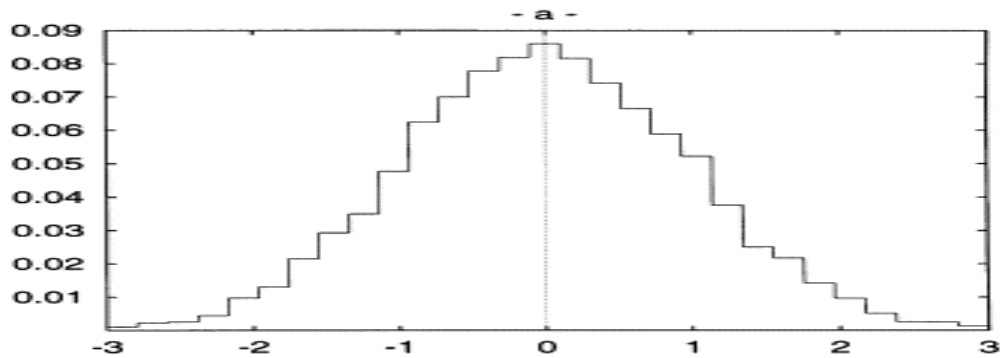


Expansion

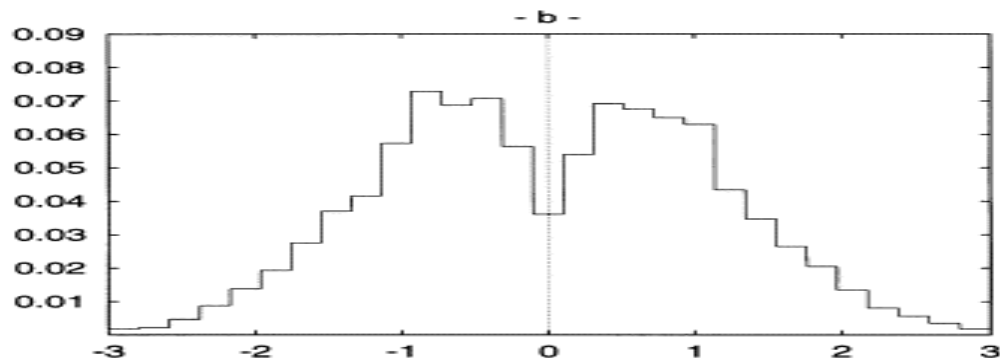


Contraction

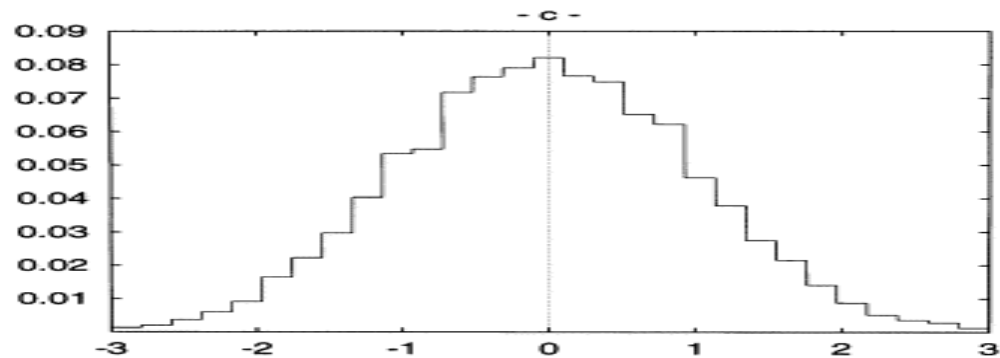
Empirical distribution of the mean local field



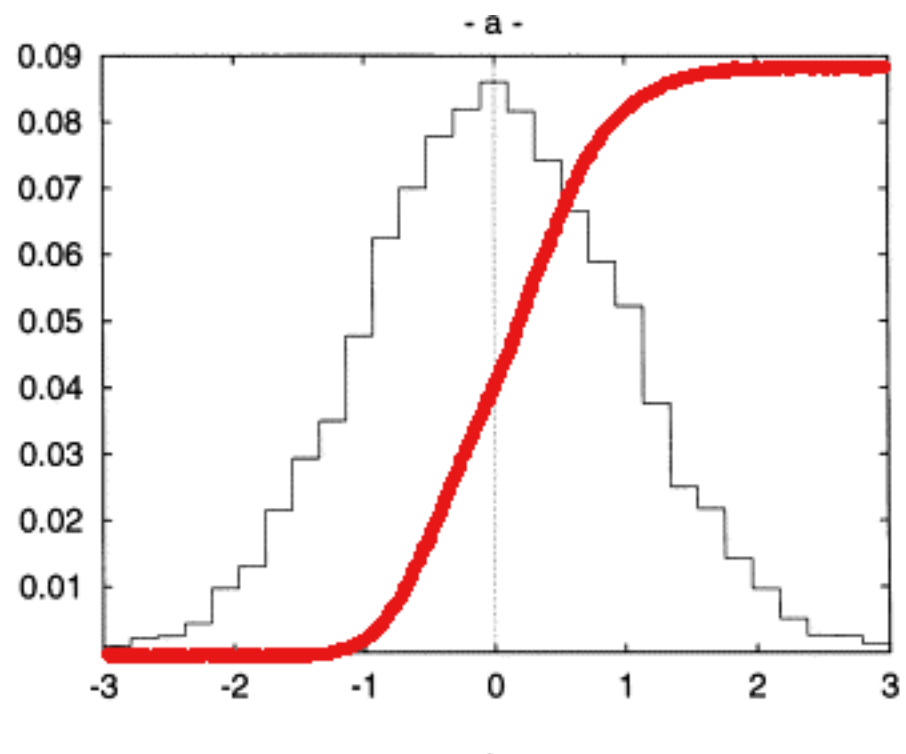
Before learning for the stimulus-forced dynamics.

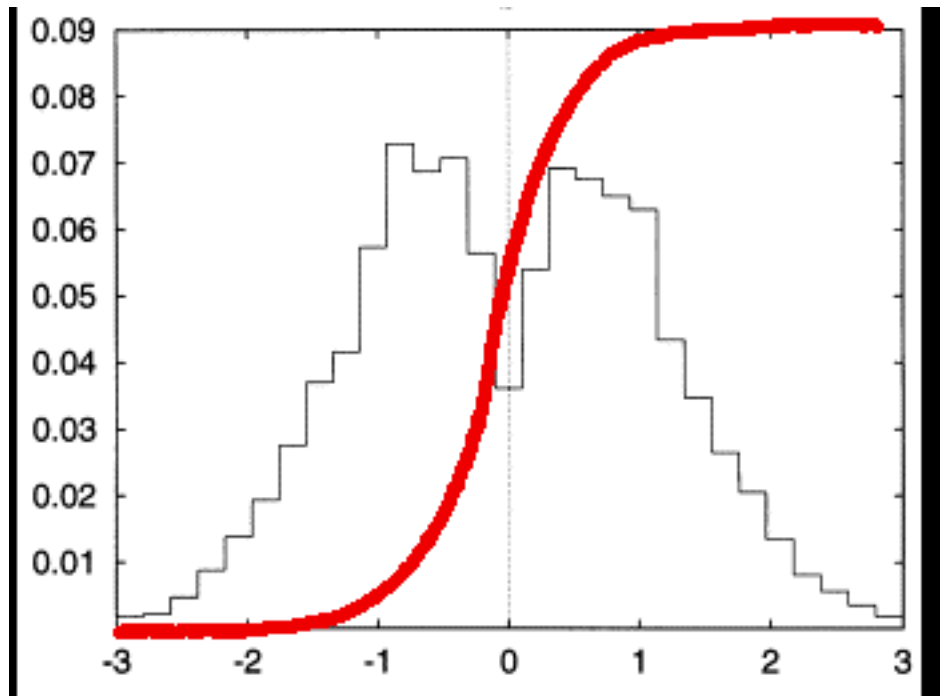


After learning for the stimulus-forced dynamics.



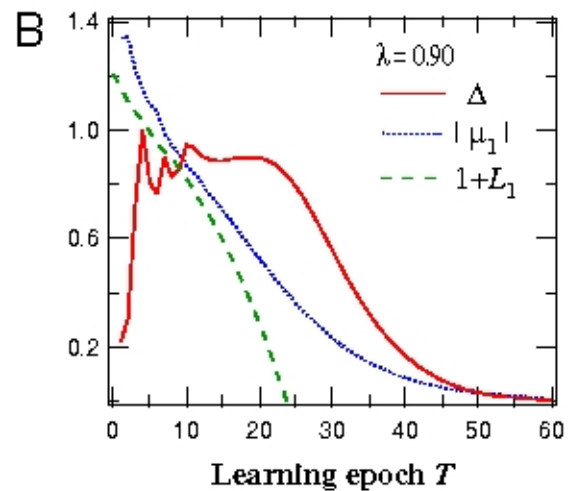
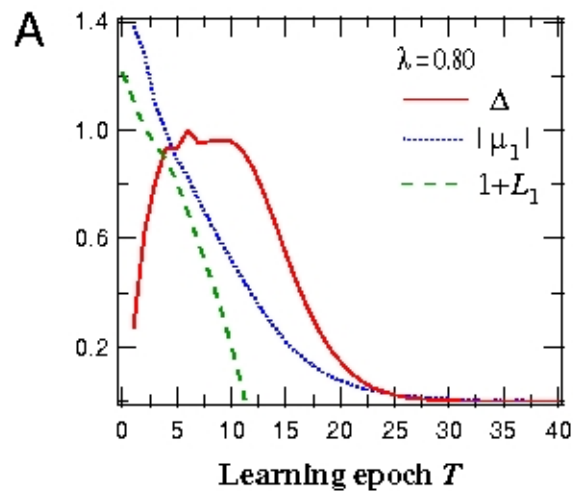
After learning, with the dynamics induced by another random non-learned stimulus

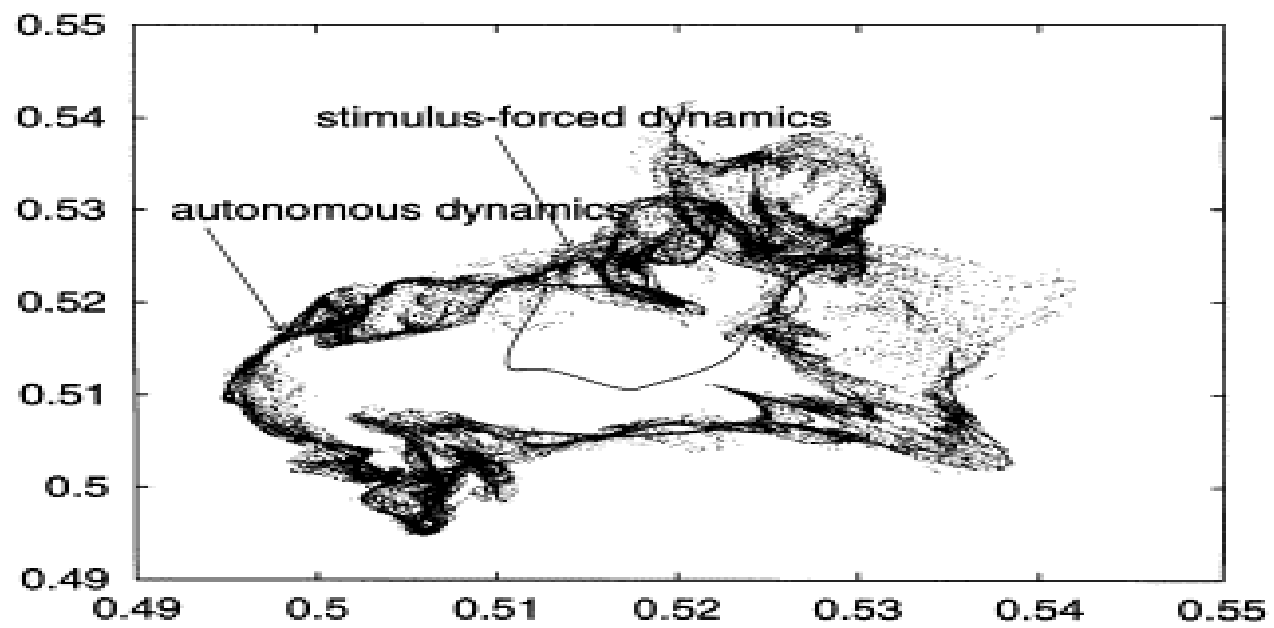
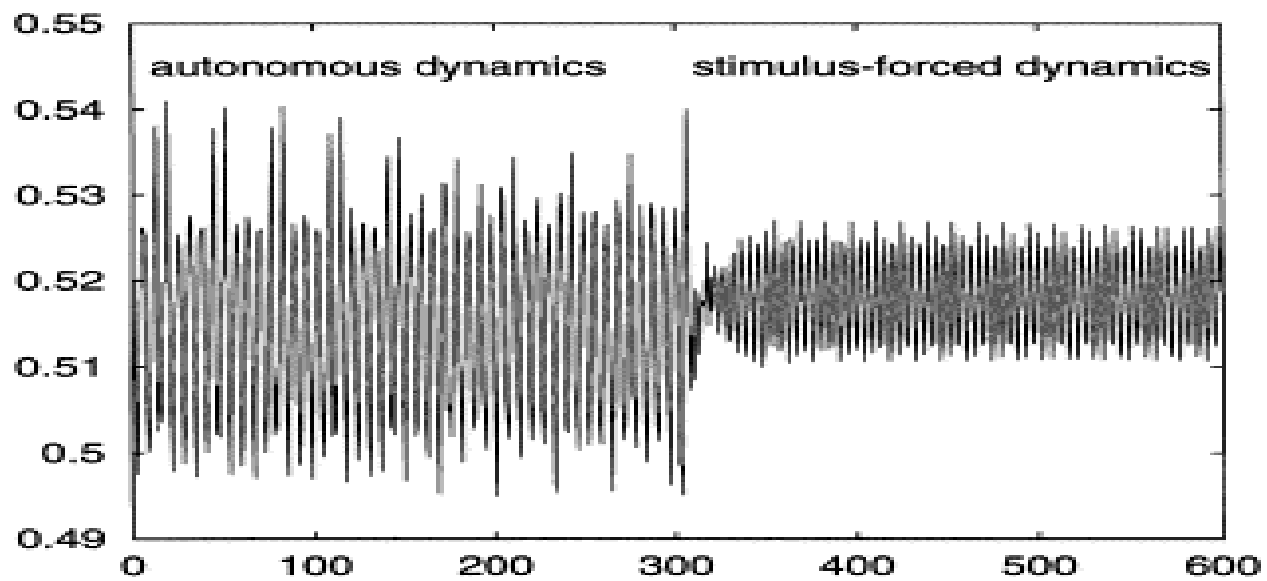




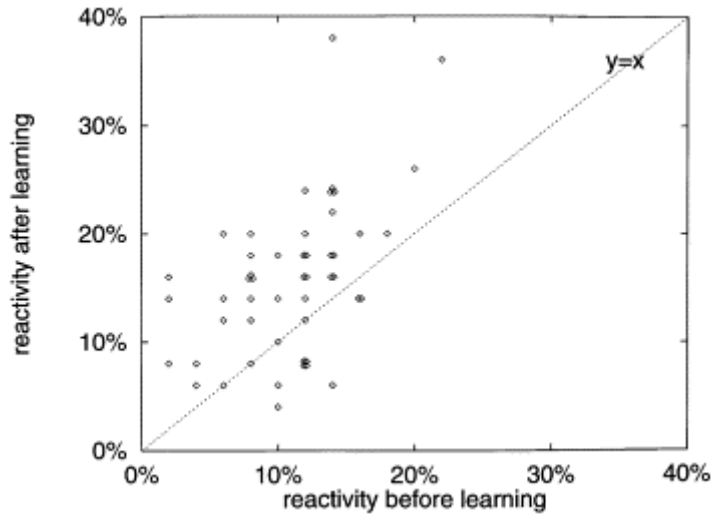
Generic effect of Hebbian learning.

$$\Delta^{(T)} = \frac{1}{N} \sqrt{\sum_{i=1}^N \left[\langle f'(u_i) \rangle^{(T)} - \langle f'(u'_i) \rangle^{(T)} \right]^2}$$



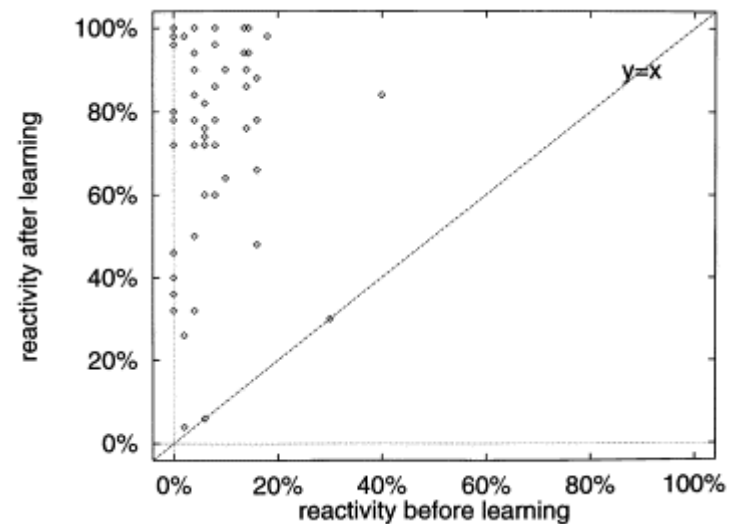


Dauce E., Quoy M., Cessac B., Doyon B. and Samuelides M. "Self-Organization and Dynamics reduction in recurrent networks: stimulus presentation and learning", Neural Networks, (11), 521-533, (1998).

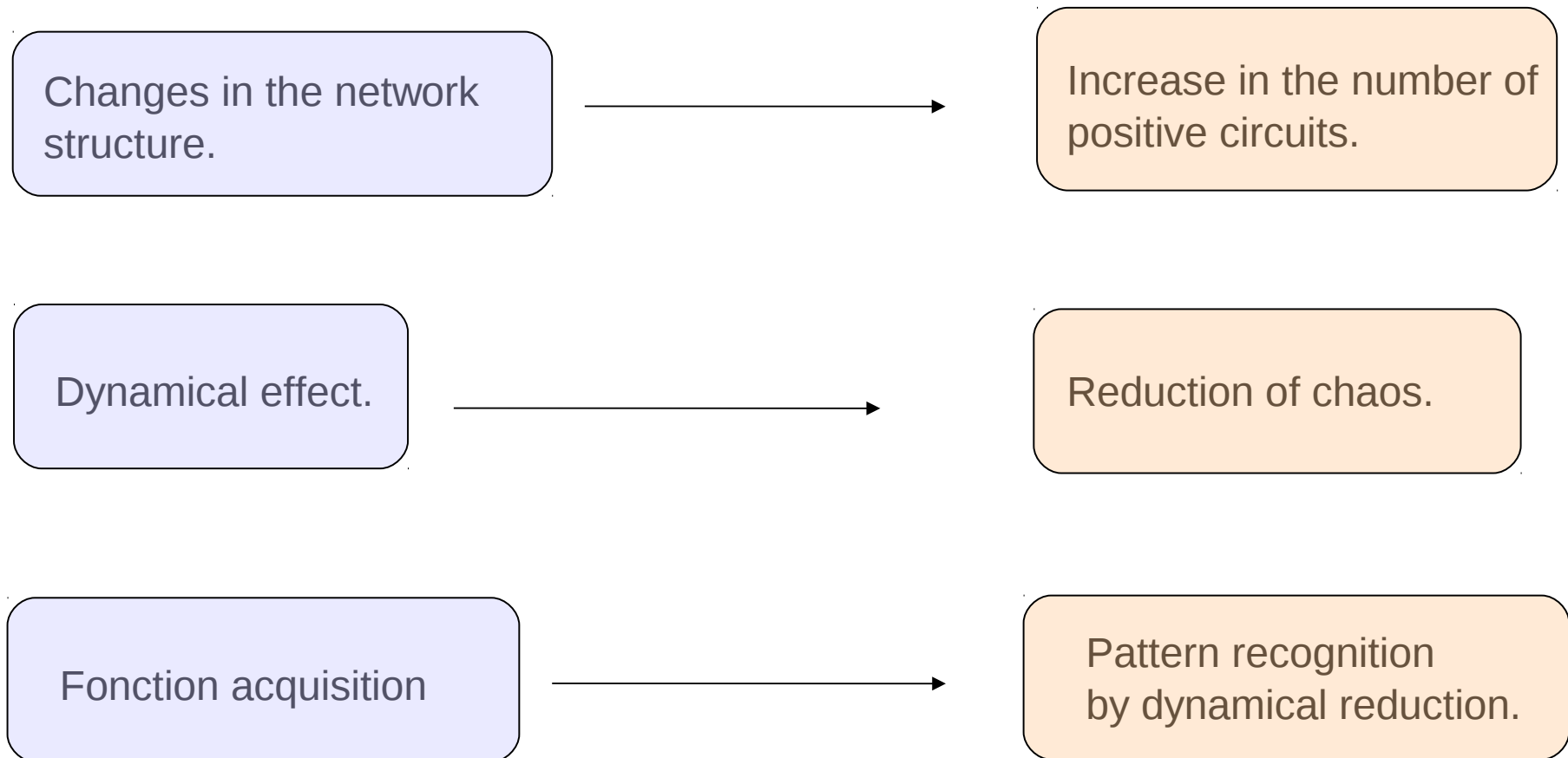


Reactivity to random stimuli on a population of 50 networks before and after learning, with $g=15$.

Reactivity to noisy versions (noise=20%) of the learned stimulus on a population of 50 networks before and after learning, with $g=15$.



Generic effect of Hebbian learning.



A movie



*Is there a bio physically relevant
mechanism allowing to stabilize the
network at the edge of chaos ?*

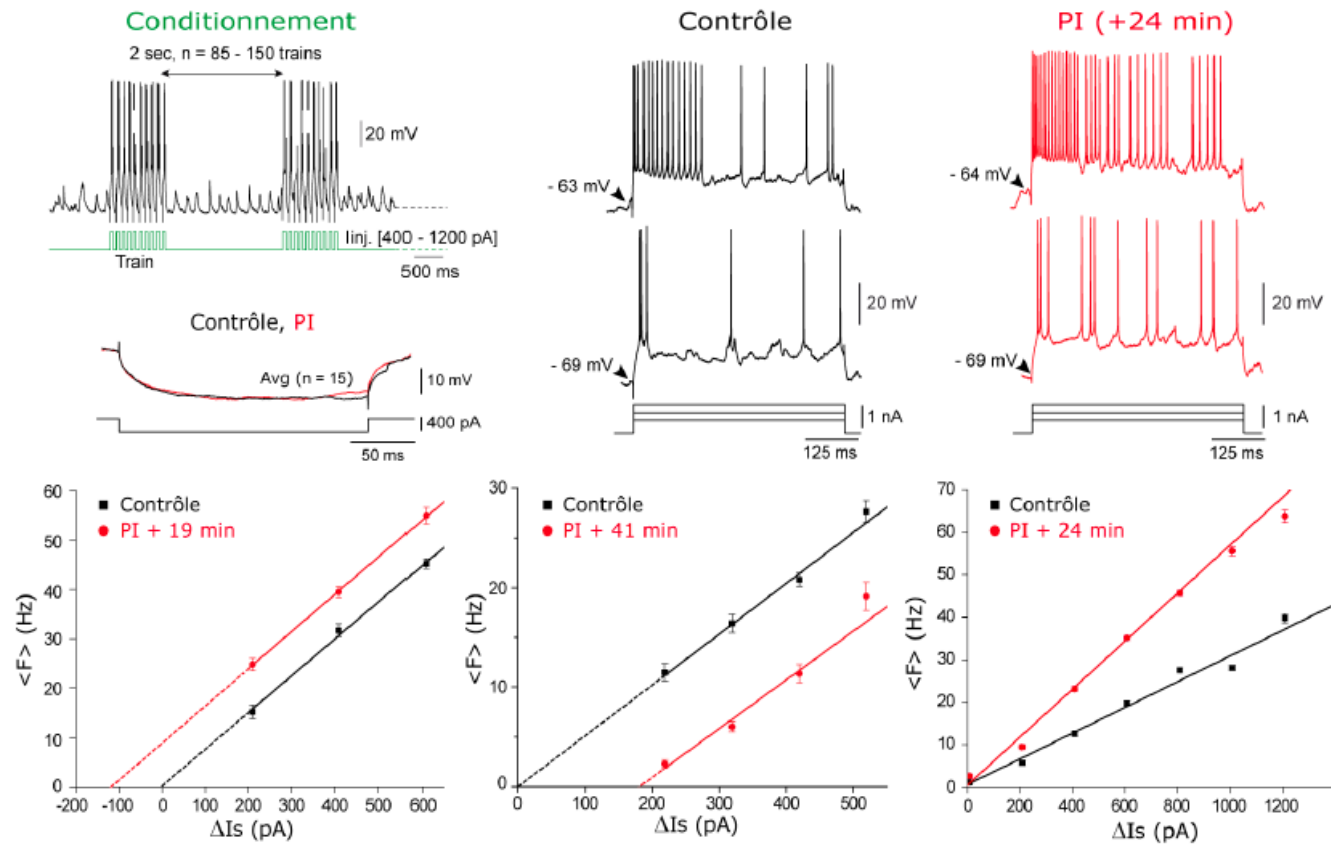
Intrinsic plasticity

- Persistent modification of a neuron's **intrinsic electrical properties** by neuronal or synaptic activity.
- Mediated by changes in the expression level or biophysical properties of **ion channels** in the membrane
- Can affect such diverse processes as synaptic integration, subthreshold signal propagation, spike generation....
- regulator of synaptic plasticity underlying learning *and memory*
- ***Homeostatic regulation.***

(Collective phenomena by which neurons alter their intrinsic or synaptic properties to maintain a target level of electrical activity).

Intrinsic plasticity in vivo (courtesy H. Berry)

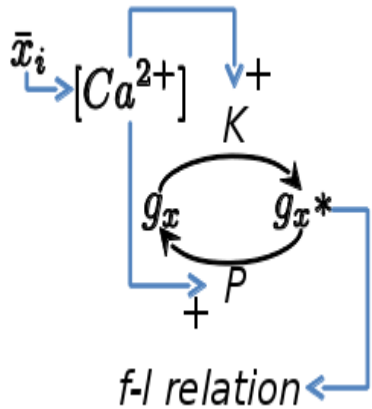
motor or
somato
sensory
cortex



Paz JT, Mahon S, Tiret P, Genet S, Delord B, et al. (2009) J Physiol 587: 3189–3205.

activity-dependent modifications of the threshold or the
slope of the f-I relation

Molecular mechanisms of IP (courtesy H. Berry)

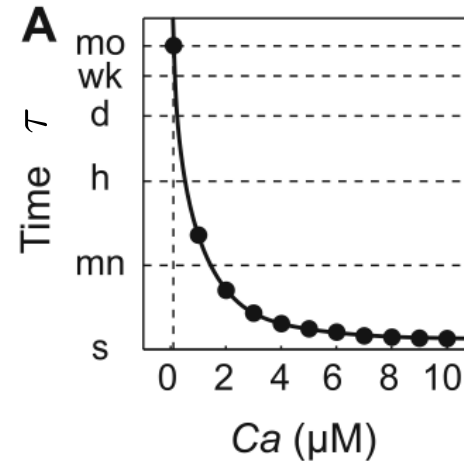
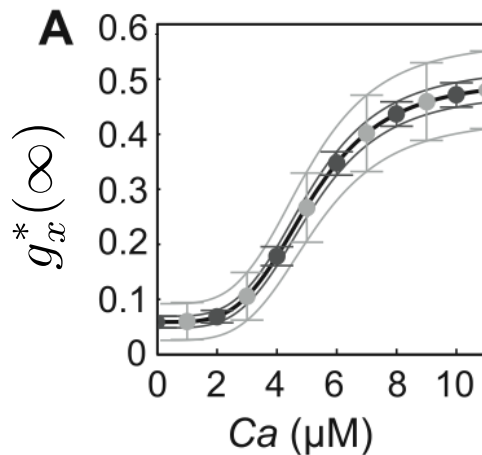


$$K(Ca) = K_{max} \frac{Ca^n}{Ca^n + K_K^n}, \quad P(Ca) = P_{max} \frac{Ca^n}{Ca^n + K_P^n}$$

$$dg_x^*/dt = K(Ca)(1 - g_x^*) - P(Ca)g_x^*$$

$$\Rightarrow g_x^*(\infty) = \frac{K(Ca)}{K(Ca) + P(Ca)}; \quad \tau = \frac{1}{K(Ca) + P(Ca)}$$

Delord et al. *PLoS Comput. Biol.* (2007)



- activity-dependent plasticity & memory in single molecular module
- activity-dependent time scale

Molecular mechanisms of IP

From Naudé et al., Plos Comp Bio , 2013.

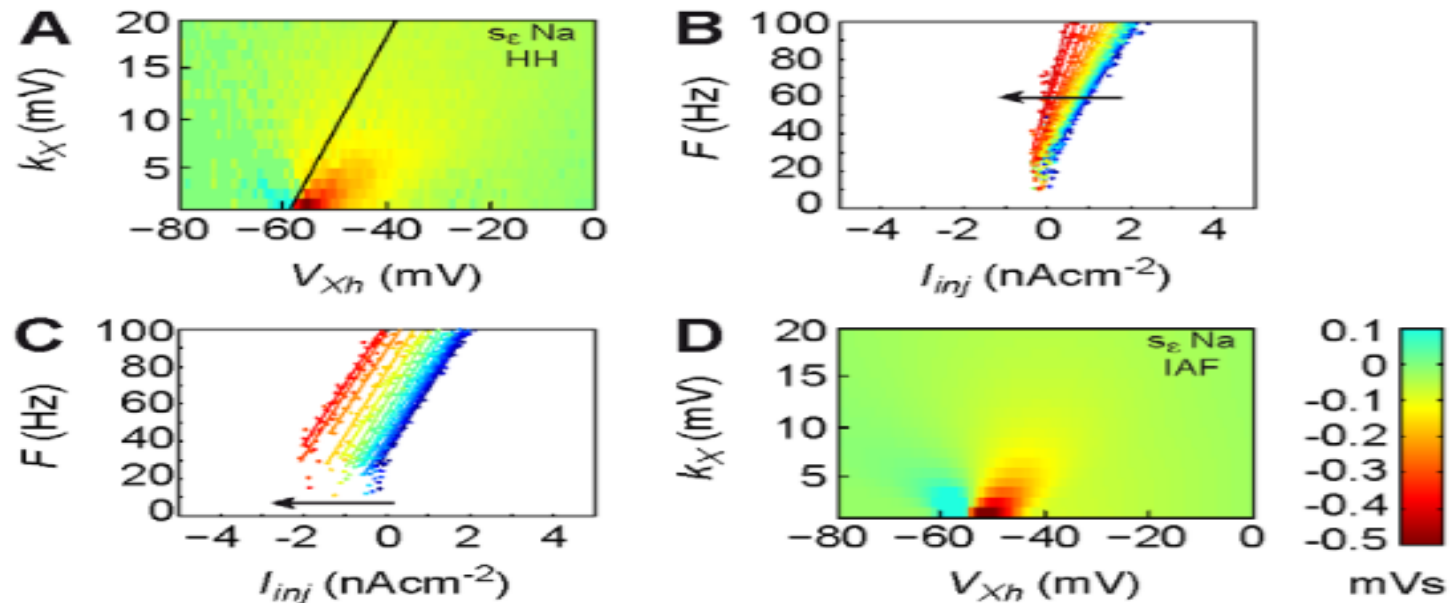


Figure 3. Inverse gain sensitivities of the standard HH model.

(A) Inverse gain sensitivity map of the standard HH model, s_e (mVs). Black line: isoline $x_\theta = 0.25$. Colorbar as in (D). (B) Increasing the maximal conductance g_X of the sodium conductance in the standard HH model with $V_{Xh} = -55$ mV and $k_X = 1$ mV increases the gain (decreases the inverse gain) of the $f-I$ (arrow). Blue to red curves: $g_X = [0.1g_X^{\text{sup}} - g_X^{\text{sup}}]$ with $g_X^{\text{sup}} = 4 \cdot 10^{-2} \text{ mS cm}^{-2}$. (C) Increasing the maximal conductance g_X of the sodium conductance in the standard HH model with $V_{Xh} = -62$ mV and $k_X = 1$ mV decreases the gain (increases the inverse gain) of the $f-I$ but this effect is masked by the much larger change of firing frequency due to modification of the threshold. Blue to red curves: $g_X = [0.1g_X^{\text{sup}} - g_X^{\text{sup}}]$ with $g_X^{\text{sup}} = 2.3 \cdot 10^{-2} \text{ mS cm}^{-2}$. (D) Theoretical inverse gain sensitivity map derived from the pre/post-spike IAF theory with sodium conductance. $I = 0.15 \text{ nA cm}^{-2}$, $k_C = 2.75$.

doi:10.1371/journal.pcbi.1002349.g003

Although generated at a **molecular scale**, synaptic and intrinsic plasticity induce prominent effects at **all scales** in the brain (neurons, neural networks, brain area).

However, at the moment, there is **no mathematical theory** to handle properly these effects.

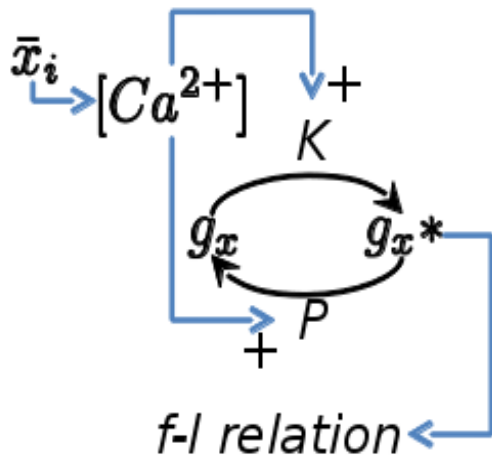
Let us see some mathematical results and obstructions toward such a theory by studying the SNT.

Dynamical effects of intrinsic plasticity at the network level.

Jeremie Naude, Bruno Cessac, Hugues Berry, and Bruno Delord, "Effects of Cellular Homeostatic Intrinsic Plasticity on Dynamical and Computational Properties of Biological Recurrent Neural Networks"

J. of Neuroscience, 2013.

Dynamical effects of intrinsic plasticity at the network level.



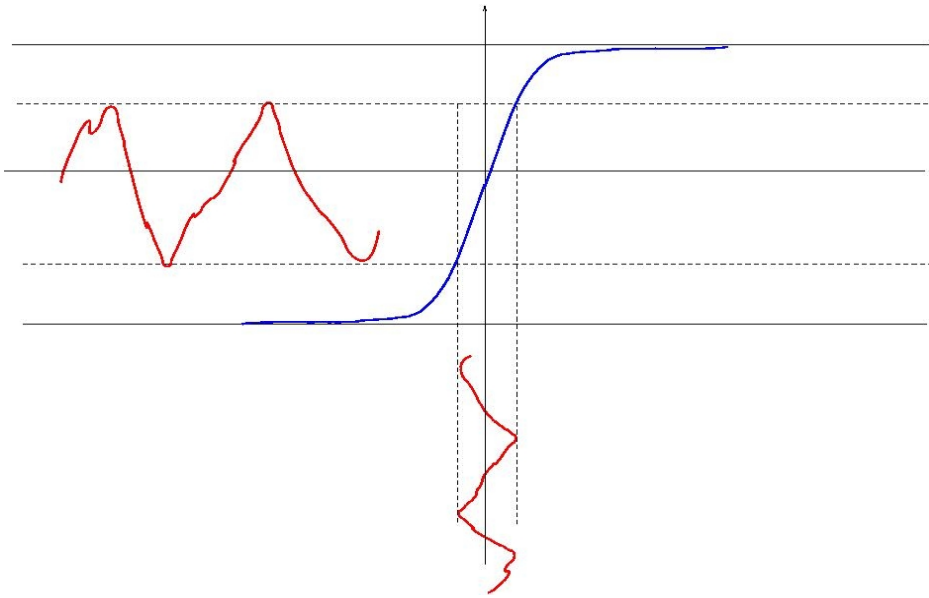
$$x_i^{(T)}(t+1) = f \left(\sum_j w_{ij} x_j^{(T)}(t) - \theta_i^{(T)} \right)$$

$$\theta_i^{(T+1)} = \theta_{max} \left(4g_i^{*(T+1)} - 1 \right)$$

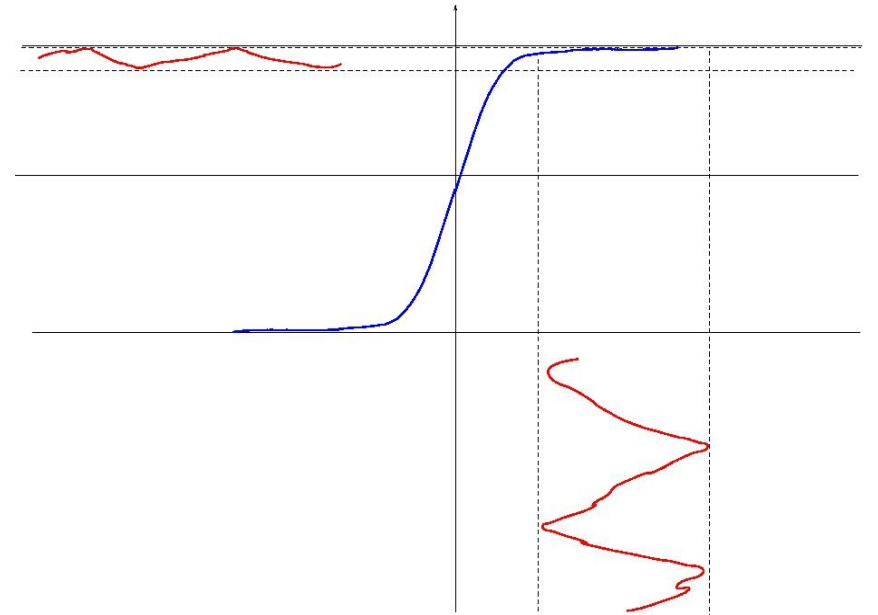
$$g_i^{*(T+1)} = K_i^{(T)} \left(1 - g_i^{*(T)} \right) - P_i^{(T)} g_i^{*(T)}$$

$$K_i^{(T)} = K_{max} \frac{(\bar{x}_i^{(T)})^4}{(\bar{x}_i^{(T)})^4 + K_K^4}$$

$$P_i^{(T)} = P_{max} \frac{(\bar{x}_i^{(T)})^4}{(\bar{x}_i^{(T)})^4 + K_P^4}$$

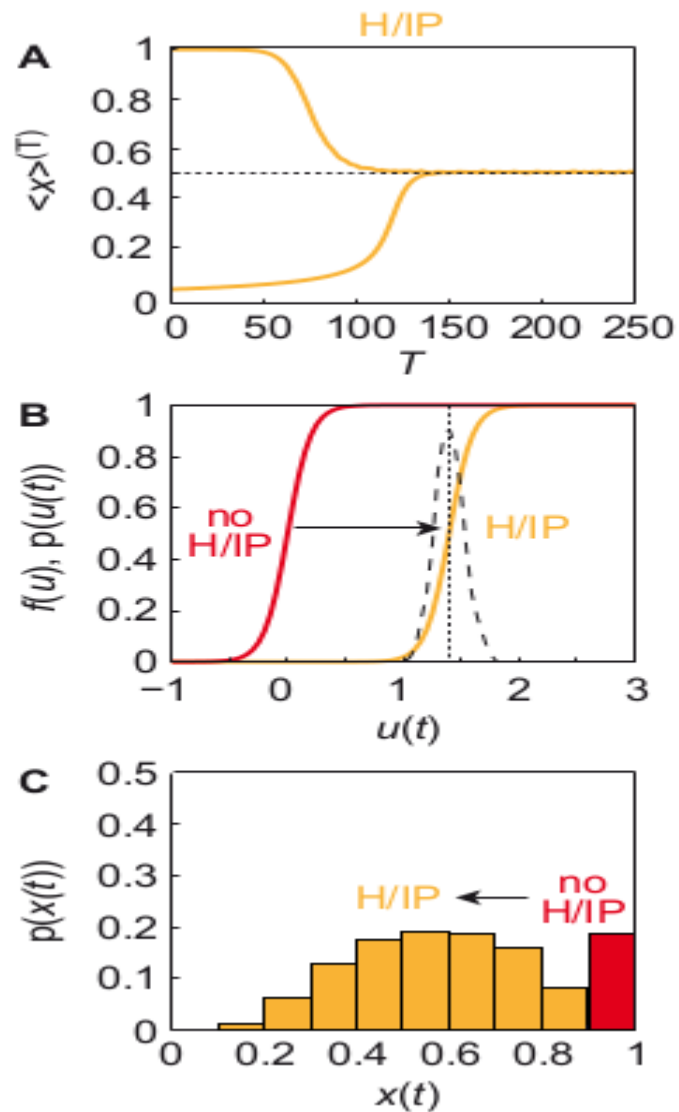


Expansion



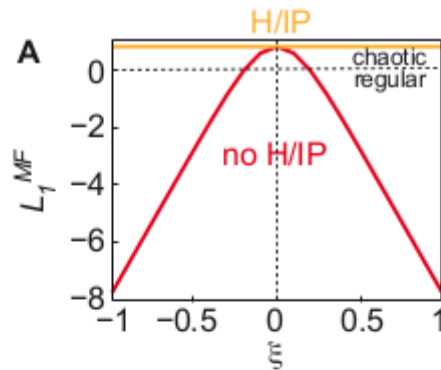
Contraction

Dynamical effects of intrinsic plasticity

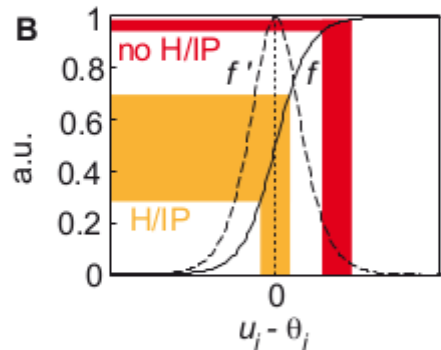


Dynamical effects of intrinsic plasticity

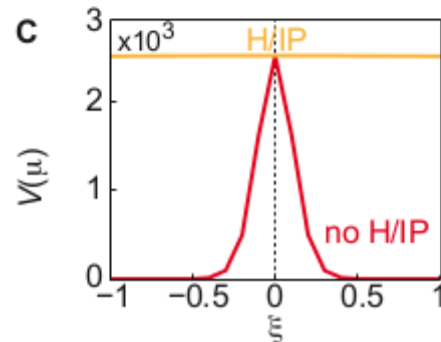
Dynamic Mean-field theory



Largest Lyapunov exponent computed with the dynamical mean-field approach, as a function of the input amplitude, in the presence (yellow) or absence (red) of HIP.

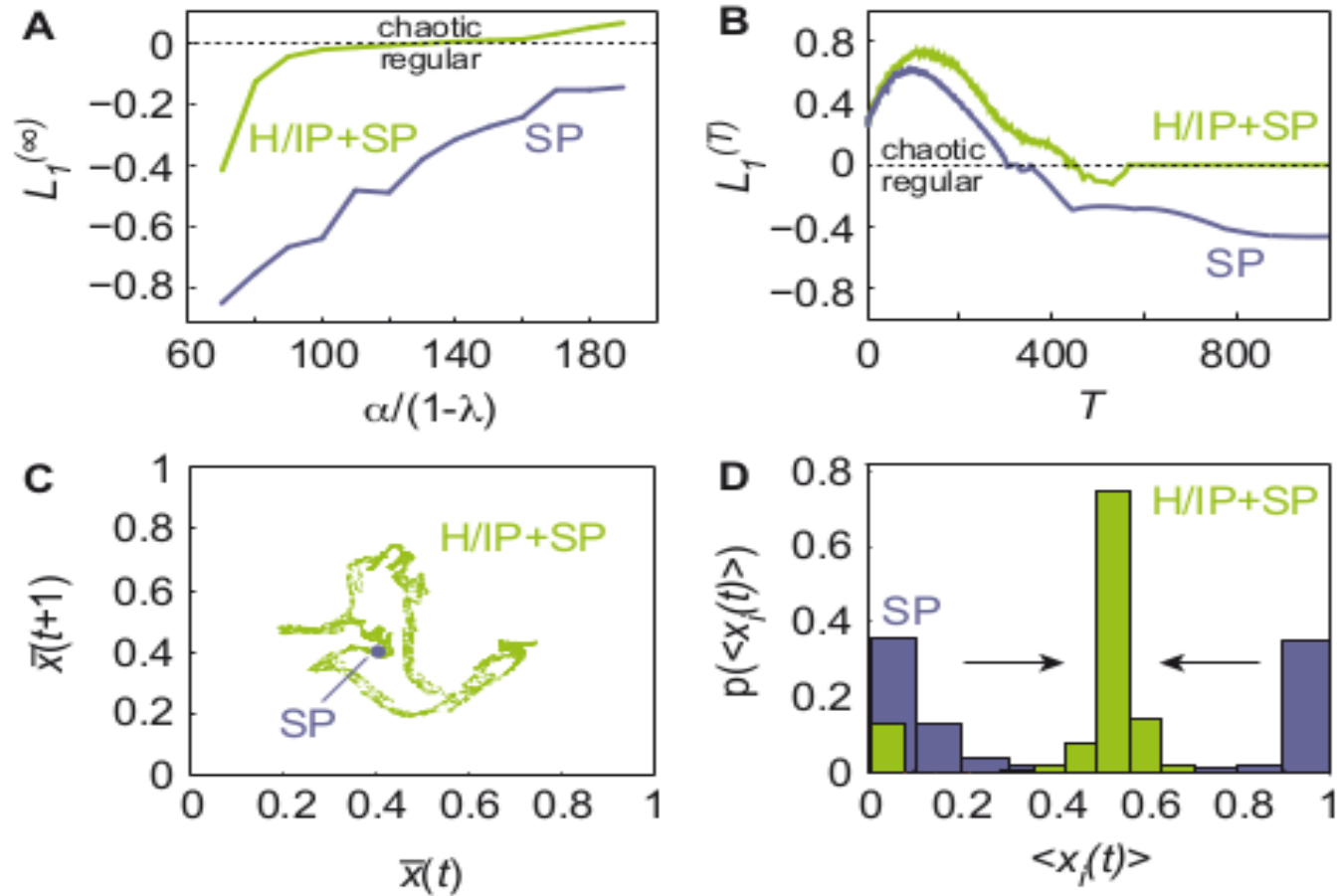


The neuronal transfer function, (black curve) and its derivative (black dashed curve), as a function of the local field. In the absence of HIP, input fluctuations (red bands) give rise to a contracted output fluctuations. On the contrary, in the presence of HIP, output fluctuations are amplified, i.e. the difference between the input and the neuron's threshold (yellow bands).

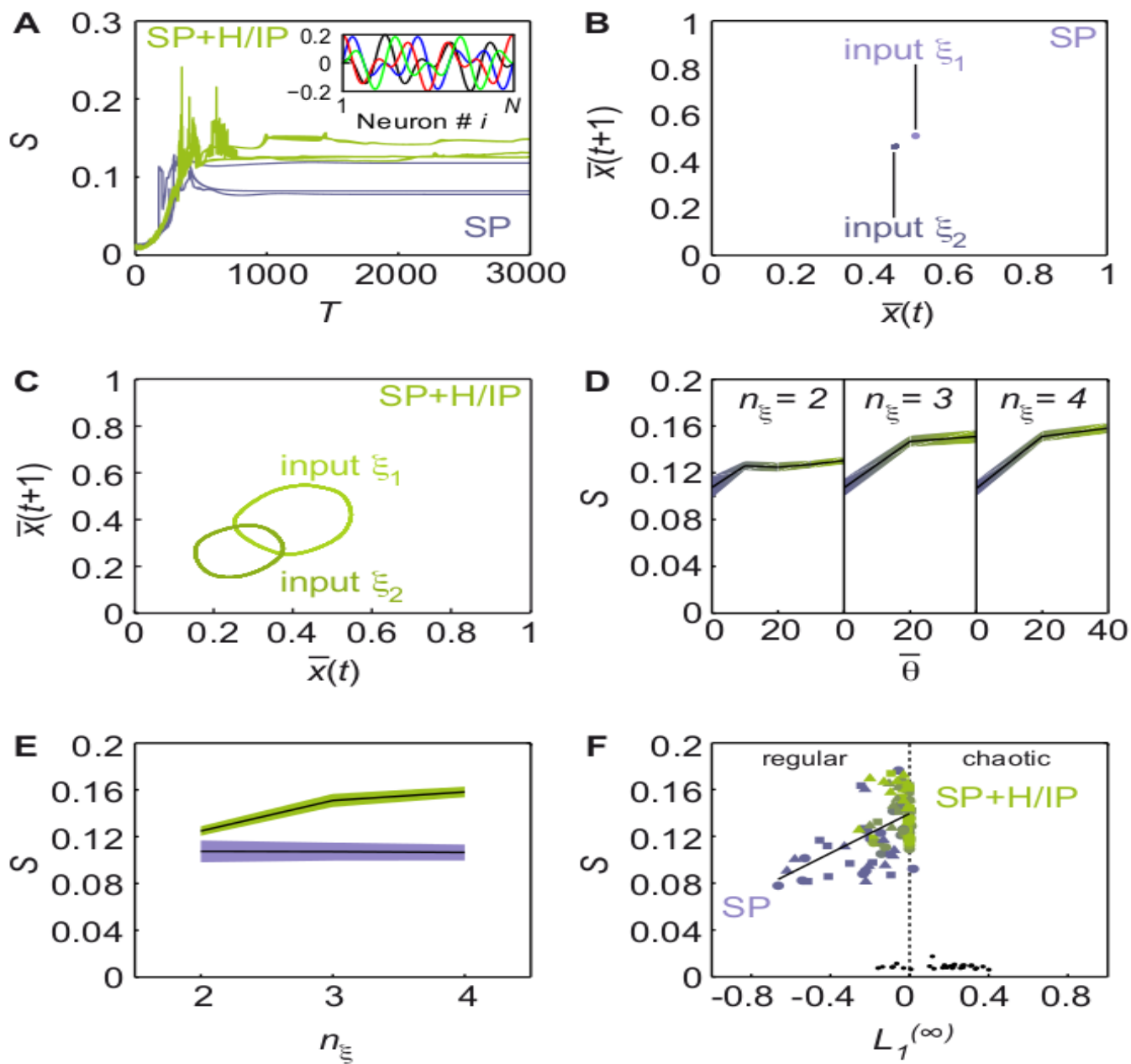


Local field variance at the end of the simulation in the presence (yellow curve) or absence (red curve) of HIP. (B-C)

Combining Hebbian and intrinsic plasticity.



Combining Hebbian and intrinsic plasticity.



Conclusion

Synaptic and Intrinsic plasticity are interesting examples of *multi scales* dynamics.

We have a *qualitative understanding* of the conjugated of SP and IP on this model dynamics, but a *mathematical theory* is still missing.

Missing steps :

Dynamic mean-field theory for correlated weights (the challenge is not to obtain the equations but rather to study them and their links with the finite system dynamics).

Finite size effects.

Developping such a theory could provide methods for the analysis of many other examples of dynamical systems with *adaptation and memory*.