



Does Boson Sampling need Fault-Tolerance?

Anthony Leverrier, Raul Garcia-Patron

► To cite this version:

Anthony Leverrier, Raul Garcia-Patron. Does Boson Sampling need Fault-Tolerance?. Quantum Information Processing - QIP 2014, Feb 2014, Barcelone, Spain. . hal-01094201

HAL Id: hal-01094201

<https://inria.hal.science/hal-01094201>

Submitted on 11 Dec 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Does Boson Sampling need Fault-Tolerance?



Anthony Leverrier¹ & Raúl García-Patrón²

¹INRIA Paris

²Université Libre de Bruxelles

ULB

ABSTRACT

BosonSampling is a problem where a quantum computer offers a provable speedup over classical computers. Its main feature is that it can be solved with current linear optics technology, without the need for a full quantum computer.

We investigate whether an experimentally realistic BosonSampler can really solve BosonSampling without any fault-tolerance mechanism.

More precisely, we study how the unavoidable errors linked to an imperfect calibration of the optical elements affect the final result of the computation.

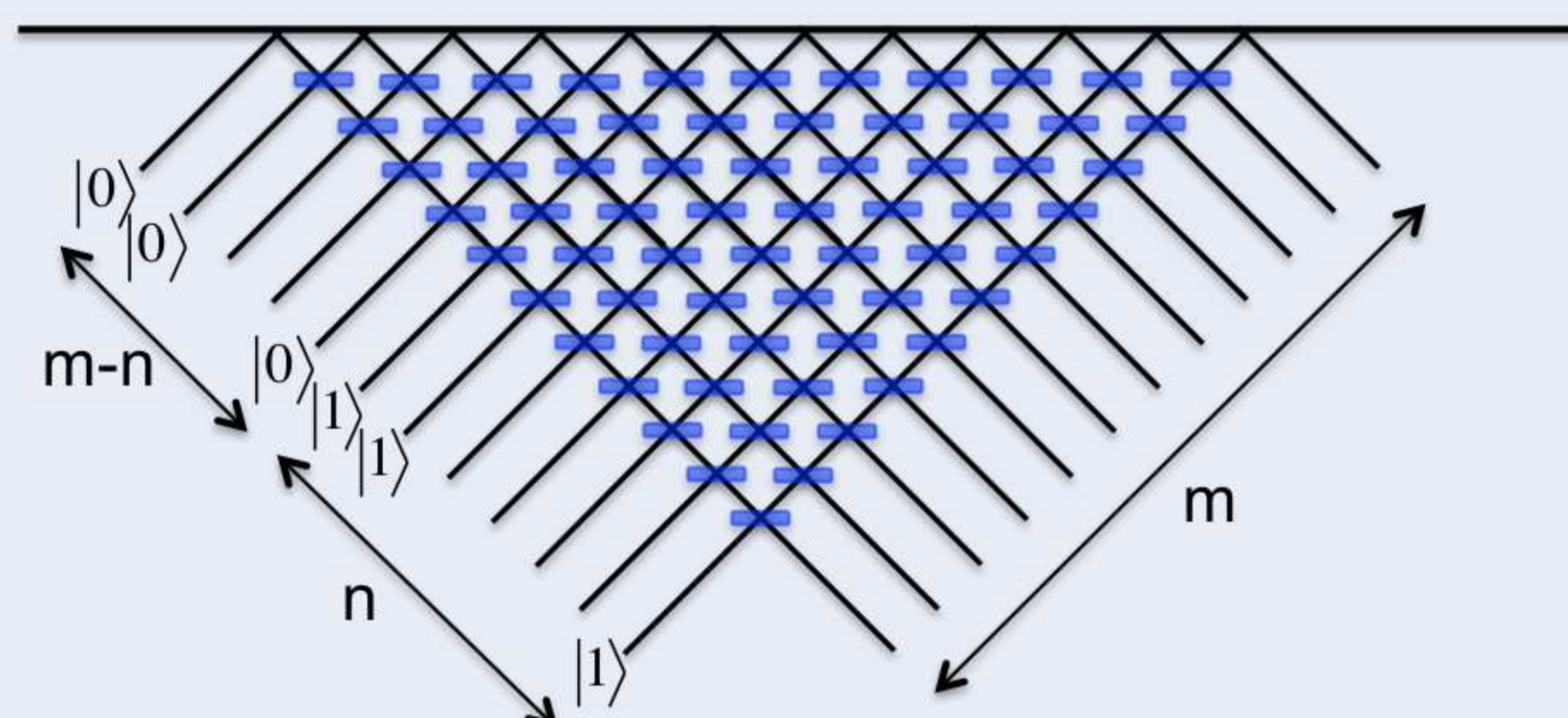
We show that the fidelity of each optical element must be at least $1 - O(1/n^2)$, where n refers to the number of single photons in the scheme. Such a requirement seems to be achievable with state-of-the-art equipment.

THE SETUP

BosonSampling(m, n):

INPUT: a unitary matrix U drawn from the Haar measure on $U(m)$

OUTPUT: a sample (s_1, \dots, s_m) from the distribution obtained by measuring the output modes of the following network in the photon number basis. More precisely, m input modes (n of which contain a single photon) are processed through an array of beamsplitters and phase-shifters that implement the unitary U (on the annihilation operators).



$$\Pr_U[s_1, s_2, \dots, s_m] = \frac{|Per(U_{s_1 \dots s_m})|^2}{s_1! s_2! \dots s_m!}$$

where s_i is the number of photons in the i th output mode and U_{s_1, \dots, s_m} is the $n \times n$ submatrix obtained by taking the first n rows of U and repeating the i th column s_i times.

Aaronson and Arkhipov [1] showed that BosonSampling is hard for a classical computer, and provide evidence for hardness even in the approximate version of the problem, where we want to sample ϵ -close to the ideal one in trace distance.

CAN A REALISTIC BOSONSAMPLER SOLVE THE PROBLEM?

A realistic experimental BosonSampler [2-5] will be subject to imperfections:

- losses [6]
- single photons not perfectly indistinguishable [7]
- imperfect detection efficiency [6]
- optical network not implementing exactly the desired transformation [8]

Reck et al [9] proposed a scheme to implement the unitary U as a network of beamsplitters and phase-shifts containing $O(m^2)$ beamplitters.

Question: how do errors made at each optical component propagate through the circuit? And how do they affect the output distribution?

MODEL FOR NOISE

The noise affects each (complex) beamsplitter identically and independently:

- the beamplitter U_k acting on modes k_1 and k_2 is replaced by $\Phi(U_k)$ acting on the same two modes such that
- $\Phi(U_k) \Phi(U_k^\dagger) = \exp(i\epsilon h_k)$ with h_k drawn from the Gaussian Unitary Ensemble (GUE)

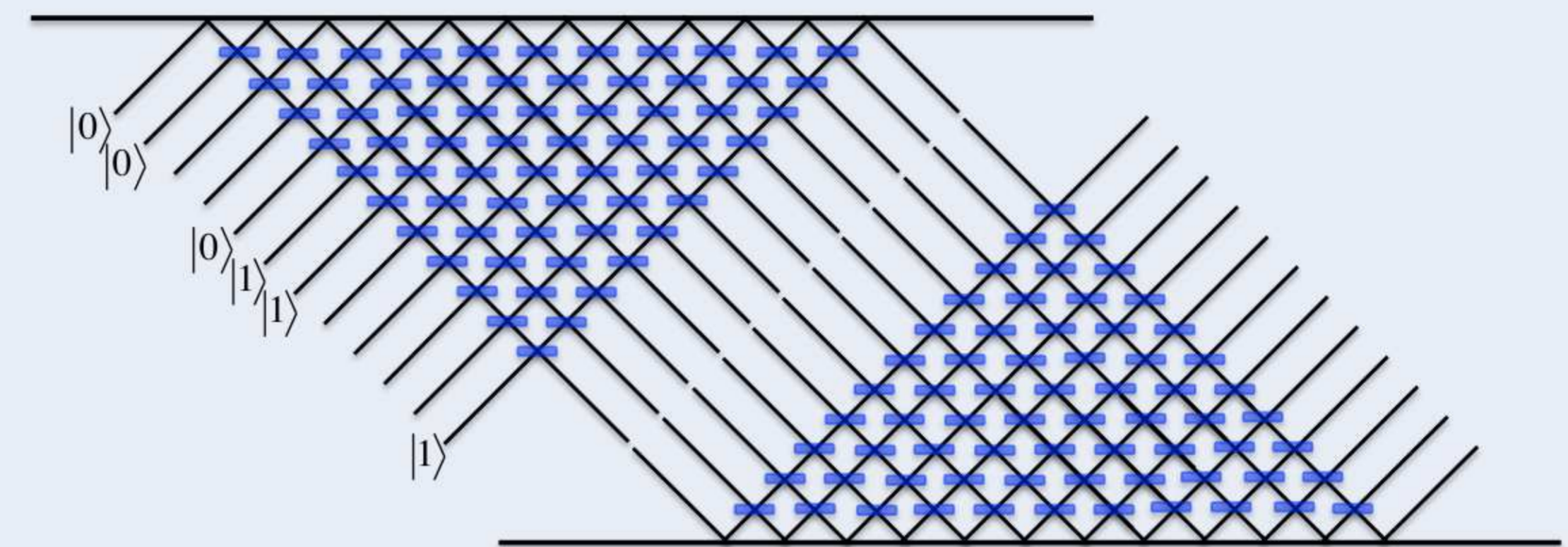
$$h_k = \begin{pmatrix} \alpha_k & \gamma_k \\ \gamma_k^* & \beta_k \end{pmatrix} \text{ with } \alpha_k, \beta_k \sim \mathcal{N}(0, 1)_{\mathbb{R}}, \quad \gamma_k \sim \mathcal{N}(0, 1/2)_{\mathbb{C}}$$

Goal: value of $E_U E_\Phi \left\| \Pr_U - \Pr_{\Phi(U)} \right\|_1$

Unfortunately, very hard question, because of the permanent, and because the sum contains exponentially many terms, all exponentially small.

A MORE TRACTABLE PROBLEM

We consider the network U , concatenated with its inverse U^\dagger . Ideally, this network should implement the identity channel. We compute: $E_U E_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^\dagger)\Phi(U)} \right\|_1$



$$E_U E_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^\dagger)\Phi(U)} \right\|_1 = 2 - 2E_U E_\Phi \left| Per \left(\left[\Phi(U^\dagger)\Phi(U) \right]_{n \times n} \right) \right|^2$$

where $[A]_{n \times n}$ is the $n \times n$ upper-left minor of A .

Main result: In the regime where $m=O(n^2)$, and $n^2 \epsilon^2 \ll 1$,

$$E_U E_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^\dagger)\Phi(U)} \right\|_1 = \Omega(n^2 \epsilon^2).$$

Proof sketch:

- The matrix $\Phi(U^\dagger)\Phi(U)$ can be written $\exp(i\epsilon H_N)$ where H_N is the N th step of a random walk on Hermitian matrices, and N is the number of beamsplitters.
- The Baker-Campbell-Hausdorff formula gives $H_N = \sum \Phi(U_n^\dagger) \dots \Phi(U_k^\dagger) h_k \Phi(U_k) \dots \Phi(U_n)$
- The permanent of the $n \times n$ upper-left minor of $\exp(i\epsilon H_N)$ can be computed via $E_x \left| \left((I_n - \Pi_n) H_N \Pi_n x \right) \right|^2$ where $x = (e^{i\theta_1}, \dots, e^{i\theta_n})$ with θ_k uniform on $[0, 2\pi]$ and $\Pi_n = \text{diag}(1, \dots, 1, 0, \dots, 0)$.
- The expectations are handled with matrix concentration bounds due to Tropp [10].

CONCLUSION

The error does not scale too badly. For “reasonable” parameters, say $m = 1000$ modes, and $n=30$ single photons, the average fidelity per gate required to maintain the error on the order of a few percent is 0.999.

While such numbers certainly are challenging, they should not be a major problem in forthcoming BosonSampling experiments.

REFERENCES

- [1] S. Aaronson, A. Arkhipov. The computational complexity of linear optics. *Theory of Computing*, 9(4):143-252, 2013.
- [2] M. Broome, et al. Photonic boson sampling in a tunable circuit. *Science*, 339(6121):794-798, 2013.
- [3] M. Tillmann, et al. Experimental boson sampling. *Nature Photonics*, 7:540, 2013.
- [4] J. Spring, et al. Boson sampling on a photonic chip. *Science*, 339(6121):798-801, 2013.
- [5] A. Crespi, et al. Experimental boson sampling in arbitrary integrated photonic circuits. *Nature Photonics*, 7:545, 2013
- [6] P. Rohde, T. Ralph. Error tolerance of the boson-sampling model for linear optics quantum computing. *Phys. Rev. A*, 85(2):022332, 2012.
- [7] P. Rohde Optical quantum computing with photons of arbitrarily low fidelity and purity. *Phys. Rev. A*, 86(5):052321, 2012.
- [8] A. Leverrier and R. García-Patrón. Does boson sampling need fault-tolerance? [arXiv:1309.4687](https://arxiv.org/abs/1309.4687), 2013.
- [9] M. Reck, et al. Experimental realization of any discrete unitary operator. *Phys. Rev. Lett.*, 73(1):58, 1994.
- [10] J. Tropp. User-friendly tail bounds for sums of random matrices. *Foundations of Computational Mathematics*, 12(4):389-434, 2012.

INRIA Rocquencourt, Domaine de Voluceau, B.P. 105, 78153 Le Chesnay Cedex, France
anthony.leverrier@inria.fr

Quantum Information and Communication, Ecole Polytechnique de Bruxelles, CP 165, Université Libre de Bruxelles, 1050 Bruxelles, Belgium
rgarciap@ulb.ac.be