Several Forms of Fuzzy Analogical Reasoning

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Abstract

We present a general framework representing analogy, on the basis of a link between variables and measures of comparison between values of variables. This analogical scheme is proven to represent a common description of several forms of reasoning used in fuzzy control or in the management of knowledge-based systems, such as deductive reasoning, inductive reasoning or prototypical reasoning, gradual reasoning.

1. Introduction

Analogy is a natural means of drawing a conclusion in human reasoning. In artificial intelligence, analogy is also an explored domain, analogical reasoning and case-based reasoning have extensiveley been studied. In both approaches, the definition of resemblances is crucial and very often given in a prior way, but there exist very few studies on adapting an existing solution to a new piece of information.

Analogical reasoning has been formalized in a fuzzy set based approach in various directions [6][18], providing solutions for this adapting phase. Approximate reasoning has been presented by L.A. Zadeh as a method of automatic reasoning as close as possible to human reasoning [22]. Then, it seems natural that there is some relationship between approximate reasoning and analogy, which is at the root of most human reasoning processes. In this paper, we give a general analogical scheme which allows to regard several reasoning methods in a common framework.

2. Analogical Scheme

We consider two variables X and Y, which may be simple or compound, defined on universes $\mathbb{X}$ and $\mathbb{Y}$. Let us denote by $F(\mathbb{X})$ and $F(\mathbb{Y})$ respective sets of fuzzy sets of $\mathbb{X}$ and $\mathbb{Y}$, which are either the respective sets $[0,1]^{\mathbb{X}}$ and $[0,1]^{\mathbb{Y}}$ of all fuzzy sets of $\mathbb{X}$ and $\mathbb{Y}$, or subsets of $[0,1]^{\mathbb{X}}$ and $[0,1]^{\mathbb{Y}}$.

For a given relation β on $[0,1]^{\mathbb{X}} \times [0,1]^{\mathbb{Y}}$ and two relations R on $[0,1]^{\mathbb{X}} \times [0,1]^{\mathbb{X}}$ and S on $[0,1]^{\mathbb{Y}} \times [0,1]^{\mathbb{Y}}$, an *analogical scheme* is a function $\Re_{\beta RS} : F(\mathbb{X}) \times F(\mathbb{Y}) \times [0,1]^{\mathbb{X}} \to [0,1]^{\mathbb{Y}}$ satisfying :
$\forall B \in F(\mathbb{X})$ and $\forall C \in F(\mathbb{Y})$ such that $B\beta C$,
$\forall B' \in [0,1]^{\mathbb{X}}$ such that BRB',
(i) $C = \Re_{\beta RS}(B, C, B)$
(ii) $C' = \Re_{\beta RS}(B, C, B')$ satisfies ($B'\beta C'$ and CSC')
(Figure 1)

We can interprete this scheme as follows, as soon as β, R and S are defined properly : if B and C are known to be linked by β, and if B′ resembles B, we are able to find C′ such that B′ and C′ are linked by β and C′ resembles C.

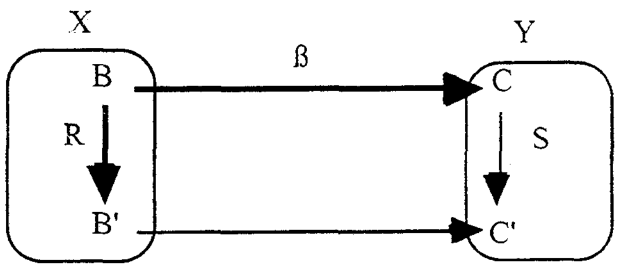

Figure 1. Analogical scheme

For instance, B is a characterization of the attribute X of an object and Y is the class of this object and the link β says that if, for an object, X is characterized by B then its class Y is C. Then, if another object corresponds to a characterization B′ of X not very far from B, then the class we must assign to this new object is not very far from C.

Depending on the choice we make for β, R, S, the general definition of an analogical scheme yields various types of reasoning used in artificial intelligence.

3. Analogical Scheme for Inductive and Prototype-Based Reasoning

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Let us suppose that we are given a population of objects E, characterized by values of a compound variable $X = (X_1, \dots X_m)$, where $X_1, \dots X_m$ are elementary attributes, and a class Y, defined on $\mathcal{Y} = \{y_j / j \in J\}$. We suppose that, for a finite subset of E, used as a training set, values of X and Y are known. For other objects of E, the only value of X is known and the value of Y is to be determined.

For instance, in the well-known iris database of Fisher, objects are described by means of four attributes (the sepal length X_1, the sepal width X_2, the petal length X_3 and the petal width X_4) and they are associated with a class Y referring to a type of iris plant (setosa, versicolor or virginica). We have $\mathcal{Y}$= {Iris-Setosa, Iris-Versicolor, Iris-Virginica}. Such database could be downloaded from the ftp site of the University of California at Irvine, (ftp://ftp.ics.uci.edu/pub/machine-learning-databases).

Inductive learning consists in constructing a tree of attributes from the training set, in such a way that a path from the root of the tree to a leaf can be expressed as an if-then rule (R_j) for any $j \in J$. Prototype-based reasoning uses the training set to identify prototypical values of the attributes in each class $\{y_j\}$. Then, the membership of elements of E to y_j can be regarded as the satisfiability of a rule (R_j) for any $j \in J$.

Examples of rules we can obtain for the iris database are the following :

(R1): if (petal-width is *Small*) then Iris-Setosa

(R2): if (petal-length is *Medium*) or ((petal-length is *Long*) and (petal-width is *Small*)) then Iris-Versicolor

(R3): if (petal-length is *Long*) and (petal-width is *High*) then Iris-Virginica

In both cases, we can use a fuzzy set based knowledge representation, allowing to avoid too strict limits of the values of attributes for a given class, and also to use in the same system either numerical ("25 km") or symbolic ("far") values of a given attribute.

We present briefly the learning phase in inductive learning and in prototype-based reasoning, then we exhibit their common analogical scheme for the classification or decision-making phase. We use the frameworks introduced in [4] and [16].

3.1. Fuzzy inductive learning

The aim of inductive learning is to find general rules enabling us to classify any object of E, *i.e.* to generalize the knowledge obtained by the observation of objects of E in the training set to determine the value of Y for any objects of E, when their only value of X is known. The common inductive learning method is based on the construction of a decision tree from the training set. A decision tree is composed by three kinds of elements: nodes, edges and leaves. A node is associated with a question on the values of an attribute X_i and each edge going out of a node is associated with a particular value (or modality) of X_i. A leaf, which is a terminal node, is labeled with a modality y_j of the class Y. A path is composed of nodes linked by edges and ends in a leaf.

To build a decision tree is equivalent to choose an efficient order on the questions to ask on the values of the attributes for an object in order to determine its class. Usually, a question related to an attribute is selected by means of a measure of discrimination from the set of all possible questions. An example of such a measure is Shannon's measure of entropy, used for instance in the most common algorithm of construction of decision trees, the ID3 algorithm [13]. The modalities of the chosen attribute split the training set into subsets of objects. On each subset, another question regarding an attribute is selected until all the objects of the subset pertain to a single class.

Classical decision trees correspond to symbolic trees, built from training sets where all the attributes take their values in a finite set. To handle other existing kind of attributes, such as numerical attributes or numerical-symbolic attributes, new methods are introduced, either to construct decision trees or to use them in a generalization process [12]. When considering that the symbolic values of a numerical-symbolic attribute are fuzzy modalities on the numerical universe of its values, particular methods from fuzzy set theory to treat these values enable to take into account this kind of attributes. These methods enable to build *fuzzy decision trees* [14], [19], [11], [20], [1], [4]. Most of these methods are based on the ID3 algorithm and use particular techniques to take into account the imprecision in the data. Differences between them lie essentially in the choice of a new measure of discrimination to use during the construction of a fuzzy decision tree and in the discretization method to construct the fuzzy modalities associated with edges. The chosen measure takes into account the discriminating power of an attribute and, also, fuzzy modalities for the numerical-symbolic attributes.

The use of such fuzzy decision trees to classify new objects of E is based on an extension of the classic method of utilization of decision tree. In addition to that, it enables to associate more than one class to an object, each class weighted by a membership degree [4]. A decision tree (either basic or fuzzy) is considered as a rule base. All the questions associated with nodes of a path of the tree are aggregated into a set of questions by means of the AND operator, and all the set of paths ending in a leaf labeled by the same value y_j of the class Y are aggregated into a rule R_j, by means of the OR operator. Thus, a set of *if-then* rules R_j is obtained, the premises of these rules are the aggregated sets of questions and their conclusion is the corresponding value y_j of the class Y.

3.2. Fuzzy prototype-based reasoning

The aim of prototype-based reasoning is to construct a fuzzy prototype for each class. A fuzzy prototype synthesizes a class and enables to generate a set of objects because of the information it contains [21]. The power of description of a prototype can be used for a classification process.

The notion of prototype is linked to the notion of typicality [17], [21]. The construction of a prototype needs to determine the typicality of each value appearing in a learning database.

We consider that the degree of typicality of an object depends positively on its total resemblance to other objects of its class (internal resemblance) and on its total dissimilarity to objects of other classes (external dissimilarity) [15].

We suppose that there exists a partition given on the set of objects E composed by crisp classes y_j. The typicality of the value B of an attribute X_i of an object of the class y_j is computed as follows:

Step 1. Compute the resemblance $r(B, B_l)$ between B and the value B_l of the attribute X_i for any example of the same class y_j. The global resemblance R(B) relative to the set of values of X present in examples, is obtained in aggregating the degrees $r(B, B_l)$ computed as above described.

Step 2. Compute the dissimilarity $d(B, B_l)$ between B and the value B_l of the attribute X for any example of class y_k different from y_j. The total dissimilarity D(B) relative to the set of values of X_i present in examples, is obtained in aggregating the degrees $d(B, B_l)$ computed as above described.

Step 3. The aggregation of this two values, R(B) et D(B), gives the typicality T(B) of B, according to the attribute X_i for the class y_j.

The fuzzy prototype is composed by the most typical values for each attribute of a considered class. This means that a fuzzy prototype is a virtual object described by means of the same attributes as those pertaining to the learning database.

A prototype can be considered as a rule describing a class [3]. The classification process is based on a comparison between the object to be classified and a prototype. The question is: *does the new object satisfy a prototype?* The computed degrees of satisfiability are aggregated in order to obtain a total degree of satisfiability of a new object for a prototype.

3.3. Analogical scheme

The if-then rules obtained by both methods are of the following form :

(R_j) : if$\left(X^j_{i_1(1)}\text{ is }B^j_{i_1(1)}\text{ and }X^j_{i_1(2)}\text{ is }B^j_{i_1(2)}\text{ and}\ldots\right)$

or ... $\left(X^j_{i_n(1)}\text{ is }B^j_{i_n(1)}\text{ and }X^j_{i_n(2)}\text{ is }B^j_{i_n(2)}\text{ and}\ldots\right)$

then Y is C_j,

with $C_j = \{y_j\}$, $i_p(1) < i_p(2) < \ldots$, and

$\left\{X^j_{i_1(1)}, X^j_{i_1(2)}\ldots\right\} \subseteq \{X_1,\ldots X_m\}\ldots$

$\left(X^j_{i_n(1)}, X^j_{i_n(2)}\ldots\right) \subseteq \{X_1,\ldots X_m\}$

Let us consider $F(\mathbb{Y})=\{\ C_j / j \in J\}$ and $F(\mathbb{X})$ the set of all corresponding values $(B_1,\ldots B_m)$ of X appearing in all the rules, such that $B_{i_p(1)} = B^j_{i_p(1)}$, $B_{i_p(2)} = B^j_{i_p(2)}$ ... for $1 \le p \le n$, $1 \le j \le m$, with B_u equal to $\mathbb{X}$ if no index $i_{p(1)}, i_{p(2)},\ldots$ is equal to u, which means that X_u has no influence on the identification of Y in this case ($1 \le u \le m$).

The link β is defined by :

$(B_1,\ldots B_m)\beta C \Leftrightarrow$ i) or ii) holds with :

i) ∃ an object in E with value of X equal to $(B_1,\ldots B_m)$ and value of Y equal to C

ii) $\exists R_j$ such that $B_{i_p(1)} = B^j_{i_p(1)}, B_{i_p(2)} = B^j_{i_p(2)},\ldots$ with p = 1 or ... n.

Let us consider a measure of satisfiability r [5], for instance defined by :

$$r(B,B') = \min_k \frac{\int_X \min(B_k(x), B'_k(x))dx}{\int_X B_k(x)dx},$$

where the min operator can be replaced by another aggregation operator.

We define the relation R as :

$\forall B = (B_1,\ldots B_m) \in [0,1]^{\mathbb{X}}\quad \forall B' = (B'_1,\ldots B'_m) \in [0,1]^{\mathbb{X}}$

$BRB' \Leftrightarrow r(B,B') > 0$

In the case where a crisp decision or class must be identified, the relation S is the identity :

$CSC' \Leftrightarrow C = C'$

and we consider the function

$\Re_{\beta RS}\ :\ F(\mathbb{X}) \times F(\mathbb{Y}) \times [0,1]^{\mathbb{X}} \to [0,1]^{\mathbb{Y}}$

defined as follows :

$\Re_{\beta RS}(B,C,B') = C \Leftrightarrow r(B,B') = \underset{B'' \in F(\mathbb{X})}{\perp} r(B'',B')$

where ⊥ is an aggregation operator, generally the t-conorm *maximum*.

Then it is easy to prove that $\Re_{\beta RS}$ is an analogical scheme.

This means, that for an object of E with a description B' of X, we look for the description B of X available in the rules which is as close as possible to B', and we choose its associated class C to assign to this object.

In the case where we can provide a fuzzy decision or class as a result of the reasoning, the relation S is defined from a resemblance relation s by
$\forall C \in [0,1]^{\mathbb{Y}} \ \forall C' \in [0,1]^{\mathbb{Y}} \ CSC' \Leftrightarrow s(C,C') > 0$, with for instance :

$$s(C,C') = \sup_j \min(C(y_j), C'(y_j))$$

We consider now the function $\Re_{\beta RS}$ defined as follows :

$$\forall j \in J \ \ \Re_{\beta RS}(B, C_j, B') = C' \Leftrightarrow$$
$$r(B,B') = \underset{\{B''/B'' \in F(\mathbb{X}), B''\beta C_j\}}{\perp} r(B'',B')$$
and $C'(y_j) = r(B,B')$

It can be proven that $\Re_{\beta RS}$ is an analogical scheme.

This means that, for an object of E with a description B' of X, we look for a fuzzy class Y, i.e. a fuzzy subset C' of $\mathbb{Y}$ obtained by assigning to each possible decision y_j in $\mathbb{Y}$ a membership degree equal to the satisfiability of the description B of X available in the rules which has the greatest satisfiability measure with B', and such that B is linked with $C_j = \{y_j\}$ (i.e. $B\beta C_j$).

4. Analogical scheme for deductive reasoning

We consider $F(\mathbb{X})$ and $F(\mathbb{Y})$ respective finite subsets of $[0,1]^{\mathbb{X}}$ and $[0,1]^{\mathbb{Y}}$ and (R_j) a base of rules of the form "if X is B_j then Y is C_j", with B_j in $F(\mathbb{X})$ and C_j in $F(\mathbb{Y})$, $j \in J$, with B_j and C_j normalized fuzzy sets. With the same example as in section 3, we can have :
if (petal-width is *Small*) then Iris-Setosa

The link β is defined by :
$B\beta C \Leftrightarrow$ i) or ii) holds with :
i) $\exists$ an object in E with value of X equal to B and value of Y equal to C
ii) $\exists R_j$ such that $B = B_j$ and $C = C_j$

The relations R and S are defined from measures of satisfiability r and s respectively defined on $[0,1]^{\mathbb{X}}$ and $[0,1]^{\mathbb{Y}}$, in such a way that there exist two thresholds ρ and σ in $[0,1]$ such that

$$\forall B \ \ \forall B' \in [0,1]^{\mathbb{X}} \ \ BRB' \Leftrightarrow r(B,B') \geq \rho$$
$$\forall C \ \ \forall C' \in [0,1]^{\mathbb{Y}} \ \ CSC' \Leftrightarrow s(C,C') \geq \sigma.$$

The function $\Re_{\beta RS}$ is defined as follows :

$\Re_{\beta RS}(B,C,B') = C' \Leftrightarrow C'$ is obtained from B, C, B' by means of the so-called compositional rule of inference.

$$\forall y \in \mathbb{Y} \ \ C'(y) = \sup_{x \in \mathbb{X}} T(B'(x), I(x,y)),$$

where $I(x,y)$ is a fuzzy implication and T is a t-norm.

It can be proven that $\Re_{\beta RS}$ is an analogical scheme with the following choices for the relations :

1) Measures of satisfiability :

$$r(B,B') = \inf_{x \in \mathbb{X}} \min(1 - B'(x) + B(x), 1)$$
$$s(C,C') = \inf_{y \in \mathbb{Y}} \min(1 - C'(y) + C(y), 1)$$

R and S are defined by any equal thresholds $\rho = \sigma$ in [0, 1], for the following fuzzy implications :

$I(x,y) = 1 - B(x) + B(x)C(y)$ (Reichenbach)
$I(x,y) = \max(1 - B(x), C(y))$ (Kleene – Dienes)
$I(x,y) = \min(1 - B(x) + C(y), 1)$ (Lukasiewicz)

We choose the Lukasiewicz t-norm
T(a,b) = max(a+b-1, 0).

2) Measures of satisfiability

$$r(B,B') = 1 - \sup_{\{x \in \mathbb{X} / B(x)=0\}} B'(x)$$
$$s(C,C') = 1 - \sup_{\{y \in \mathbb{Y} / C(y)=0\}} C'(y)$$

R and S are defined by any equal thresholds $\rho = \sigma$ in [0, 1], for the following fuzzy implications :

$$I(x,y) = \begin{cases} 1 & \text{if } B(x) \leq C(y) \\ 0 & \text{otherwise} \end{cases} \quad \text{(Rescher – Gaines)}$$
$$I(x,y) = \begin{cases} 1 & \text{if } B(x) \leq C(y) \\ C(y) & \text{otherwise} \end{cases} \quad \text{(Brouwer – G del)}$$
$$I(x,y) = \begin{cases} \min\left(\frac{C(y)}{B(x)}, 1\right) & \text{if } B(x) \neq 0 \\ 1 & \text{otherwise} \end{cases} \quad \text{(Goguen)}$$

We choose the Lukasiewicz t-norm
T(a,b) = max(a+b-1, 0).

3) Measures of similitude

$$r(B,B') = \sup_{x \in \mathbb{X}} \min(B(x), B'(x))$$
$$s(C,C') = \sup_{y \in \mathbb{Y}} \min(C(y), C'(y))$$

R and S are defined by any equal thresholds $\rho = \sigma$ in [0, 1], for the following fuzzy implication :
$I(x,y) = \min(B(x), C(y))$ (Mamdani)
We choose the Zadeh t-norm T(a,b) = min(a, b).

4) Measures of similitude

$$r(B,B') = \sup_{x \in \mathbb{X}} (B(x).B'(x))$$
$$s(C,C') = \sup_{y \in \mathbb{Y}} (C(y).C'(y))$$

R and S are defined by any equal thresholds $\rho = \sigma$ in [0, 1], for the following fuzzy implication :
$I(x, y) = B(x).C(y)$ (Larsen)
We choose the product t-norm $T(a,b) = \min(a, b)$.

This can be interpreted as follows. If there exist a rule "if X is B then Y is C", such that B′ resembles B at least at the level ρ in [0, 1], then the compositional rule of inference yields a result C′ which resembles C also at least at the same level ρ, and the resemblance between C and C′ is the same as the comparison between B and B′, since the measure of satisfiability or similitude which is involved is the same. This proves that the compositional rule of inference works in an analogical way.

In the case of Mamdani and Larsen implications, which do not correspond to extensions of the implication in classical logic, but are known to be useful in fuzzy control, we obtain the same property of equivalence between the use of the compositional rule of inference (well-known fuzzy control methodology) and an analogical scheme.

5. Analogical scheme for gradual reasoning

Gradual knowledge is very common in knowledge-based systems, generally expressed as rules of the form "the more X is B, the more Y is C", or "the more X is B, the more Y is C is certain". There exist several approaches to such graduality [7][9][10]. Here, we restrict ourselves to a graduality represented by linguistic modifiers [2][7], such as "really" or "relatively", for instance. Linguistic modifiers are useful to modulate the fuzzy values of attributes, by weakening or reinforcing the meaning of a given value, for instance "small" yielding "very small" or "relatively small".

A modifier m modulates the characterization B of X by creating a new characterization B′ = mB with membership function obtained by means of a transformation t_m as such that $\forall x \in X \;\; B'(x) = t_m(B(x))$ [2]. We use for instance modifier m_α [7] defined as follows. Other definitions would give analogous results [8].

$$\forall x \in X \;\; m_\alpha B(x) = B(x+\alpha) \text{ if } x+\alpha \in X$$
$$m_\alpha B(x) = B(X^-) \text{ if } x+\alpha < X^-$$
$$m_\alpha B(x) = B(X^+) \text{ if } x+\alpha > X^+$$

if X is supposed to be ordered, with smallest element X^-, greatest element X^+. If we apply this modifier to fuzzy descriptions of X, constituting a fuzzy partition of X, we obtain two kinds of behavior, either reinforcing ("really") or weakening ("relatively"), according to α being positive or negative and to the position of B in the ordered list of classes of the partition (Figure 2).

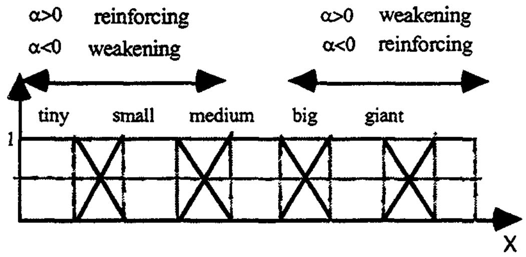

Figure 2. Effect of a modifier on fuzzy descriptions of X

The concept of graduality we use here corresponds to the fact that, if we weaken (or reinforce) the value of a variable X, then we weaken (or reinforce) also the value of a variable Y linked to X. For instance "the more we separate the iris rhizomes, the more flowers we obtain". This is expressed as "if the value of X is reinforced, then the value of Y is reinforced". It corresponds to the idea that a rule "if X is m_αB, then Y is $m_{\alpha'}$C" stems from a rule "is X is A then Y is B"

The graduality is defined by the fact that :
• the same modifiers (with linked values of parameters) are used for the descriptions of both variables,
• variations regarding membership functions are equal, i.e. :

$$\forall x \in X \;\; \forall y \in Y \;\; (C(y) = B(x)) \Rightarrow$$
$$\left(m_{\alpha'}C_\gamma(y) - C_\gamma(y) = m_\alpha B_\gamma(x) - B_\gamma(x)\right)$$

where γ indicates the right or left hand part of the function.

We consider respective finite subsets F(X) and F(Y) of $[0,1]^X$ and $[0,1]^Y$ and (R_j) a base of rules of the form "if X is B_j then Y is C_j", with B_j in F(X) and C_j in F(Y), $j \in J$, B_j and C_j normalized fuzzy sets with trapezoidal membership functions.

The link β is defined by :
$B\beta C \Leftrightarrow$ i) or ii) holds with :
i) $\exists$ an object in E with value of X equal to B and value of Y equal to C
ii) $\exists R_j$ such that $B = B_j$ and $C = C_j$

We consider two operations defined by the inverse of the addition of fuzzy intervals, respectively on X and Y, denoted by $r : [0,1]^X \times [0,1]^X \rightarrow [0,1]^X$ and $s : [0,1]^Y \times [0,1]^Y \rightarrow [0,1]^Y$. The relations R and S are defined from r and s in such a way that there exist two thresholds $\rho \in [0,1]^X$ and $\sigma \in [0,1]^Y$ satisfying :

$$\forall B \;\; \forall B' \in [0,1]^X \;\; BRB' \Leftrightarrow r(B,B') = \rho, \;\; \rho \in [0,1]^X$$
$$\forall C \;\; \forall C' \in [0,1]^Y \;\; CSC' \Leftrightarrow s(C,C') = \sigma, \;\; \sigma \in [0,1]^Y,$$

We define a function $\Re_{\beta RS}$ as follows :

$\Re_{\beta RS}(B, C, B') = C' \Leftrightarrow B' = m_{\alpha}B, C' = m_{\alpha'}C$,

with $\alpha=\phi(\rho)$, σ is obtained from α and from the difference between support and kernel of B and C [8], and $\alpha'=\phi(\sigma)$.

It can be proven that $\Re_{\beta RS}$ is an analogical scheme.

This result can be extended to other kinds of modifiers. It expresses the fact that gradual reasoning can be regarded as a progressive passage from a reference given in a rule and other rules obtained from this reference by using linguistic modifiers. The link between gradual reasoning and analogical reasoning corresponds to the utilization of a relationship between variations of X and variations of Y expressed in gradual knowledge to infer a value of Y from a given value of X. The links between this kind of graduality and interpolation [8] would lead to another form of analogical scheme for interpolative reasoning.

6. Conclusion

We have presented a general definition of analogy through a fonction considered as an analogical scheme, depending on a link between variables and relations generally defined from measures of proximity, resemblance, satisfiability and, more generally measures of comparison. This definition shows that several forms of reasoning in a fuzzy environment are based on the same approach and work in an analogical way. We will explore several other forms of reasoning from this point of view, such as interpolative reasoning, analogical reasoning, case-based reasoning...

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