

# A Cancellation Operator Suitable for Identification of Nonlinear Volterra Models

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## Abstract

We present a new identification method for nonlinear possibly singular Volterra models based on a suitable operatorial transformation of the problem with the property that some nonlinear terms are cancelled, allowing to get an equivalent formulation well adapted to least square identification.

## Keywords

Nonlinear Volterra Equation, Diffusive Representation, Pseudo-Inverse.

## 1 Introduction

Consider a nonlinear Volterra model of the form:

$$H(\partial_t)X = F(X) + G(X)u, \quad (1)$$

where  $u(t)$ ,  $X(t) \in \mathbb{R}$ ,  $t > 0$ ,  $F$  and  $G$  are possibly singular functions defined on  $\mathbb{R}$ , and  $H(\partial_t)$  is an invertible causal convolution operator of diffusive type [3] such that  $|\frac{H(i\omega)}{\omega}| \rightarrow 0$  when  $|\omega| \rightarrow \infty$ . We suppose that problem (1) is well-posed.

Let  $\psi_X$  the  $\gamma$ -diffusive representation of  $X$ , that is the unique solution of the following problem on  $(t, \xi) \in \mathbb{R}^{+*} \times \mathbb{R}$  [3]:

$$\partial_t \psi_X = \gamma \psi_X + X, \quad \psi_X(0, \cdot) = 0, \quad (2)$$

with  $\gamma : \mathbb{R} \rightarrow \mathbb{C}^-$ . With  $\mu$  the  $\gamma$ -symbol associated with  $H(\partial_t) \circ \partial_t^{-1}$ , the  $\gamma$ -diffusive formulation of equation (1) is [3]:

$$\langle \mu, \gamma \psi_X + X \rangle_{\Delta'_\gamma, \Delta_\gamma} = F(X) + G(X)u. \quad (3)$$

The problem under consideration in the sequel is to build estimations of  $H(\partial_t)$  via the identification of the  $\gamma$ -symbol  $\mu$  of  $H(\partial_t) \circ \partial_t^{-1}$  from (noised) data  $(\tilde{X}, \tilde{u}) = (X^k + \eta^k, u^k)_{k=1:n}$ , with  $(X^k, u^k)$  solution of (1) and  $\eta^k$  continuous measurement noises.

## 2 The $\mathfrak{D}_\varepsilon$ -transformed problem

Given  $\mathbf{X}, \mathbf{Y}$  two Banach spaces, let  $(x, y) \in (C^0([t_0, T]; \mathbf{X}))^n \times (C^0([t_0, T]; \mathbf{Y}))^n$  and  $\varepsilon \in \mathbb{R}^+$ ; we denote:

$$\Omega_{x,\varepsilon} := \bigcup_{i,j} \{(i, j)\} \times \Omega_{x,\varepsilon}^{i,j}, \quad \text{where } \Omega_{x,\varepsilon}^{i,j} := \{(t, \tau) \in [t_0, T]^2; \|x^i(t) - x^j(\tau)\| \leq \varepsilon\}.$$

The  $\varepsilon$ -cancellation operator  $\mathfrak{D}_\varepsilon$  is defined by:

$$(x, y) \mapsto \mathfrak{D}_\varepsilon(x, y) : \quad \begin{array}{ccc} \Omega_{x,\varepsilon} & \rightarrow & \mathbf{Y} \\ (i, j, t, \tau) & \mapsto & y^i(t) - y^j(\tau). \end{array} \quad (4)$$

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**Proposition 2.1.** (1)  $\mathfrak{D}_\varepsilon$  is continuous. (2) For any continuous  $f : \mathbf{X}_0 \subset \mathbf{X} \rightarrow \mathbf{Y}$ , we have the *cancellation property*:  $\mathfrak{D}_0(x, f \circ x) = 0$ .

Thanks to proposition 2.1 and by application of operator  $\mathfrak{D}_0(X, \cdot)$  to both members of the equation, model (3) is rewritten ( $n = 1$ ):

$$\begin{cases} \langle \mu, \mathfrak{D}_0(X, \gamma \psi_X + X) \rangle = \mathfrak{D}_0(X, G(X)u) \\ \langle \mu, \gamma \psi_X(t_0, \cdot) + X(t_0) \rangle - F(X(t_0)) = G(X(t_0))u(t_0). \end{cases} \quad (5)$$

The interest of this new formulation is that up to the quantity  $F_0 := F(X(t_0))$ , the  $\gamma$ -symbol  $\mu$  can be identified *independently* of  $F$  by (for simplicity,  $G$  is here supposed to be known):

$$(\mu^*, F_0^*) = \mathcal{Y}_\varepsilon^\dagger \left( \mathfrak{D}_\varepsilon(\tilde{X}, G(\tilde{X})\tilde{u}), G(\tilde{X}(t_0))\tilde{u}(t_0) \right),$$

where  $\mathcal{Y}_\varepsilon^\dagger$  is the pseudo-inverse, in a suitable sense, of the operator  $\mathcal{Y}_\varepsilon$  defined by:

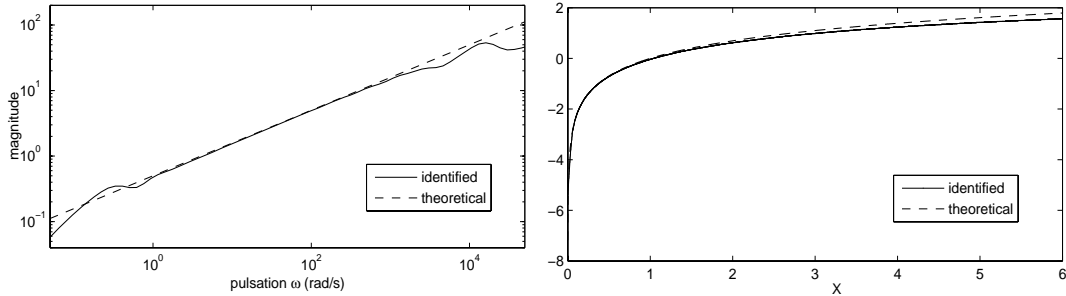
$$(\mu, F_0) \mapsto \left( \left\langle \mu, \mathfrak{D}_\varepsilon(\tilde{X}, \gamma \psi_{\tilde{X}} + \tilde{X}) \right\rangle, -F_0 + \left\langle \mu, \gamma \psi_{\tilde{X}}(t_0, \cdot) + \tilde{X}(t_0) \right\rangle \right). \quad (6)$$

By means of standard regression methods, the function  $F$  can then be easily estimated from the "pseudo graph" of  $F$ :

$$\mathcal{G}_F = \bigcup_{k,j} \left\{ \left( \tilde{X}^k(t_j), \left\langle \mu^*, \gamma \psi_{\tilde{X}^k}(t_j, \cdot) + \tilde{X}^k(t_j) \right\rangle - G(\tilde{X}^k(t_j))\tilde{u}^k(t_j) \right) \right\}. \quad (7)$$

### 3 A concrete example

This method has been tested on data elaborated from numerical simulation of the following model of spherical flames [2]:  $X \partial_t^{1/2} X = X \ln(X) + u$ ,  $u \geq 0$ ,  $X \geq 0$ . The identified symbol  $H^*(i\omega) = \int \frac{\mu^*(\xi)}{i\omega - \gamma(\xi)} d\xi$  [3] and function  $F^*$  are given here-after. More details and extensions will be found in [1] and in a further publication.



(a) Frequency response of  $H^*(i\omega)$ .

(b) Identified  $F^*$ .

### References

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