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An on-line classification approach of visitors' movements in 3D virtual museums

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Abstract. Recommender systems, in virtual museums and art galleries, providing the personalization and context awareness features require the off-line synthesis of visitors' behaviors therein and the off-line training stage of those synthetic data. This paper deals with the simulation of four visitors' styles, i.e., ant, fish, grasshopper, and butterfly, and the classification of those four styles using an Adaptive Neuro-Fuzzy Inference System (ANFIS). First, we analyze visitors' behaviors related to a visit time and an observation distance. Then, the proposed synthesis procedure is developed and used in the off-line training stage of ANFIS. The training and testing data are the average and variance of a set of visitors' attention data computing by the proposed function of the visit time and observation distance variables. Therefore, the trained ANFIS can identify the behavior style of an on-line visitor using the training set of synthetic data and its memberships can describe degrees of uncertainty in behavior styles.

Key words: 3D virtual environment; Recommender system; Visitor movement; Visitor behavior; Visualization; Adaptive Neuro-Fuzzy Inference Systems (ANFIS)

1 Introduction

Currently, the number of existing virtual museums in Second Life (SL) is greater than 150 locations, characteristics of which vary tremendously in terms of scale and size, artifact types and exhibitions, media technologies, and target visitors, as reported by Urban [1]. An immersively interactive feature embedded in SL can attract the attention of up to 30 official organizations existing in Real Life (RL). One of them is the Smithsonian Latino Center opened the Smithsonian Latino Virtual Museum (LVM) in March 19, 2009 which provides the vast and rich collections, research and scholarship, exhibitions and educational activities related to U.S. Latinos and Latin America. This trend leads to the need of a conceptual framework of digital museums in the 3D virtual world, which was designed, developed and evaluated in [2, 3].

Recommender modules, such as, guide avatar and personal route generation modules, were designed in the conceptual framework proposed by Sookhanaphibarn

and Thawonmas [3] to achieve high visitors' satisfaction. The guide avatar addressed in [4] can create a new text for an individual visitor, at each time when he/she views an object. The guide avatar never repeats information which it has already expressed. For example, if a visitor moves from one artifact to another artifact, both of which belong in the classical period; the guide avatar will not repeat the description on classical period at the latter location. A personal route for a museum tour assists visitors in making a decision where they should stop over at and how long they should take. Since visitors in SL can teleport to any place with a constantly short time, the personal route generation module for a museum tour is a novel concept in the personalization issue.

Visitors in an art gallery have their stereotypical movement, which is categorized in four styles as proposed by Veron and Levasseur [5]. Four visiting styles based on an animals' behavior metaphor are ant, fish, grasshopper, and butterfly styles. The ant visitors spend quite a long time to observe all exhibits by walking closer to exhibits but avoids empty spaces. The fish visitors prefer to move to and stop over at empty spaces but avoid areas near exhibits. The grasshopper visitors spend a long time to see selected exhibits but ignore the rest of exhibits. The butterfly visitors observe all exhibits but spend varied times to observe each exhibit. Identifying their visiting styles can take advantage of recommender systems in virtual museums as mentioned in [6–9].

In this paper, we propose an on-line classification system of four visitor styles based on their definitions, which are summarized in [5, 10]. This is based on our hypothesis that a visiting style can be simulated by using mathematic functions. To validate our hypothesis, we conduct an adaptive neuro-fuzzy inference system (ANFIS) trained by the synthetic visitor data in the off-line stage and tested by the new visitor data in the on-line stage. ANFIS is a hybrid learning algorithm to identify the membership function parameters of four visiting styles. The contributions of this work are (a) the novel simulation method of visiting styles in art galleries and museums in SL, (b) the proposed function to measure the visitors' attention of each exhibit, (c) the classification using ANFIS based on the average and variance of a set of visitors' attention data of all exhibit, and (d) our implications to applications of these findings.

2 Four visiting styles

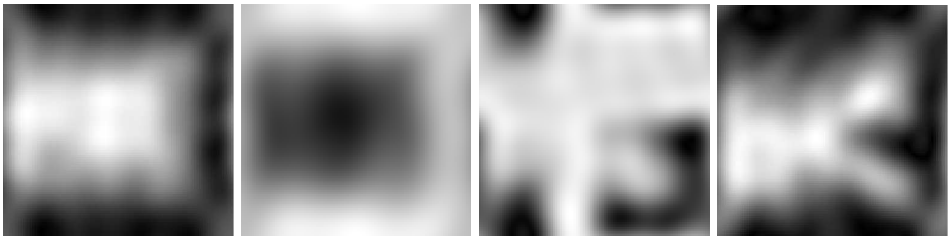
2.1 Definition

Chittaro and Ieronutti [10] described four visiting styles based on results from their visualization tool, where black highlights areas more traveled, white identifies the areas less traveled and different shades of gray are used to identify intermediate previous situations, as shown in Table 1. Corresponding to four visiting styles, the aforementioned researches [5, 7, 10] described them beneficial to a physical environment design in museums as shown in Table 1. Note that Fig. 1 shows the visualization of a visitor in the art gallery, where all exhibits are placed on the wall. Without loss of generality, an art gallery shown in Fig. 2

is used as the museum map in our synthetic data with 12 exhibits hanging in three sides of the room.

Table 1. Visualization and recommended environment design of four visiting styles

Styles	**Visualization**	**Recommendation**
Ant	The center of most space on the map is colored in black, and there are no large differences in the time different exhibits have been seen as shown in Fig. 1 (a).	They need to be guided by a powerful rationale and this is the reason why they usually follow the path proposed by the museum curator.
Fish	The areas near all exhibits on the map are colored in black as shown in Fig. 1 (b) .	They prefer a holistic observation area, such as the center of the room.
Grasshopper	The areas near some exhibits are colored with highly variable shades of gray due to the fact that this visitor spends a variable time to observe different exhibits and ignores the rest of them as shown in Fig. 1 (c).	They move directly to their selective exhibits.
Butterfly	The areas near all exhibits are colored with different shades of gray and some are colored with black, but less regularly than those of the ant visitors as shown in Fig. 1 (d).	They refuse to follow the given path designed by other people and prefer their own route.



(a) Ant style (b) Fish style (c) Grasshopper style (d) Butterfly style

Fig. 1. Visualization of visitor styles.

2.2 Algorithms of Data Synthesis

To synthesize the visit map, there are two input data as follows: a plan of museum, which illustrates the location of all exhibits, and his/her visitor type. We propose a synthesis approach for four visiting styles through the following steps:

1. Selective exhibit randomness
2. Preference weight randomness
3. Visit time distribution
4. Noise synthesis
5. Smoothing filter

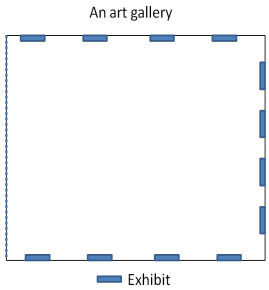


Fig. 2. The plan of an art gallery which all exhibits are hung on the wall.

There are four proposed synthesis procedures, i.e., ant, fish, grasshopper, and butterfly styles as described in Fig. 3 and their notations as defined in Table 2. All four synthesis procedures have one input argument, H or a set of exhibits, and one output, V or a visit map. Our synthetic visit maps have darkest colored pixels equal to 1 and brightest colored pixels equal to 0. The difference between the grasshopper and the butterfly styles is the number of stopovers at exhibits, i.e., the former stops fewer than the latter does. The selective exhibit randomness step is used in these two visiting styles using a β probability density function since two parameter γ and β can control the PDF curve corresponding to the grasshopper and butterfly paradigm of the number of stopovers. The β PDF of a sample x is written below:

$$f(x;\gamma,\beta) = \frac{1}{B(\gamma,\beta)} x^{\gamma-1} 1 - x^{\beta-1} x, \tag{1}$$

where $B(\cdot)$ is the Beta function as defined in [11]. In cases of both ant and fish styles, the selective exhibit randomness are not applied because the ant visitor stops at all exhibits and moves following the exhibit path proposed by a museum curator and the fish visitor stops at none of exhibits and moves only to the center of the room.

The preference weight randomness step generates varying stopover times at exhibits. Both ant and fish styles show no significant difference among their preferences of exhibits. Second, the visit time distribution functions are different among visiting styles. Third, noise synthesis is conducted using a Gaussian PDF. The implication of noise is randomly short stops in the exhibition area. Then, the smooth filtering such as a low pass filter is applied in order to discard the sharp detail and remove the noise. An example of the ant, fish, grasshopper, and butterfly visit maps derived by using the procedure is shown in Fig. 1 (a)-(d), respectively.

The aforementioned synthesis procedures require to construct a Voronoi partition of the set of pixels in a museum area. This is a division of the museum

area into regions around each exhibit; a region is the set of pixels that are closer to that exhibit than to any other. Let $\mathbf{h}_k$ be the position of exhibit k and let $H = \{\mathbf{h}_k | 1 \leq k \leq M\}$ be a set of all positions of exhibits, where M is a total exhibit. Let $\mathbf{u}$ be the position of a visitor. The exhibit $\mathbf{h}_{k^*}$ nearest to a visitor $\mathbf{u}$ is calculated by the following

$$k^* = \text{argmin}_k \|\mathbf{h}_k - \mathbf{u}\| \quad where \quad 1 \leq k \leq M \tag{2}$$

where $\|\mathbf{h}_k - \mathbf{u}\|$ is the Euclidean distance between exhibit $\mathbf{h}_k$ and visitor $\mathbf{u}$.

Table 2. Notations used in the synthesis procedures of four visiting styles

$U = \{(i_1, i_2) \| 1 \leq i_1 \leq m,\ 1 \leq i_2 \leq n\}$:	The set of pixels in a museum area
$V = \{v(\mathbf{i}) \| \mathbf{i} \in U\}$:	Visit map
$H = \{h(\mathbf{k}) \| 1 \leq k \leq M\}$:	A set of all positions of exhibits
$v(\mathbf{i})$:	The visitor's stopover time at pixel $\mathbf{i}$, called *Visit time*
$\mathbf{h}_k$:	The position of exhibit k
M:	A total exhibit
$S(\mathbf{h}_k)$:	A set of pixels $\mathbf{i}$ belonging to $\mathbf{h}_k$
w_k:	The preference weight of exhibit $\mathbf{h}_k$

3 Classification system of visitors‘ behaviors in 3D virtual museums

3.1 Proposed function of visitor's attention at each exhibit

Considering the visualization scheme of visiting styles in [10, 12], we can discard the temporal information in trajectories data of visitor movement in a museum. Given a visit map, V, it is obvious that a dark colored area is a key role to identify the visiting styles because it illustrates a long stopover. However, the dark colored area near or far from an exhibit is considered as a parameter to measure the visitors' attention to the exhibit. Given a museum area, U, the observation distance and map are defined as follows.

1. Observation distance $o(\mathbf{i})$ is defined as the visitor's fuzzy distance from the nearest exhibit
2. Observation map is defined as a set of $o(\mathbf{i})$, where $\mathbf{i} \in U$

To compute the observation distance, the observation distance function $\Psi(\mathbf{h}_k, \mathbf{u})$ is defined as an exponential function of the distance from an exhibit $\mathbf{h}_k$ to a visitor $\mathbf{u}$ as follows:

$$\Psi(\mathbf{h}_k, \mathbf{u}) = e^{-\rho \|\mathbf{h}_k - \mathbf{u}\|^2} \tag{3}$$

ρ is a weighted constant. In the same manner of an Gaussian-based activation function, this weighted constant is obtained from the variance of the Euclidean

Procedure: Synthesis of *Ant* visiting style
Input: H
Output: V
Step 1: Preference weight randomness is set by using the Gaussian PDF with the mean closer to 1 and smallest variance.
Step 2: Visit time $v(\mathbf{i})$ at pixel $\mathbf{i}$ is defined by using the following equation:
$v(\mathbf{i}) = w_k e^{-\rho\|\mathbf{h}_k - \mathbf{i}\|^2}$ where $\mathbf{i} \in S(\mathbf{k})$
Step 3: Smoothing filter is applied to the visit map obtained from Step 2.

Procedure: Synthesis of *Fish* visiting style
Input: H
Output: V
Step 1: Preference weight randomness is set by using the Gaussian PDF with the mean closer to 1 and smallest variance.
Step 2: Visit time $v(\mathbf{i})$ at pixel $\mathbf{i}$ is defined by using the following equation:
$v(\mathbf{i}) = w_k(1 - e^{-\rho\|\mathbf{h}_k - \mathbf{i}\|^2})$ where $\mathbf{i} \in S(\mathbf{k})$
Step 3: Smoothing filter is applied to the visit map obtained from Step 2.

Procedure: Synthesis of *Grasshopper* visiting style
Input: H
Output: V
Step 1: Selective exhibit randomness is decided by using Beta PDF with $\gamma = 1$ and $\beta = 5$.
$P(\eta) = \frac{\eta^{\alpha-1}(1-\eta)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)}\Gamma(\alpha+\beta)$
where Γ is the gamma function and the parameters α and β are related through
$\alpha = \mu\gamma$ and $\beta = (1-\mu)\gamma$ where $\gamma = \frac{\mu(1-\mu)}{\sigma} - 1$.
The output of the PDF is the number of selected exhibits, as denoted by L.
Then, then selective exhibit is defined as a binary vector where f_i is an element of the binary vector and $\sum_i f_i$ must be L.
Step 2: Preference weight randomness is set by using the Gaussian PDF with the mean closer to 1 and small variance.
Step 3: Visit time $v(\mathbf{i})$ at pixel $\mathbf{i}$ is defined by using the following equation:
$$v(\mathbf{i}) = \begin{cases} w_k e^{-\rho\|\mathbf{h}_k - \mathbf{i}\|^2} & \text{if } \mathbf{i} \in S(\mathbf{k}) \text{ and } f_k = 1 \\ 0 & \text{otherwise} \end{cases}$$
Step 4: Noise synthesis is performed by using Gaussian PDF.
Step 5: Smoothing filter is applied to the visit map obtained from Step 4.

Procedure: Synthesis of *Butterfly* visiting style
Input: H
Output: V
Step 1: Selective exhibit randomness is decided by using Beta PDF with $\gamma = 5$ and $\beta = 1$.
$P(\eta) = \frac{\eta^{\alpha-1}(1-\eta)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)}\Gamma(\alpha+\beta)$
where Γ is the gamma function and the parameters α and β are related through
$\alpha = \mu\gamma$ and $\beta = (1-\mu)\gamma$ where $\gamma = \frac{\mu(1-\mu)}{\sigma} - 1$.
The output of the PDF is the number of selected exhibits, as denoted by L.
Then, then selective exhibit is defined as a binary vector where f_i is an element of the binary vector and $\sum_i f_i$ must be L.
Step 2: Preference weight randomness is set by using the Gaussian PDF with the mean closer to 1 and variance higher than the ant's.
Step 3: Visit time $v(\mathbf{i})$ at pixel $\mathbf{i}$ is defined by using the following equation:
$$v(\mathbf{i}) = \begin{cases} w_k e^{-\rho\|\mathbf{h}_k - \mathbf{i}\|^2} & \text{if } \mathbf{i} \in S(\mathbf{k}) \text{ and } f_k = 1 \\ 0 & \text{otherwise} \end{cases}$$
Step 4: Noise synthesis is performed by using Gaussian PDF.
Step 5: Smoothing filter is applied to the visit map obtained from Step. 4.

Fig. 3. Synthesis procedures of four visiting styles

distance between exhibit $\mathbf{h}_k$ and pixel $\mathbf{i}$, for all $\mathbf{h}_k$ and $\mathbf{i}$, as shown in Eq.(4).

$$\mu = mean\left\{\bigcup_{\mathbf{h}_k \in H \,\wedge\, \mathbf{i} \in U} \|\mathbf{h}_k - \mathbf{i}\|\right\} \tag{4a}$$

$$\sigma^2 = mean\left\{\bigcup_{\mathbf{h}_k \in H \,\wedge\, \mathbf{i} \in U} (\|\mathbf{h}_k - \mathbf{i}\| - \mu)^2\right\} \tag{4b}$$

$$\rho = 2\sigma^2 \tag{4c}$$

Fig. 4 shows a graph plotting of $\Psi(\mathbf{h}_i, \mathbf{u})$ and $\|\mathbf{h}_k - \mathbf{v}\|$ where σ^2 is 0.07. The observation distance is calculated as follows:

$$o(\mathbf{i}) = \Psi(\mathbf{h}_k, \mathbf{i}) \quad where \quad \mathbf{i} \in S(\mathbf{h}_k) \tag{5}$$

The visitor's attention at exhibit $\mathbf{h}_k$ is computed by using the visit time and the observation distance belonging to $S(\mathbf{h}_k)$, as shown below:

$$A(\mathbf{h}_k) = mean\left\{\bigcup_{\mathbf{i} \in S(\mathbf{h}_k)} \{o(\mathbf{i})\, v(\mathbf{i})\}\right\} \tag{6}$$

where $S(\mathbf{h}_k)$ is a set of pixels $\mathbf{i}$ belonging to $\mathbf{h}_k$, as defined by Eq.(2), and $o(\mathbf{i}) \geq \delta$. δ is a threshold in the range from 0 to 1, which its implication is a observation distance limit.

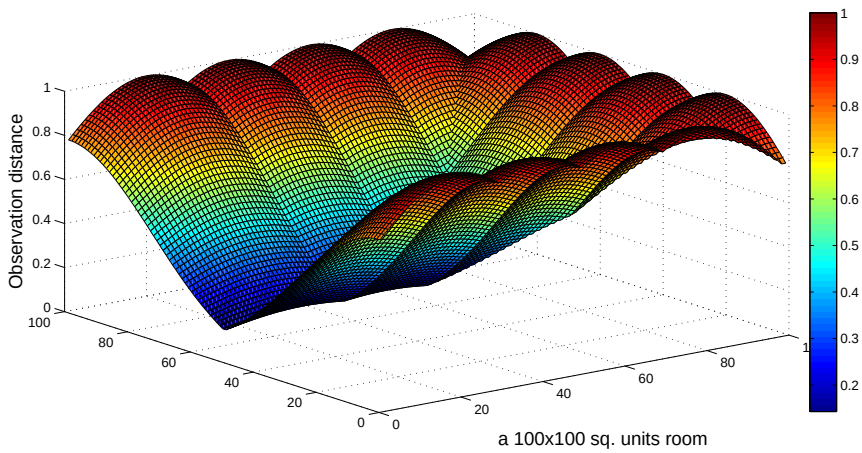


Fig. 4. Observation distance function of Eq.(3) where $H = \{(1,20),(1,40),(1,60),(1,80),(20,100),(40,100),(60,100),(80,100),(100,20),(100,40),(100,60),(100,80)\}$.

3.2 Framework of classification system

An overview of the proposed classification system is divided into on-line and off-line stages corresponding to learning and testing models, respectively. These

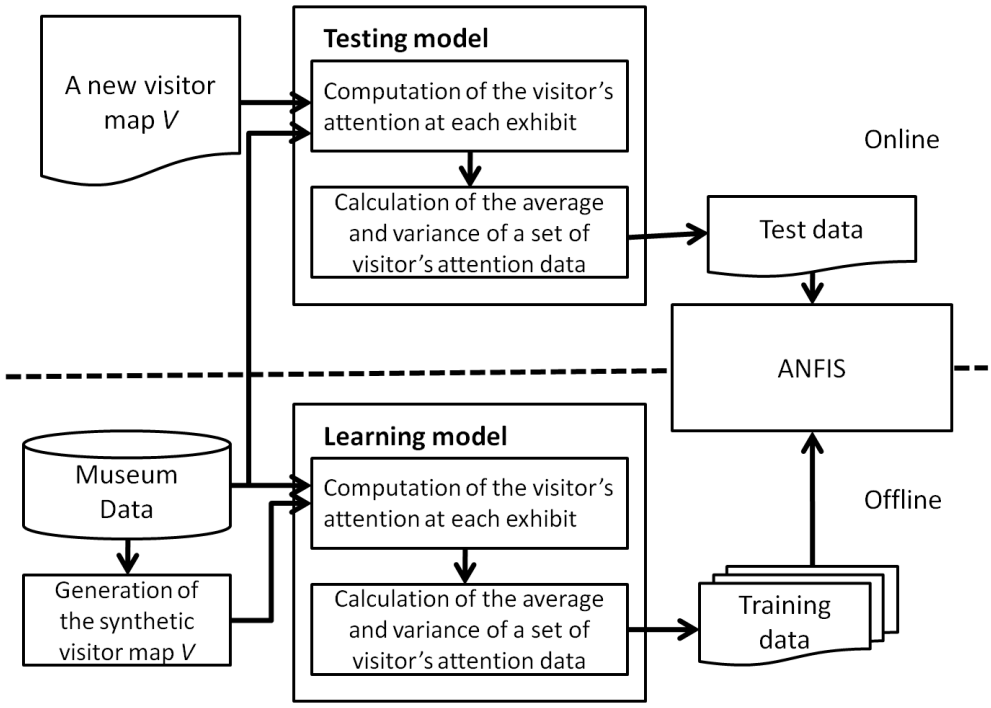


Fig. 5. Framework of the proposed classification system in off-line and on-line stages

models deal with the museum data, which is a set of the exhibits' positions in a museum area, and the visit data, which is two dimensional time-series data. This framework performs the learning model by generating the virtual samples of visitors belonging to all four visiting styles as mentioned in Section 2.2. The virtual samples are represented by the visualization of a set of visitors' stopover-time data in the museum area, denoted by V as described in Table 2 and Fig. 3.

The training and test data of ANFIS are the average and variance of a set of visitor's attention data, which the visitor's attentions $A(\mathbf{h}_k)$ for all k are calculated by using Eq.(6). For example, there are 400 visitors each of which has the average and variance of their attentions as shown in Fig. 6. The clusters of four visitor types indicate that the clusters of ant and fish visitor types are isolated, which their probabilities found in [6] are 30% and 20%, respectively. The selective exhibit randomness is a cause of an overlapping area between grasshopper and butterfly clusters with the result that the Beta PDFs of both visitor types are a common area. However, the probability of grasshopper visitors found in the physical world is only 10% as reported in [6]. Therefore, a fuzzy inference system is used to classify the overlapped clusters and generate a linguistic rule-based classfier.

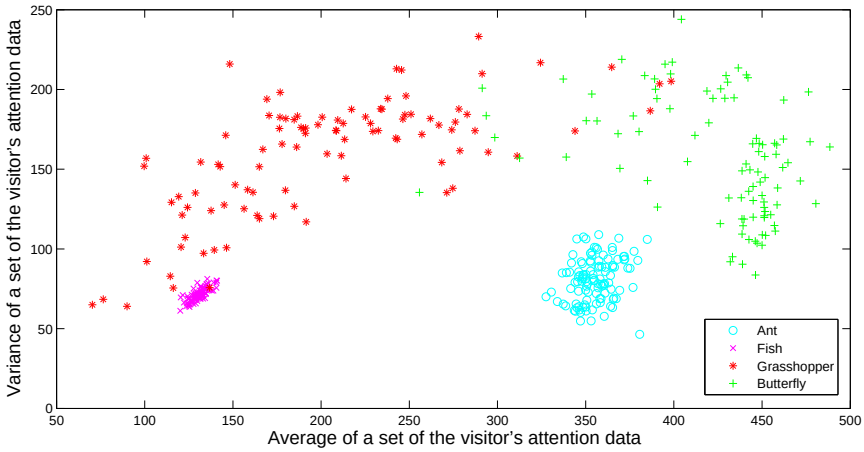


Fig. 6. 400 visitors of four clusters are plotted on the average and variance of their attention at all exhibits.

A fuzzy inference system is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made, or patterns discerned. ANFIS includes the neuro-adaptive learning method that works similarly to that of neural networks. The neuro-adaptive learning techniques provide a method for the fuzzy modeling procedure to learn information about a data set. The training data set is a set

of input-output data, which the input data is the average and variance of a set of attention data obtained from the synthetic visit maps and the output is their visiting styles in the off-line stage, whereas the test data is from a new visitor in the on-line stage.

4 Experimental results

In our experiment, we generated the total of 100 synthetic data by using the aforementioned procedures. Using the uniform distribution to generates the data portion, the percentages of the four styles are 21, 22, 25, and 32, respectively. In the off-line stage, we used the spline-based curve, i.e., a built-in membership function in MATLAB. Its parameters a and d locate the feet of the curve, while b and c locate its shoulders as shown in Fig. 7. The number of membership functions are four per variable; therefore, four by four rules are produced from two variables, i.e., the average and variance of a visitor's attention. The ANFIS outputs are four classes corresponding to ant, fish, grasshopper, and butterfly styles. In the off-line stage, we trained the ANFIS against the synthetic data and obtained the training error of 4%. Fig. 8 shows 16 produced rules built by spline curves. The degree of color intensity in spline curves indicate the fuzzy degree of the average or variance in according to rules. Fig. 9 and 9 show the ANFIS outputs comparing with their targets versus their averages and their variances, respectively.

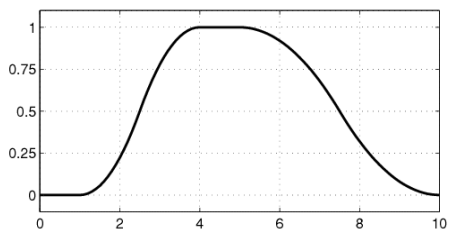
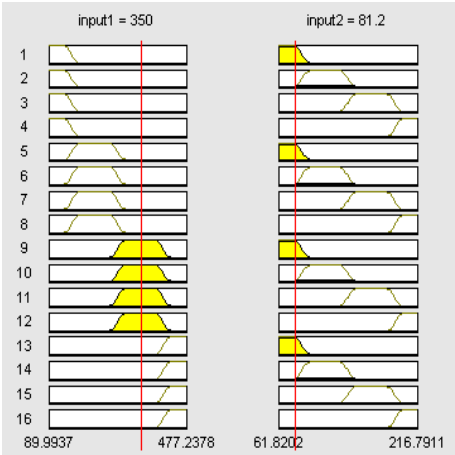
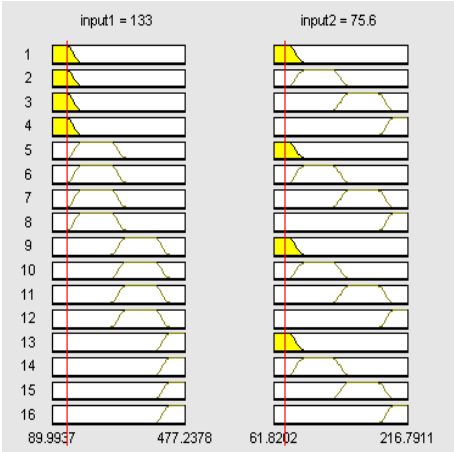


Fig. 7. Spline-curve-based membership function with the parameters $[a, b, c, d] = [1, 4, 5, 10]$.

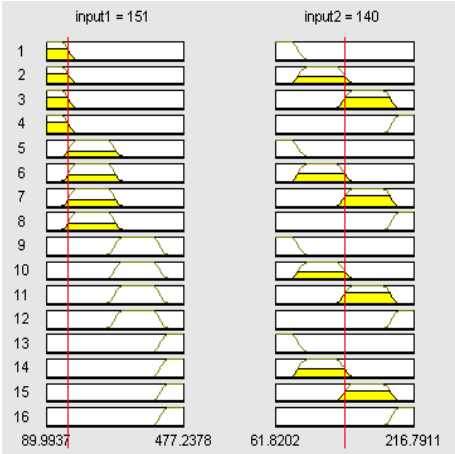
The on-line stage is tested against 36 avatars in RDAP (Ritsumeikan Digital Archiving Pavilion). RDAP is a virtual gallery displaying the kimono textiles of 19 objects. All avatars intentionally explored the gallery following their assigned styles. In this experiment, the number of ant, fish, grasshopper, and butterfly visitors are equally set to 9. Then, the classification accuracy of on-line stage is 83%.



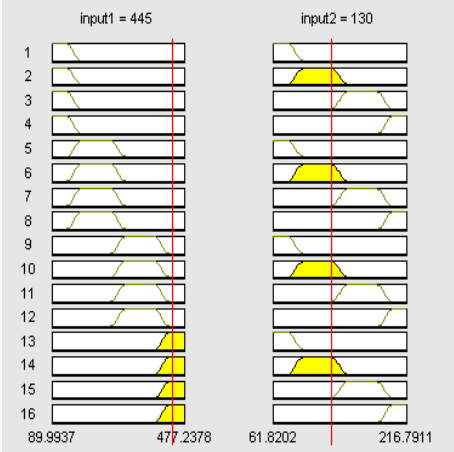
(a) Ant [349.90 81.22]



(b) Fish [132.80 75.63]



(c) Grasshopper [151.27 140.16]



(d) Butterfly[444.98 130.34]

Fig. 8. Four examples of 16 spline-curve-based rules combining of two ANFIS inputs, [average variance]: (a) Ant, (b) Fish, (c) Grasshopper, and (d) Butterfly.

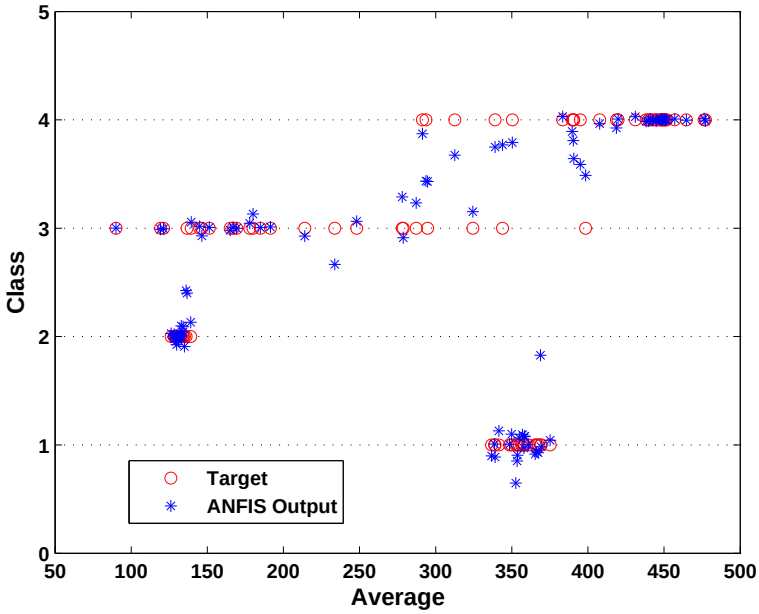


Fig. 9. Training results versus their average of 100 samples.

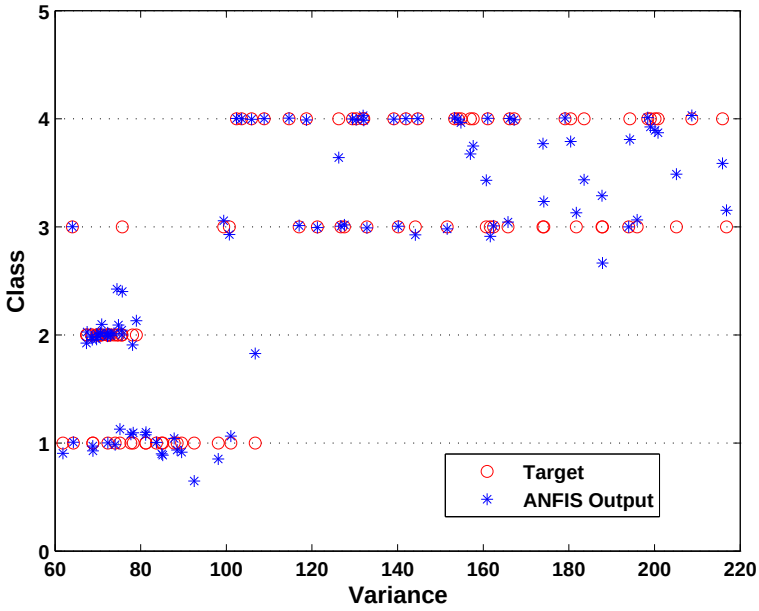


Fig. 10. Training results versus their variance of 100 samples.

5 Conclusions and discussions

In this paper, we propose the on-line classification system that its off-line stage can be trained by using the synthetic data. We, also, introduce the algorithms of generating the synthetic data of four visiting styles as defined in [5, 10]. We, then, conduct an adaptive neuro-fuzzy inference system (ANFIS) trained by the synthetic visitor data in the off-line stage and tested by the new visitor data in the on-line stage.

Since the overlapping between two clusters of grasshopper and butterfly, we exploit fuzzy paradigm for dealing with continuous and imprecise variables. Two reasons of using ANFIS are the followings. First, fuzzy inference system (FIS) employing fuzzy if-then rules that can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analyses. Second, ANFIS uses a hybrid learning algorithm to identify parameters of Sugeno-type fuzzy inference systems.

In our future work, we will implement the on-line classification in Second Life in order to investigate user behaviors in 3D virtual museums comparing with those in real museums. A recommendation of personal route map is one of applications when a 3D virtual museum can classify the visitor style of a user. The system can recommend new artifacts, exhibitions, and events and navigate a route direction by using our proposed function of measuring the visitors' attention of each exhibit.

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