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# On the Optimization of Riemann-Stieltjes-Control-Systems with Application in Vehicle Dynamics

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# On the Optimization of Riemann-Stieltjes-Control-Systems with Application in Vehicle Dynamics

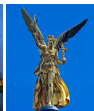
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Fotos: <http://de.wikipedia.org/wiki/Mnchen>

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# Content

Problem Description

Impulsive Multi-Body-Systems

Riemann-Stieltjes-Control-Systems

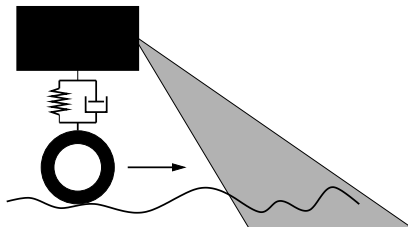
Optimization of Riemann-Stieltjes-Control-Systems

Numerical Examples

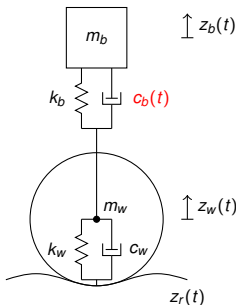
## Proactive–Chassis–Control

Setting:

- ▶ **Input:** Vertical Road Model delivered e.g. by Preview Sensors  $z_r(t) \ t \in [0, T]$
- ▶ **Cost Function:** Comfort and safety characteristics (acceleration body, road-to-wheel-force,...)
- ▶ **Controls:** Damping coefficients (electrorheological dampers, ...)



## Classical Model



### Classical quartercar model

- ▶ two masses ( $m_w, m_b$ )
- ▶ spring-damper connection between body ( $z_b$ ) and wheel ( $z_w$ )
- ▶ spring-damper connection between wheel and road ( $z_r$ )
- ▶  $k_b, k_w, c_w$  are constants
- ▶  $c_b \in [c_{min}, c_{max}]$  is the control

Model delivers twice continuously differentiable state

⇒ Mechanical optimal control problem (easy to solve)

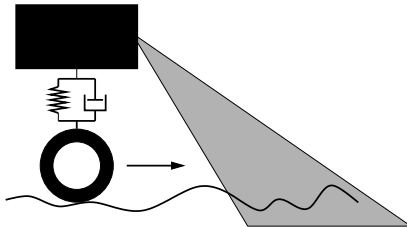
# Optimal Control Problem (OCP)

$$\min_{u \in \mathbb{U}[0, T]} J(u) = \int_0^T \mu_b F_{bw}^2 + \mu_c F_{wr}^2 dt$$

$$\text{s.t.} \quad \begin{pmatrix} m_b \ddot{z}_b \\ m_w \ddot{z}_w \end{pmatrix} = \begin{pmatrix} F_{bw} \\ F_{wr} - F_{bw} \end{pmatrix}$$

$$z_b(0) = z_{b0}, z_w(0) = z_{w0}, v_b(0) = v_{b0}, v_w(0) = v_{w0}$$

$$z_r, v_r \in C^1[0, T]$$

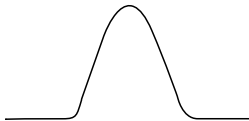


## Singular events

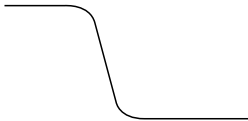
What if the road undergoes rapid changes?

Examples:

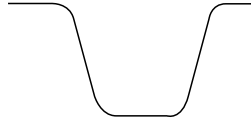
Speedbump



Step



Pothole

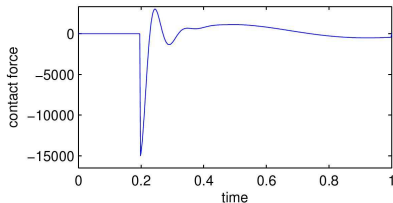
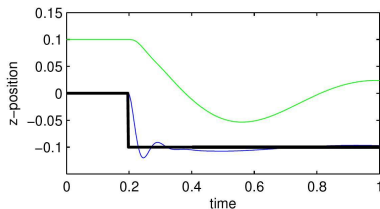


### Definition (Singular Event)

A singular event is an individual, short and rapid change in the road surfaces, with a deflection amount that is considerably higher than the common road disturbances

## Singular events

Behaviour of the classical model at singular events



Negative contact force at the transition

Problem: **Model does not incorporate the possible occurring contact loss**

Way out: Remove spring-damper connection between wheel and road.



## Contact formulation

Possibilities:

- ▶ Full FEM-Formulation (pde-ode-coupling)
- ▶ Commercial Tire Libraries (Fraunhofer CDTire)
- ▶ Reduced Formulation using Hertzian stress (tyre deflection approximated by Hertzian stress)

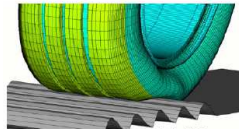
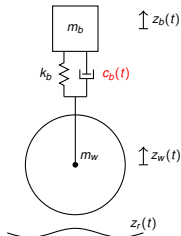


Bild: ITWM, 2009

Or...

- ▶ **Masspoint impact model** using coefficients of restitution (COR)



## Impact model with COR

### Requirements

- ▶ Wheel above road

$$(z_w - z_r) \geq 0$$

- ▶ Contact force positive

$$F_i \geq 0$$

- ▶ Complementarity condition

$$(z_w - z_r)F_i = 0$$

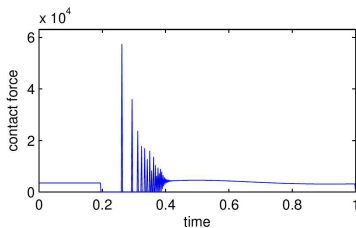
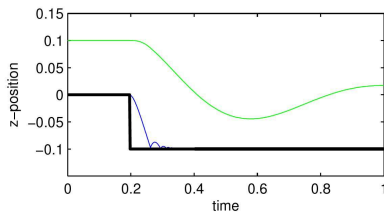
- ▶ Newtonian impact equation with coefficient of restitution  $\epsilon$

$$F_i(v_w^+ + \epsilon v_w^-) = 0$$

Behaviour of a system with an impact at time  $\tau$ , ( $F_i(\tau) > 0$ ):

$$v(\tau^+) = -\epsilon v(\tau^-) \quad \text{for} \quad z_w(\tau^-) = z_r(\tau^-), v_w(\tau^-) \leq 0$$

## Impact model with COR



### Challenges:

- ▶ Contact detection
- ▶ Incorporate friction
- ▶ Control optimization

Objective: [Closed Formulation of impact model with COR](#) to obtain sensitivity information

# Impulsive Multi-Body-Systems

Regarding the system

$$M(q)\ddot{q} + b(q, \dot{q}) + g(q) = F_i(q)$$

with

- ▶  $q \in \mathbb{R}^n$  – generalized coordinates
- ▶  $\dot{q} \in \mathbb{R}^n$  – generalized velocities
- ▶  $b(q, \dot{q}) \in \mathbb{R}^n$  – coupling forces
- ▶  $g(q) \in \mathbb{R}^n$  – weight forces
- ▶  $F_i(q)$  – impuls and contact forces

Transformation to first order system with  $x = [q, \dot{q}]^T$ :

$$\dot{x} = F(x) + F_i(x).$$

- ▶  $F(x)$  – continuous Forces of the system
- ▶  $F_i(x)$  – large impact Force for the short contact time

## Impulsive update

Systemformulation using

- ▶ Continuous dynamic

$$\dot{x}(t) = F(x(t)) \quad \text{contact free}$$

- ▶ Jump condition for velocities

$$\Delta x(\tau) = S(x(\tau)) \geq 0 \quad \text{if contact occurs}$$

⇒ hybrid system formulation

Transformation into an integral equation

$$x(t) = x_a + \int_a^t F(x(\tau)) d\tau + \int_a^t d\alpha(x)(\tau)$$

leads to a Riemann-Stieltjes-Integral

## Integral System Equation

### Definition

Let  $\alpha(x)(t) \in BV([a, b], \mathbb{R}^n)$ ,  $F : \mathbb{R}^{2n} \rightarrow \mathbb{R}^{2n}$  und  $G : \mathbb{R}^{2n} \rightarrow (\mathbb{R}^{2n} \times \mathbb{R}^n)$ . Let  $x : [a, b] \rightarrow \mathbb{R}^n$  be the function that solves the Integral equation

$$x(t) = x_a + \int_a^t F(x(\tau))d\tau + \int_a^t G(x(\tau))d\alpha(x)(\tau)$$
$$x_a = x(a)$$

for  $a \leq t \leq b$ . This  $x$  is called an impulsive system.

- ▶ Without contact it holds  $d\alpha(x)(\tau) = 0$
- ▶ At contact: Determine  $G(x(\tau))$  and  $d\alpha(x)(\tau)$ , the impact condition holds.

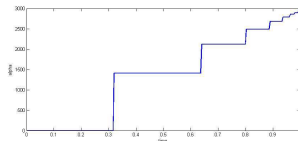
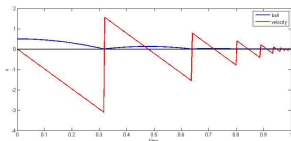
### Theorem

If  $G$  is continuous on  $[a, b]$  and  $\alpha \in BV([a, b], \mathbb{R}^n)$ , the Riemann-Stieltjes-Integral exists.

## Example: Jump condition – 1D

For the 1D problem with horizontal contact surface the jump condition writes as:

$$d\alpha(x)(\tau) = \begin{cases} 0 & x(\tau) > 0 \\ 0 & x(\tau) = 0 \ \& \ v(\tau) \geq 0 \\ (1 + \varepsilon)v(\tau^-) & \text{otherwise} \end{cases} \quad G := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



### Remark

Let  $\alpha$  be defined as above, then it holds  $\alpha \in BV([a, b], \mathbb{R}^n)$  [vgl. Natanson 1977]

## Generalization

To Do: Determination of  $\alpha$  for 3D contacts with Coulomb-Friction

General contacts means:

- ▶ Multi-body-system with  $n$  bodies and given transformation into state space  $\xi_i(q), i = 1, \dots, n$
- ▶ Parametrized contact surfaces  $c_j, j = 1, \dots, l$  with admissible region  $\nu_j$ .

Then at contact point it holds:

$$v_{ij}^{c+} = E_{ij} v_{ij}^{c-} \quad \text{with} \quad E = \text{diag}(e_{ij}^n, e_{ij}^t, e_{ij}^{t'})$$

**Given:** COR in normal direction  $e_{ij}^n$

**Unknown:** COR in tangential directions  $e_{ij}^t, e_{ij}^{t'}$



## Update with friction

Collecting active Contact points, one derives:

### Contact Optimization Problem (COP)

$$\begin{aligned}
 & \min_{e_{ij}^k, (i,j) \in I_A, k \in t, t'} && K^+(E) = \dot{q}(\tau^+)^T M \dot{q}(\tau^+) \\
 & \text{s.t.} && \dot{q}(\tau^+) = (\mathbb{I} - \bar{\mathcal{J}}\mathcal{J} + \bar{\mathcal{J}}E\mathcal{J})\dot{q}(\tau^-) \\
 & && \mathbf{p}(E) = (\mathcal{J}M^{-1}\mathcal{J}^T)^{-1}(E - \mathbb{I})\mathcal{J}\dot{q}(\tau^-) \\
 & && -\rho_{ni}(E) \leq 0 \\
 & && \|(\mathbf{p}_{ti}(E), \mathbf{p}_{t'i}(E))\|_2^2 \leq (\mu(i,j)\rho_{ni}(E))^2 \quad (\text{Coulomb-Friction})
 \end{aligned}$$

Quadratic optimization problem with nonlinear constraint

$$d\alpha(x)(\tau) = \dot{q}(\tau^+) - \dot{q}(\tau^-)$$

## Example - Extended Bouncing Ball

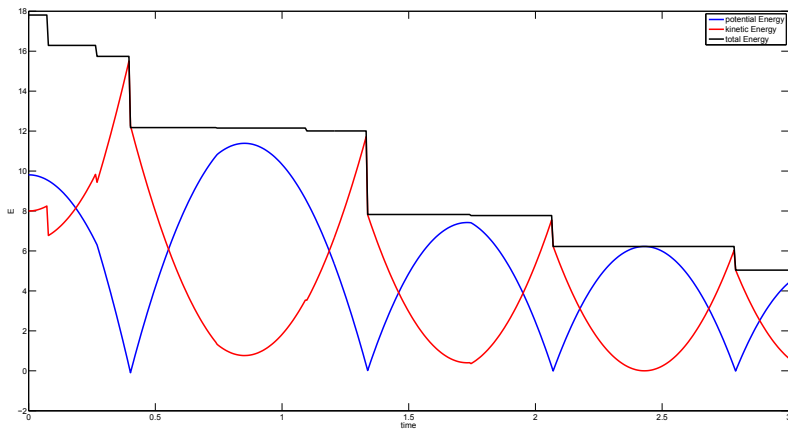


Figure: Energy of the Ball

## Extension: Riemann-Stieltjes-Control-Systems

### Definition

Let  $\alpha(x(u))(t) \in BV([a, b], \mathbb{R}^n)$ ,  $F : ([a, b] \times \mathbb{R}^{2n} \times \mathbb{U}^{n_u}) \rightarrow \mathbb{R}^{2n}$  and  $G : ([a, b] \times \mathbb{R}^{2n}) \rightarrow (\mathbb{R}^{2n} \times \mathbb{R}^n)$ . If the function  $x : [a, b] \rightarrow \mathbb{R}^n$  solves the integral equation

$$x(u)(t) = x_a + \int_a^t F(x(u)(\tau), u) d\tau + \int_a^t G(x(u)(\tau)) d\alpha(x(u))(\tau)$$
$$x_a = x(a)$$

for  $a \leq t \leq b$  it is called the solution of an impulsive system with control

Remark:

- ▶ No impulsive control

## Existence of sensitivities

**Goal:** Calculate Sensitivities w.r.t. control to obtain information for optimization.

### Existence of sensitivities

Let an impulsive System with control be given on the Interval  $[0, T]$ .  $F$  and  $G$  are supposed to be Lipschitz on  $[0, T]$ . Let  $\alpha(x(u))(\tau)$ ,  $\tau \in (0, T)$ , be an impact point given by the switching function  $\varsigma(x(u)(t))$ .  $\varsigma$  has a real zero crossing at  $t = \tau$ . It holds

$$\tau(u) = \operatorname{argmin}_{s \in (0, T)} \left\{ \varsigma(x(u)(s)) = 0 \mid \frac{d}{dt} \varsigma(x(u)(s)) \neq 0 \right\}$$

Then there exists an  $S(u)$  for that the limit

$$\lim_{\|\varepsilon\| \rightarrow 0} \frac{\|x(u + \varepsilon) - x(u) - S(u)\varepsilon\|_{\infty}}{\|\varepsilon\|_{\infty}} = 0$$

exists.

## Sketch of the proof

- ▶ Map the interval  $[0, \tau(u)]$ ,  $[\tau(u), T]$  onto  $[0, 1]$ ,  $[1, 2]$
- ▶ Due to the properties of  $\varsigma$  exists  $\frac{d\tau(u)}{du}$  in a neighborhood of  $u$
- ▶ With the decomposition property of functions of bounded variation it holds

$$\begin{aligned} & \|x(u + \varepsilon) - x(u) - S(u)\varepsilon\|_{\infty} \\ &= \|x_c(u + \varepsilon) + x_s(u + \varepsilon) - x_c(u) - x_s(u) - S_c(u)\varepsilon - S_s(u)\varepsilon\|_{\infty} \\ &\leq \|x_c(u + \varepsilon) - x_c(u) - S_c(u)\varepsilon\|_{\infty, [0,1]} + \|x_c(u + \varepsilon) - x_c(u) - S_c(u)\varepsilon\|_{\infty, [1,2]} \\ &\quad + \|x_s(u + \varepsilon) - x_s(u) - S_s(u)\varepsilon\|_{\infty} \end{aligned}$$

- ▶ Estimate the summands with mean value theorem and the lemma of Gronwall.

## Optimisation of RCS

- ▶ Additional state  $x_J$  for **continuous costs**
- ▶ Control-discretization with  $N$  equidistant controlgrid  $\mathcal{G}_u^N$
- ▶ Subdiscretization with  $M$  substeps  $\mathcal{G}_s^{N \cdot M}$
- ▶ Statevector  $x(u) \in \mathbb{R}^{2n+1} \times \mathbb{R}^{M+1}$ , Controlvector  $u = \{u_1, \dots, u_M\}$ ,
- ▶ Calculation of the sensitivity matrix using differential-quotient

$$S(u)_{(\cdot, j)} = \frac{x_J(u + \delta u_j) - x_J(u)}{\delta} \quad j = 1, \dots, M$$

and the gradient

$$\nabla J(x(u)) = S(u)^\top \dot{x}_J(u)$$

- ▶ Update Method using projected gradient-method with Armijo line-search.

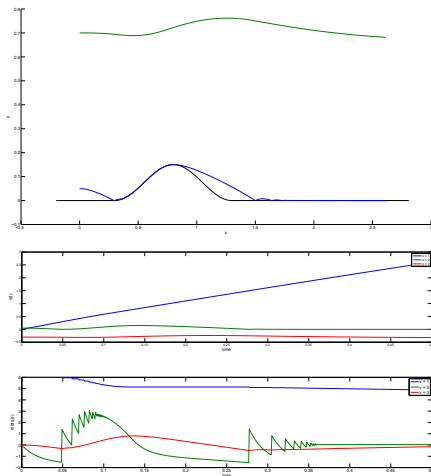
## Quartercar - An Example

Quartercar-Model with impact formulation.

$$x(t) = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ x_J \end{pmatrix} \hat{=} \begin{pmatrix} \text{x-position} \\ \text{y-position tire} \\ \text{y-position body} \\ \text{x-velocity} \\ \text{y-velocity tire} \\ \text{y-velocity body} \\ \text{continuous costs} \end{pmatrix}$$

$$x(t, x_0, u) = x_0 + \int_{t_0}^t \begin{pmatrix} x_4(\tau) \\ x_5(\tau) \\ x_6(\tau) \\ 0 \\ -gm_w + c_2(x_2 - x_1) + u(x_4 - x_3) \\ -gm_b - c_2(x_2 - x_1) - u(x_4 - x_3) \\ (-gm_b - c_2(x_2 - x_1) - u(x_4 - x_3))^2 \end{pmatrix} d\tau + \int_{t_0}^t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} d\alpha(x(\tau)),$$

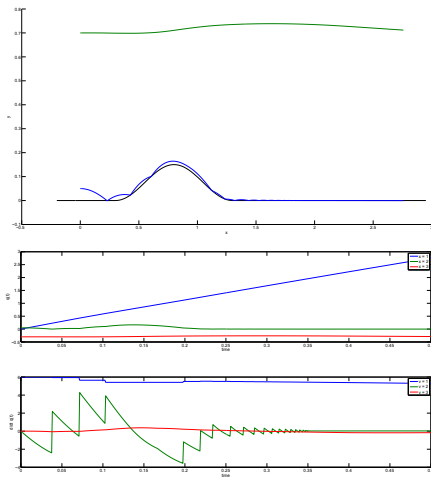
# Initial Trajectory



Top: Statespace, bottom: generalized coordinates

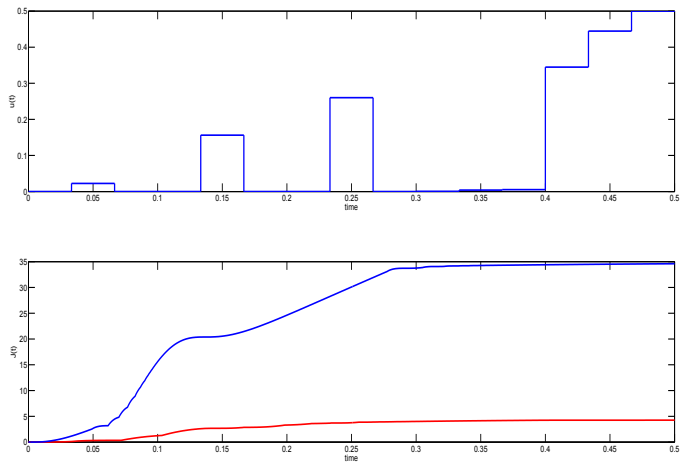


# Optimized Trajectory



Top: Statespace, bottom: generalized coordinates

# Cost-Comparison



Top: optimale control, bottom: cost function for initial and optimized control

## Example - Quarter-Car-Animation



## Conclusion

We demonstrated

- ▶ Modelling of impulsive multi-body-systems using Riemann-Stieltjes-Integrals
- ▶ Energy-consistent calculation of impacts using generalized COR
- ▶ Optimization with Gradient-Method

Open questions:

- ▶ How about points of contact? (only Hölder-continuous)
- ▶ Modelling of safety aspects in cost function
- ▶ Comparison to full finite element calculations