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Parametric sensitivity analysis and real-time optimal control using TransWORHP

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NetCo 2014, 27 June 2014

High rack warehouses





trolley

cable ropes

load-carrying equipment

Crane System

An alternative concept
for high rack warehouses



Mathematical Task

General problem formulation of optimal control

How do I have to control the motors, so that

$$\min_{u, x, t_f} \int_0^{t_f} g(x(t), u(t), t) dt$$

a system is brought from an initial position

$$\text{s.t.} \quad \dot{x}(t) = f(x(t), u(t), t), \quad t \in [0; t_f]$$

to a final position as good as possible

$$C(x(t), u(t), t) \leq 0, \quad t \in [0; t_f]$$

without overstressing?



Overview

- Motivation: Crane System
- Numerical Solution of Optimal Control Problems
- Post-Optimality Analysis
- Real-Time Solutions



Optimal Control Problem

Standard formulation and methods

$$\begin{aligned} \min_{u, x, t_f} & \int_0^{t_f} g(x(t), u(t), t) dt \\ \text{s.t.} & \dot{x}(t) = f(x(t), u(t), t), \quad t \in [0; t_f] \\ & \omega(x(0), x(t_f)) = 0 \\ & C(x(t), u(t), t) \leq 0, \quad t \in [0; t_f] \end{aligned}$$

Indirect Methods

(First optimize, then discretize)

- Reformulation to a boundary value problem using necessary conditions

Direct Methods

(First discretize, then optimize)

- Reformulation to a nonlinear optimization problem by discretization



Direct Methods

Numerical solution by discretization of time variable

$$t \in [0, t_f] \rightarrow t \in \{0 = t_1 \leq t_2 \leq \dots t_N = t_f\}$$

$$h_i = t_{i+1} - t_i, \quad i = 1, \dots, N - 1$$

Single Shooting/Multiple Shooting

Free Variables

$$u^i \approx u(t_i) \quad x^0 \approx x(0)$$

Recursive integration

$$x^i = x^i(u^1, \dots, u^{i-1})$$

Small + dense

Full Discretization

Free Variables

$$u^i \approx u(t_i) \quad x^i \approx x(t_i)$$

Integration scheme as constraints

$$0 = x^{i+1} - x^i - h_i \cdot f(x^i, u^i, t_i)$$

Large + sparse


Direct Methods

Numerical Solution by discretization of time variable and Euler's method, e.g.

$$t \in [0, t_f] \rightarrow t \in \{0 = t_1 \leq t_2 \leq \dots t_N = t_f\}$$

$$h_i = t_{i+1} - t_i, \quad i = 1, \dots, N - 1$$

$$u^i \approx u(t_i) \quad x^i \approx x(t_i)$$

$$\begin{aligned} \min_{u, x} \quad & \sum_{i=1}^{N-1} h_i \cdot g(x^i, u^i, t_i) \\ \text{s.t.} \quad & x^{i+1} = x^i + h_i \cdot f(x^i, u^i, t_i), \quad i = 1, \dots, N - 1 \\ & \omega(x^1, x^N) = 0 \\ & C(x^i, u^i, t_i) \leq 0, \quad i = 1, \dots, N \end{aligned}$$


High dimensional NLP problem



Methods of Higher Order

Implicit methods for **free**

Euler's Method

$$0 = x^{i+1} - x^i - h_i f_i$$

Trapezoidal rule

$$0 = x^{i+1} - x^i - \frac{h_i}{2} (f_{i+1} + f_i)$$

Hermite-Simpson: additional point $t_{i+\frac{1}{2}} = \frac{1}{2}(t_{i+1} + t_i)$

$$0 = x^{i+\frac{1}{2}} - \frac{1}{2}(x^{i+1} + x^i) - \frac{h_i}{8} (f_i - f_{i+1})$$

$$0 = x^{i+1} - x^i - \frac{h_i}{6} \left(f_{i+1} - 4f_{i+\frac{1}{2}} + f_i \right)$$

Precision
of solution

- ODE holds first in later iterations
- Less variables for higher order
- Stiff ODEs, DAEs => PDEs

$\mathcal{O}(h^2)$

$\mathcal{O}(h^4)$



Nonlinear Optimization

Problem formulation

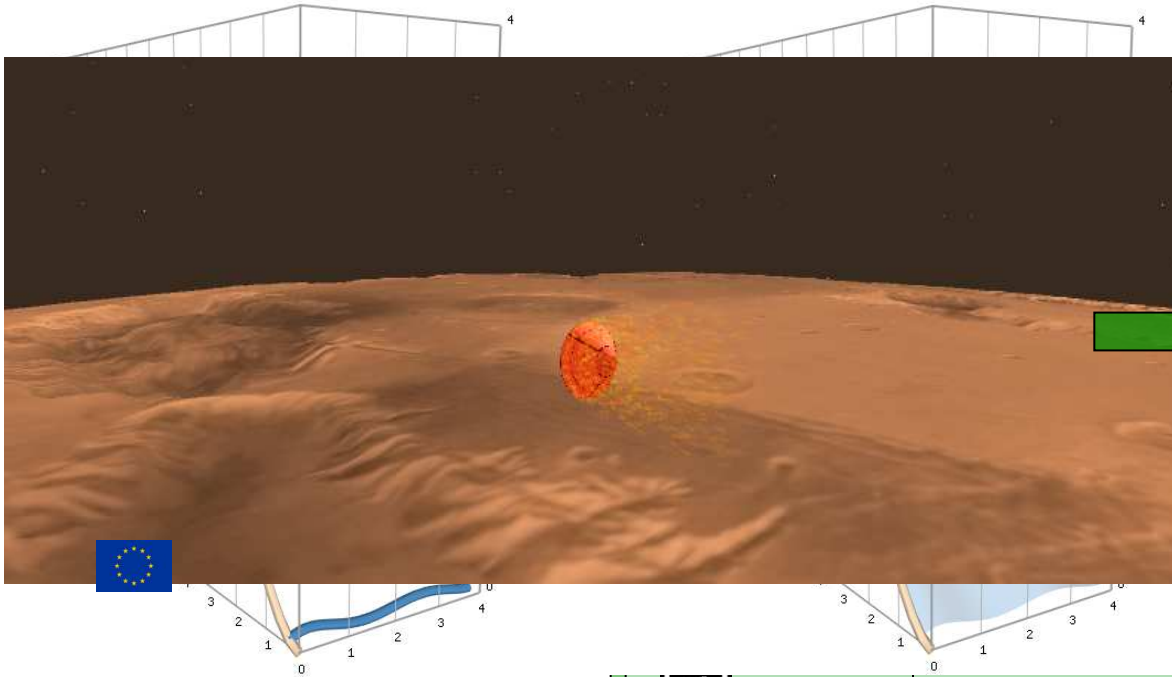
$$\begin{aligned} \min_{z \in \mathbb{R}^n} \quad & F(z) \\ \text{s.t.} \quad & G(z) \leq 0 \end{aligned}$$

$$z = (u^1, \dots, u^N, x^1, \dots, x^N)$$



The WORHP Family

We Optimize Really Huge Problems



TransWORHP

- Sparse OCP solver
 - $>10^6$ state and control variables in time
- Full discretization
- Implicit methods
- Multiple Phases
- Derivatives via operator overloading
- GUI (data analysis)

Funding



Development



WORHP

We Optimize Really Huge Problems

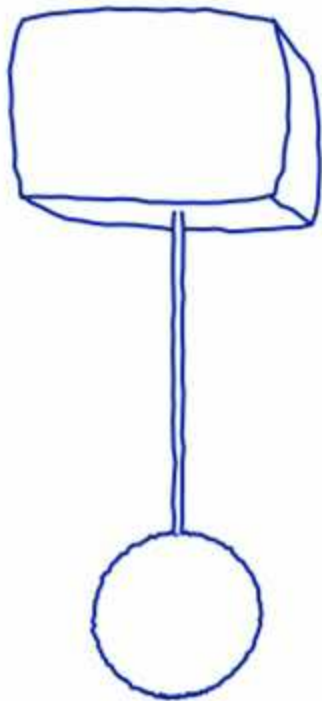
- Free for academics and testing
- Industrial standard
 - ECSS-E-40 tailored
 - TRL 6-7 (of 9)
(TRL 1-2 scientific software)
- Interfaces
 - C++
 - FORTRAN
 - MATLAB/Simulink
 - AMPL
- www.worhp.de



10^6 variables
on a smartphone



>350 users worldwide
>120 international users



$$\ddot{x} = -\frac{g}{l}x$$

Modellierung

Path Planning with TransWORHP

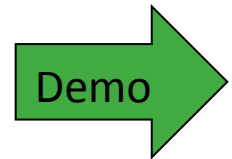
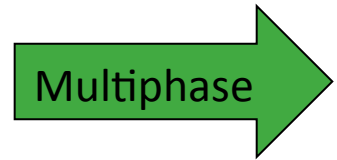
Move crane system from (S_0, L_0) to (S_f, L_f) .

$$\min_{u(t), x(t), t_f} t_f + \int_0^{t_f} u_1^2(t) + u_2^2(t) dt$$

$$\text{s.t.} \quad \dot{x}(t) = \begin{pmatrix} x_2 \\ x_5 \\ x_4 \\ x_5 - (g - x_8) \frac{x_3}{x_6} \\ u_1 \\ x_7 \\ x_8 \\ u_2 \end{pmatrix}$$

position trolley
 velocity trolley
 rel. displacement payload
 rel. velocity payload
 acceleration trolley
 length of rope
 velocity rope
 acceleration rope

$$\begin{aligned}
 x(0) &= (S_0 \ 0 \ 0 \ 0 \ 0 \ L_0 \ 0 \ 0)^T \\
 x(t_f) &= (S_f \ 0 \ 0 \ 0 \ 0 \ L_f \ 0 \ 0)^T \\
 u_i(t) &\in [u_{i,\min}; u_{i,\max}], & i = 1, 2 \\
 x_i(t) &\in [x_{i,\min}; x_{i,\max}], & i \in \mathcal{I}_c
 \end{aligned}$$

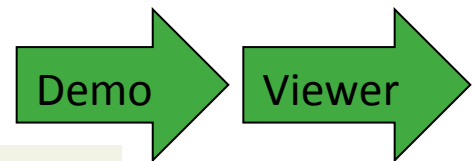




MPC with WORHP

Nonlinear Model Predictive Control

- Hexapod-Robot with control=acceleration
 - 15 state functions
 - 7 control functions
- Predicted Pseudo-Tracking
- Optimization cycle <200 ms for each MPC step



WORHP Zen

Impact of small perturbations on optimal solution

- Parametric Nonlinear Programming $(p_0, r_0, q_0) = (p_0, 0, 0)$

$$\begin{aligned} \min_{z \in \mathbb{R}^n} \quad & F(z, p) + r^T z \\ \text{s.t.} \quad & G(z, p) + q \leq 0 \end{aligned}$$

- Sensitivity derivatives of solution $\frac{dz}{dp}(p_0)$
- Extension of sensitivity theorem of Fiacco, Robinson
 - Eliminating NLP fractals (relaxation, regularisation)
 - Without measurable computational cost
 - For free: Additional sensitivity derivatives $\frac{dz}{dq}(0)$ and $\frac{dz}{dr}(0)$

$$\begin{pmatrix} \nabla_z^2 L & \nabla_z G^T \\ \Delta \nabla_z G & \Gamma \end{pmatrix} \cdot \begin{pmatrix} \frac{dz}{dp}(p_0) \\ \frac{d\lambda}{dp}(p_0) \end{pmatrix} = - \begin{pmatrix} \nabla_{zp} L \\ \Delta \nabla_p G \end{pmatrix}$$



Real-Time Optimal Control

Using sensitivity differentials of initial and final point

1. Update control for $i = 1$

$$u^{[1]}(p) = u(p_0) + \frac{du}{dp}(p_0) \cdot (p - p_0)$$

2. Measure final state deviation $\Delta q^i := x(t_f; u^{[i]}) - x_f$

3. While $\Delta q^i > \varepsilon$

- a. Update control

$$u^{[i+1]}(p) = u^{[i]}(p) + \frac{du}{dq}(0) \cdot \Delta q^i$$

- b. Update deviation, $i = i + 1$

Convergence

Theorem: Convergence of repeated correction method [B.]

Assumptions:

- KKT and SSC hold
- $F, G \in C^3$

Then there exists

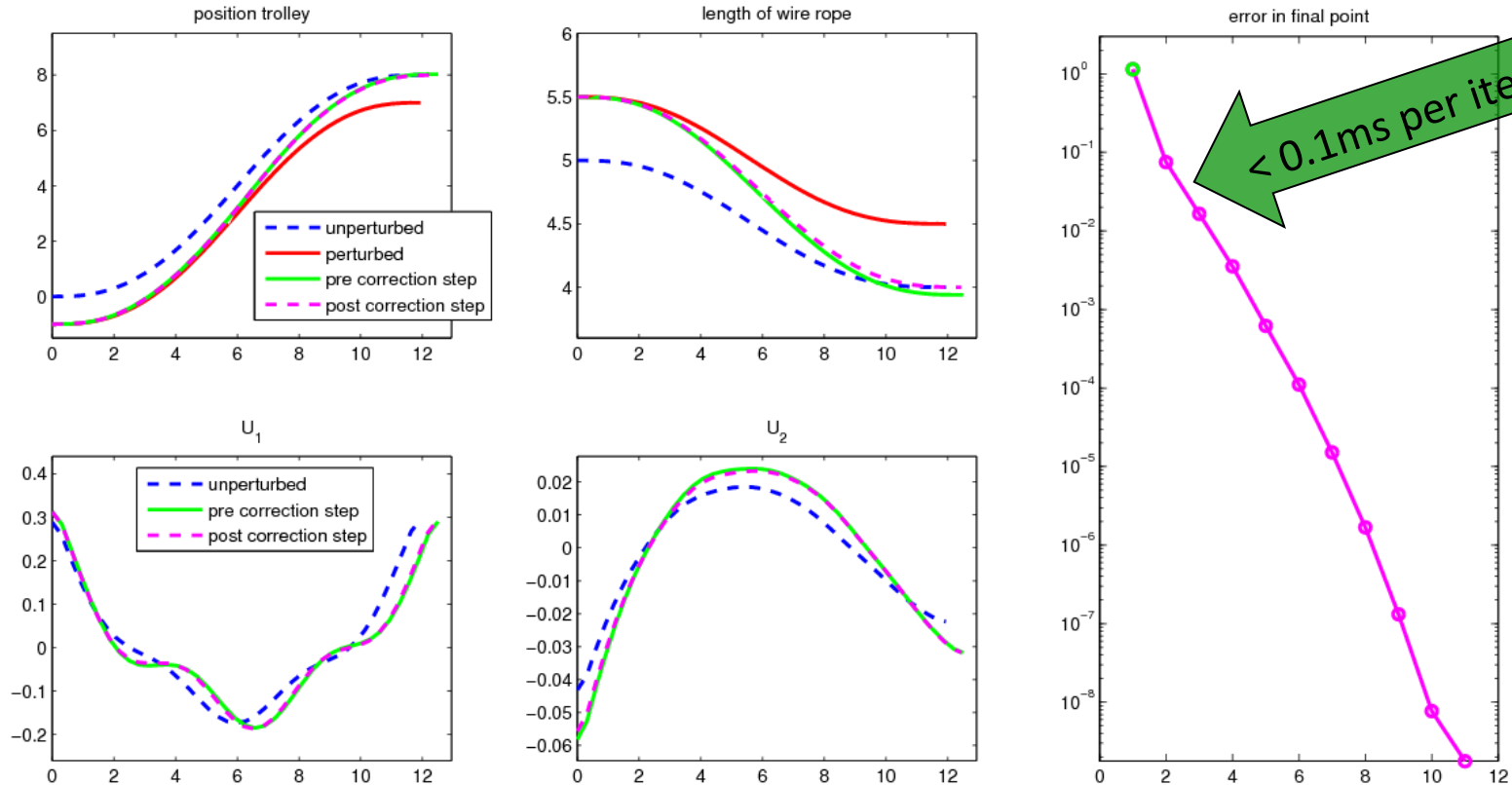
- $U(p_0)$
- $v \in \text{Ker}(\nabla_z G^a(z(p_0), p_0)), \quad \|v\| = \mathcal{O}(\|\Delta p\|^2)$

Such that for all $p \in U(p_0), \quad i=2,3,4,\dots$

$$\begin{aligned} \|z(p) - z^{[i]}(p)\| &= \|v\| + \mathcal{O}(\|\Delta p\|^3) \\ \|F(z(p), p) - F(z^{[i]}(p), p)\| &= \mathcal{O}(\|\Delta p\|^3) \\ \|G^a(z^{[i]}(p), p)\| &= \mathcal{O}(\|\Delta p\|^{i+1}) \end{aligned}$$

Convergence to a feasible point

Real-Time Optimal Control



Real-Time Optimal Control

Using sensitivity differentials of ode

1. Update control and state for $i = 1$

$$u^{[1]}(p) = u(p_0) + \frac{du}{dp}(p_0) \cdot (p - p_0) \quad x^{[1]}(p) = x(p_0) + \frac{dx}{dp}(p_0) \cdot (p - p_0)$$

2. Measure deviations $\Delta q^i := \left(x^{[i],j+1} - x^{[i],j} - h_j \cdot f(x^{[i],j}, u^{[i],j}, t_j) \right)_{j=1, \dots, N-1}$

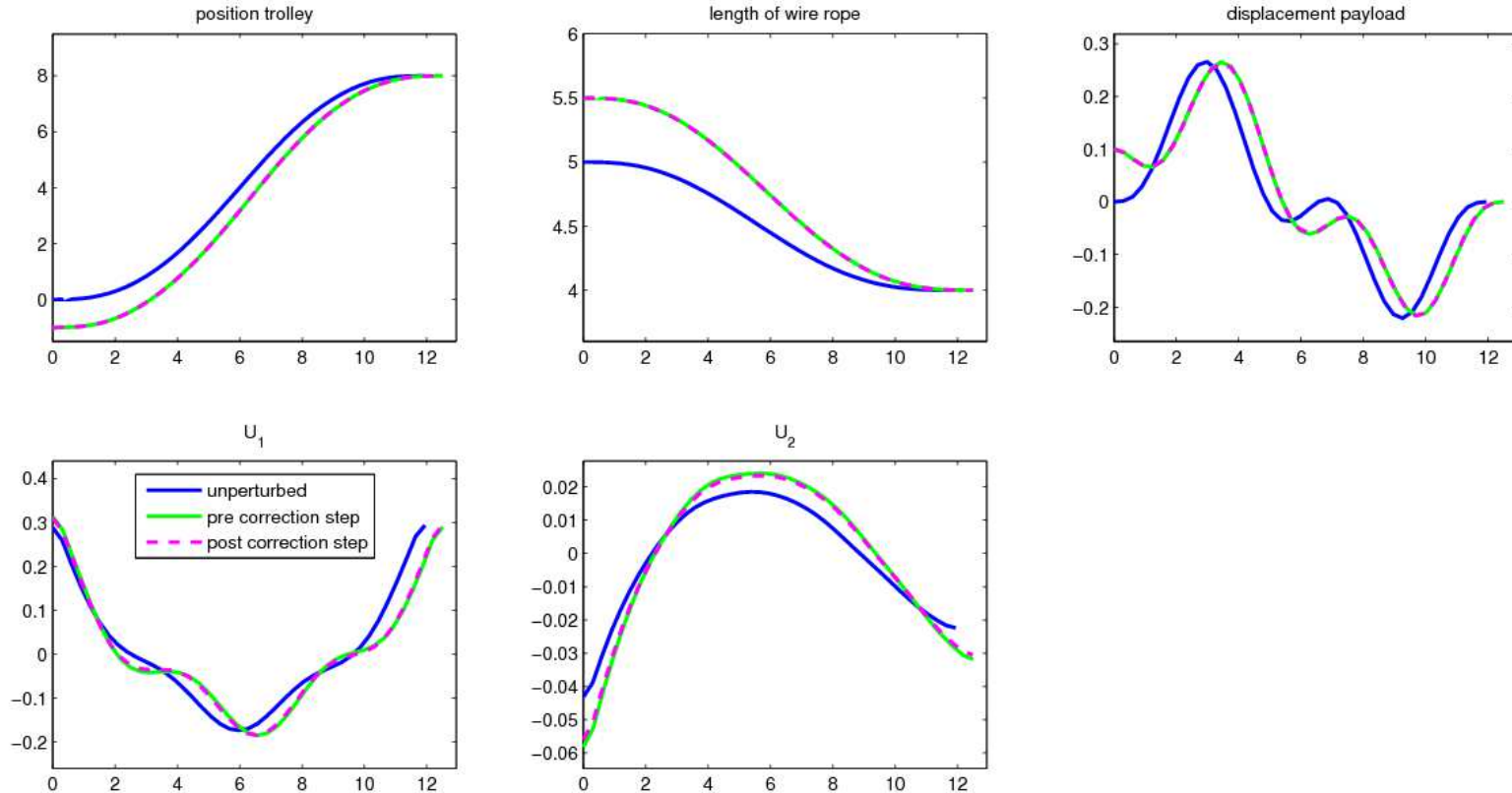
3. While $\Delta q^i > \varepsilon$

- a. Update control and state

$$u^{[i+1]}(p) = u^{[i]}(p) + \frac{du}{dq}(0) \cdot \Delta q^i \quad x^{[i+1]}(p) = x^{[i]}(p) + \frac{dx}{dq}(0) \cdot \Delta q^i$$

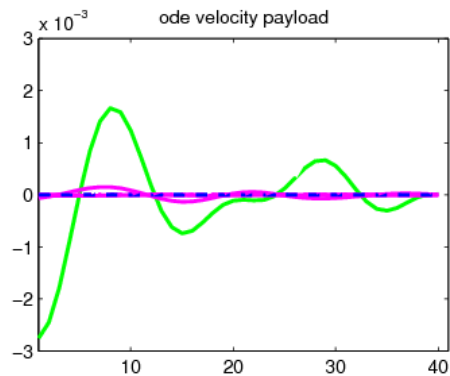
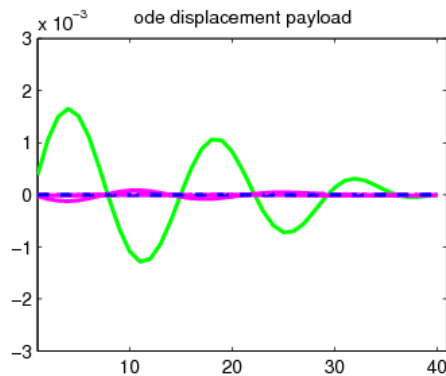
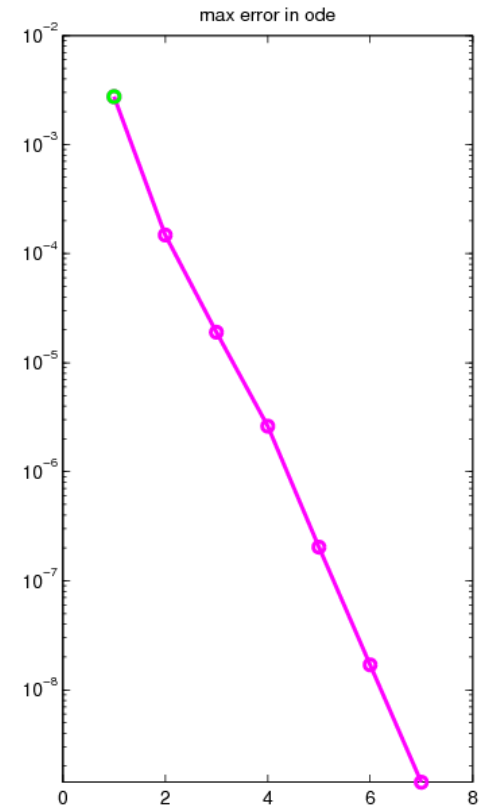
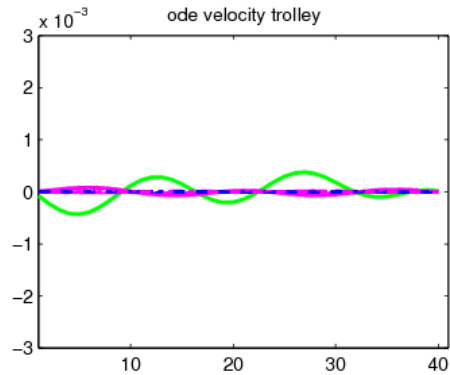
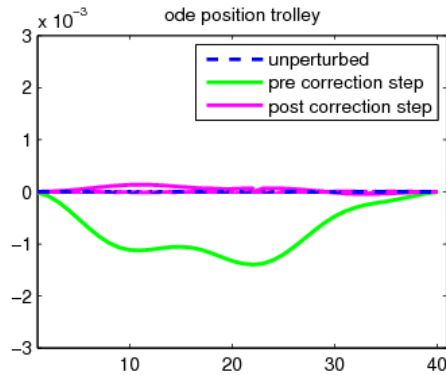
- b. Update deviation, $i = i + 1$

Real-Time Optimal Control





Real-Time Optimal Control



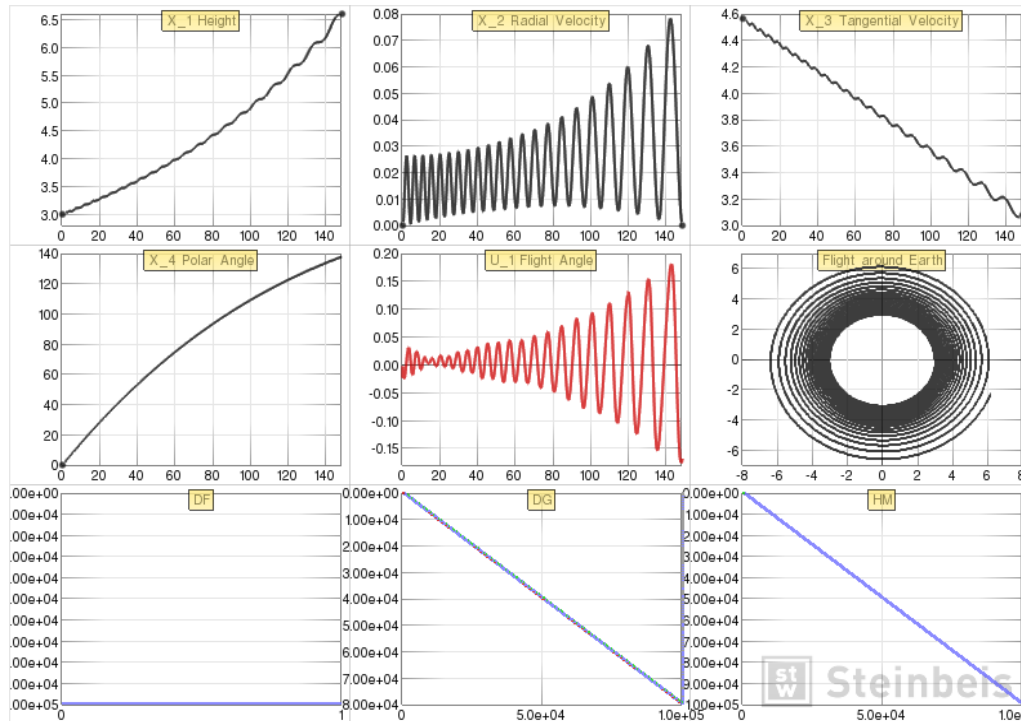
Conclusions

- WORHP Family
 - WORHP - TransWORHP - WORHP Zen
- Online and real-time solutions of OCP
 - WORHP/TransWORHP < 1000 ms
 - MPC < 100 ms
 - Real-time correction < 1 ms
- **Current Work.** Pareto Optimization, Mixed Integer, Parameter Identification under Constraints
- **Transfer.** e.g. Automotive, Logistics, Aerospace, Energy
- **Conference.** Optimisation in Space Engineering (Global Optim.)





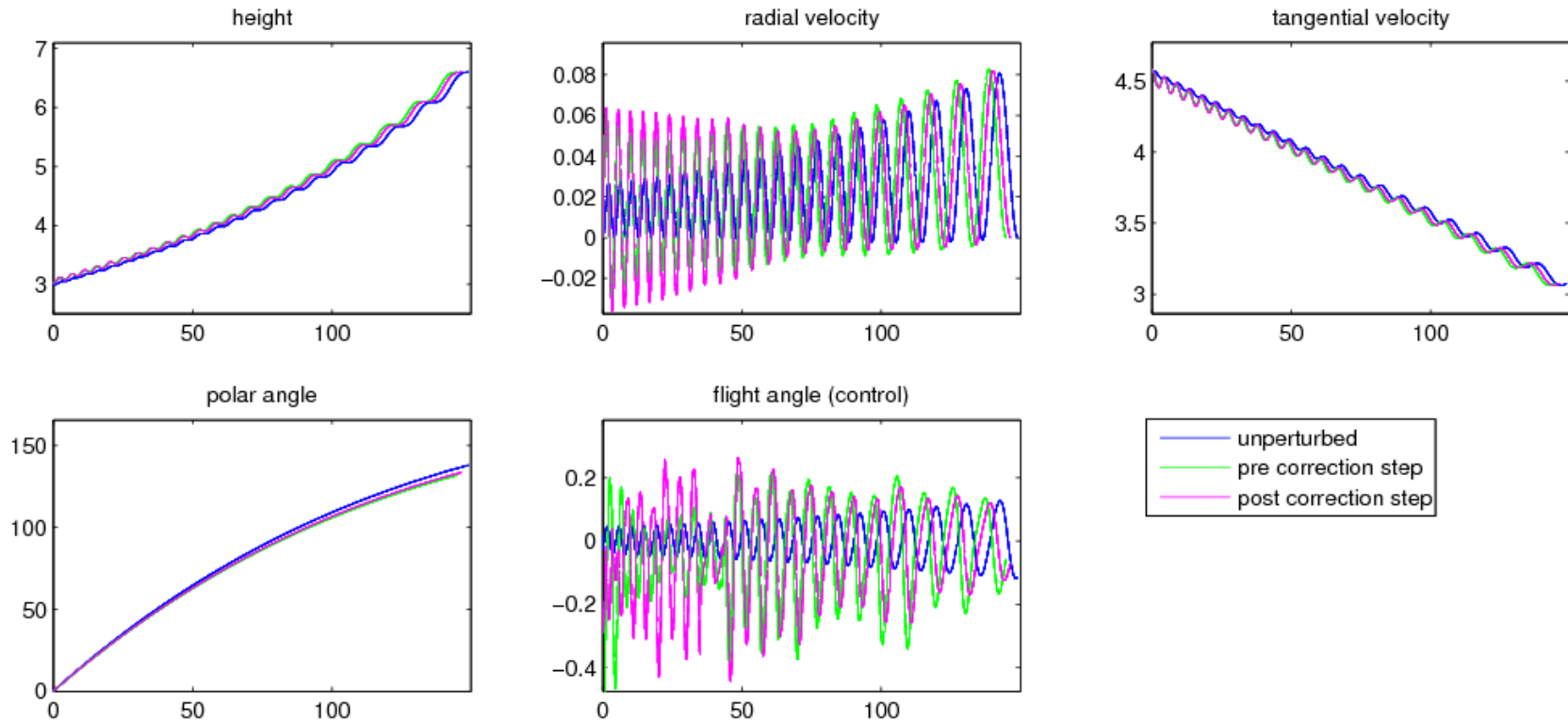
LEO-GEO-Transfer





LEO-GEO-Transfer

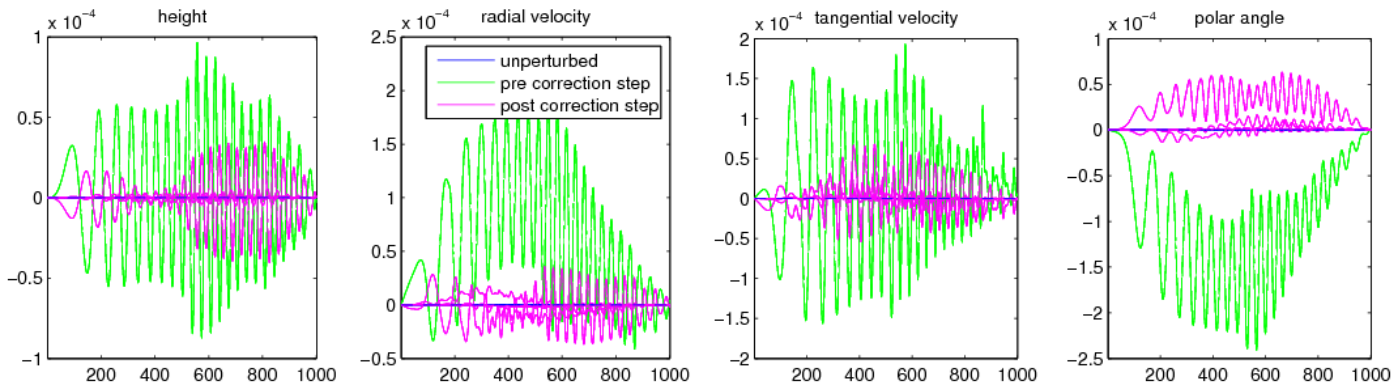
States and controls for the unperturbed and the perturbed problem





LEO-GEO-Transfer

Equations of the discretized differential equation system for the unperturbed and the perturbed problem.



Iteration	Final Time	Error
0	148.92	0.022360680
1	144.92	0.000240790
2	146.50	0.000071057
3	146.50	0.000032369
4	146.46	0.000017318
5	146.44	0.000004138
6	146.45	0.000002446
7	146.45	0.000000677
8	146.45	0.000000304
9	146.45	0.000000173
10	146.45	0.000000042
11	146.45	0.000000023
12	146.45	0.000000008
13	146.45	0.000000004
14	146.45	0.000000003