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2-Point-based Outlier Rejection for Camera-Imu Systems with applications to Micro Aerial Vehicules

Chiara Troiani¹, Agostino Martinelli¹, Christian Laugier¹ and Davide Scaramuzza²

Abstract—This paper presents a novel method to perform the outlier rejection task between two different views of a camera rigidly attached to an Inertial Measurement Unit (IMU). Only two feature correspondences and gyroscopic data from IMU measurements are used to compute the motion hypothesis. By exploiting this 2-point motion parametrization, we propose two algorithms to remove wrong data associations in the feature-matching process for case of a 6DoF motion. We show that in the case of a monocular camera mounted on a quadrotor vehicle, motion priors from IMU can be used to discard wrong estimations in the framework of a 2-point-RANSAC based approach. The proposed methods are evaluated on both synthetic and real data.

I. INTRODUCTION

Egomotion estimation is a core problem in the framework of many robotic applications. The estimation of the motion of a vehicle from the input of onboard cameras is known as Visual Odometry. In recent years, the coupling of vision and inertial sensing has received great attention by the mobile robotics community. It has been proved [1], [2], [3] that exploiting the complementarity of these sensors, it is possible to build a system capable to perform Inertial-Aided visual odometry and mapping in unknown environments.

One of the primary problems in Visual Odometry is wrong data associations. Matched features between two different camera views are usually affected by outliers. This is due to the fact that changes in viewpoint, occlusions, image noise, illumination changes and image noise are not modeled by feature-matching techniques. To perform a robust motion estimation, it is essential to remove the outliers. The outlier detection task is usually very expensive from a computational point of view and is based on the exploitation of the geometric constraints induced by the motion model.

The standard method to process datasets contaminated by outliers is RANSAC (RANdom SAMple Consensus) [4]. It is an iterative approach consisting in the generation of a solution free of outliers is found is [5]:

\[ N = \frac{\log(1 - p)}{\log(1 - (1 - \varepsilon)^s)} \]  

where \( s \) is the number of data points from which the model can be computed, \( \varepsilon \) is the percentage of outliers in the dataset, \( p \) is the requested probability of success.

Table I shows the number of iterations, \( N \), with respect to the number of points necessary to estimate the model, \( s \). The values are computed for \( p = 0.99 \) and \( \varepsilon = 0.5 \).

From Table I we can see that the number of iterations is exponential in the minimum number of points necessary to estimate the model. For this reason, it is very important to have a minimal parametrization of the model.

In this paper, we presented two methods to remove outliers between two different views of a camera rigidly attached to an Inertial Measurement Unit (IMU). This paper extends our previous contribution in [6] by including the general case of a 6DoF motion. In [6] the analysis only regarded the case of a planar motion. We also show that in the case of a monocular camera mounted on a quadrotor vehicle, motion priors from IMU can be used to discard wrong estimations in the framework of a 2-point RANSAC based approach.

The paper is organized as follows. In Section II we illustrate the related works. In Section III we describe the camera motion model and the computation of the essential matrix. Section IV provides the description of the proposed algorithms for outlier rejection. Section V describes the quadrotor motion model. Section VI illustrates the performance of our methods while Section VII draws the conclusions.

II. RELATED WORKS

The minimum number of feature correspondences needed to infer a 6 degrees of freedom (6DoF) camera motion is five. This problem was solved for the first time by Kruppa [7] in 1913. Later on, many different five-point minimal solvers have been implemented [8],[9],[10], but the six [11], seven and eight-point solvers were still the most used. The more efficient five-point minimal solver was proposed by Nister [12] in 2003 and improved by Stewenius later [13]. The advantage of the five-point algorithm is that it is suitable also for planar scenarios.

In the last twenty years, there has been a significant interest in minimal motion parametrizations in order to
Based on histogram voting.

proposed as well a single iteration outlier detection technique circular and planar, and, consequently, characterized by two
that the motion of wheeled vehicles can be considered locally
decrease to three and only two points are enough to
orientation.

from the scene) was used to fix the two angles of the
possibility of using a three-point minimal solver in a four- point RANSAC framework. The fourth point (a point distant
later enhanced in [15], where the authors demonstrated the
two out of three angles characterizing the relative orientation
[14]. The authors considered a monocular camera rigidly
freedom decrease to three and only two points are enough to
parameters. This leads to a one-point minimal solver. They
proposed as well a single iteration outlier detection technique based on histogram voting.

A comparison between five, two, and one-point RANSAC
algorithms for Visual Odometry can be found in [19].

III. EPIPOLAR GEOMETRY

When a camera is calibrated, it is always possible to
project the feature coordinates onto a unit sphere. This allows
us to make our approach independent of the camera model.

Let \( p_1 = (x_1, y_1, z_1) \) and \( p_2 = (x_2, y_2, z_2) \) be the image coordinates of a point feature seen from two camera positions
and back projected onto the unit sphere (i.e., \( \| p_1 \| = \| p_2 \| = 1 \)) (Figure 1).

The image coordinates of point features relative to two
different unknown camera positions must satisfy the epipolar
contstraint (Figure 1) [20].

\[
p_2^T E p_1 = 0
\]

where \( E \) is the essential matrix, defined as \( E = [T]_x R \). \( R \) and \( T = [T_x, T_y, T_z]^T \) describe the relative rotation and
translation between the two camera positions, and \( [T]_x \) is the skew symmetric matrix:

\[
[T]_x = \begin{bmatrix}
0 & -T_z & T_y \\
T_z & 0 & -T_x \\
-T_y & T_x & 0
\end{bmatrix}
\]

According to equation (2), the essential matrix can be
computed given a set of image coordinate points. \( E \) can then be decomposed into \( R \) and \( T \) [20].

The minimum number of feature correspondences needed to
estimate the essential matrix is function of the degrees of
freedom of the camera’s motion. In the case of a monocular
camera performing a 6DoF motion (three for the rotation
and three for the translation), considered the impossibility to
recover the scale factor, a minimum of five correspondences
is needed.

Let us consider a camera rigidly attached to an Inertial
Measurement Unit (IMU) consisting of three orthogonal
accelerometers and three orthogonal gyroscopes. The transform-
formation between the camera reference frame \( \{C\} \) and the
IMU frame \( \{I\} \) can be computed using [21]. Without loss
of generality, we can therefore assume that these two frames
are coincident \((\{I\} = \{C\})\). The \( \Delta \phi \), \( \Delta \theta \) and \( \Delta \psi \) angles
characterizing the relative rotation between two consecutive
camera frames can be calculated by integrating the high
frequency gyroscopic measurements, provided by the IMU.
This measurement is affected only by a slowly-changing drift
term and can safely be recovered if the system is in motion.

If \( R_x(\Delta), R_y(\Delta), R_z(\Delta) \) are the orthonormal rotation
matrices for rotations of \( \Delta \) about the x-, y- and z-axes, the
matrix

\[
C_0 R_{C_1} = (R_x(\Delta \phi) \cdot R_y(\Delta \theta) \cdot R_z(\Delta \psi))^T
\]

allows us to virtually rotate the first camera frame \( \{C_1\} \) into
a new frame \( \{C_0\} \) (Figure 2) having the same orientation of the second one \( \{C_2\} \).

The matrix \( C_0 R_{C_1} \) allows us to express the image coordinates relative to \( C_1 \) into the new reference frame \( C_0 \):

\[
p_0 = C_0 R_{C_1} \cdot p_1.
\]
At this point, the transformation between \{C_0\} and \{C_2\} is a pure translation
\[
T = \rho [s(\beta) \cdot c(\alpha) - s(\alpha) \cdot c(\beta)]^T
\]
\[
R = I_3,
\]
which depends only on the angles \(\alpha\) and \(\beta\) and on the scale factor \(\rho\). The essential matrix results therefore simplified:
\[
E = [T]_x R = \rho \begin{bmatrix}
0 & -s(\beta) \cdot s(\alpha) & s(\beta) \cdot c(\alpha) \\
-c(\beta) & 0 & s(\beta) \cdot c(\alpha) \\
s(\beta) \cdot c(\alpha) & -s(\beta) \cdot c(\alpha) & 0
\end{bmatrix}
\]
(7)

With \(s(\cdot)\) and \(c(\cdot)\) we denote the \(\sin(\cdot)\) and \(\cos(\cdot)\) respectively. At this point, being \(p_0 = [x_0 \ y_0 \ z_0]^T\) and \(p_2 = [x_2 \ y_2 \ z_2]^T\), the coordinates of a feature matched between two different camera frames and backprojected onto the unit sphere, we impose the epipolar constraint according to (2) and we obtain the homogeneous equation that must be satisfied by all the point correspondences.
\[
x_2(y_0c(\beta) + z_0s(\alpha)s(\beta)) - y_2(x_0c(\beta) - z_0c(\alpha)s(\beta)) +
- z_2(y_0c(\alpha)s(\beta) + x_0s(\alpha)s(\beta)) = 0.
\]
(8)

Equation (8) depends on two parameters (\(\alpha\) and \(\beta\)). This means that the relative vehicle motion can be estimated using only two image feature correspondences that we will identify as \(p_A\) and \(p_B\), where \(p_i = [x_i \ y_i \ z_i]^T\) with \(i = A, B\) and \(j = 0, 2\) indicate the direction of the feature \(i\) in the reference frame \(j\).

At this point, we can recover the angles \(\alpha\) and \(\beta\) solving (8) for the features \(p_A\) and \(p_B\):  
\[
\alpha = -\tan^{-1}\left(\frac{c_4 c_2 - c_1 c_6}{c_1 c_3 + c_4 c_5}\right),
\]
\[
\beta = -\tan^{-1}\left(\frac{c_2 c(\alpha) + c_3 s(\alpha)}{c_1 c(\alpha) - c_2 s(\alpha)}\right),
\]
(9)

where
\[
c_1 = x_2 y_0 a_0 \cdot c_2 = -y_0 y_0 z_0 a_0 + y_0 z_0 z_0 a_0, \]
\[
c_3 = -x_0 z_0 z_0 a_0, \]
\[
c_4 = x_2 b_2 y_0 - x_0 y_0 y_2, \]
\[
c_5 = -y_0 b_2 z_2 + y_0 z_2 b_0, \]
\[
c_6 = -x_0 z_2 b_0 + x_0 b_0 z_2.
\]
(10)

Finally, without loss of generality, we can set the scale factor \(\rho\) to 1 and estimate the essential matrix according to (7).

IV. OUTLIER REJECTION

Starting from the above-mentioned camera-motion parametrization, we propose two different approaches to reject outliers from a set of feature correspondences.

The former is based on estimating the angles \(\alpha^*\) and \(\beta^*\) by selecting the pick in a Hough Space and discarding the outliers by using the reprojection error. We call this method “Hough”. The second one is a 2-point RANSAC-based method with an extension to the particular case of a camera mounted on a quadrotor. The algorithm takes into account the relations between the orientation and the translation of the vehicle to remove wrong estimations. The inliers are identified using the reprojection error.

A. Hough

The angles \(\alpha\) and \(\beta\) are computed according to (9) from all the feature pairs matched between two consecutive frames and distant from each other more than a defined threshold (see VI). A distribution \(\{\alpha_i, \beta_i\}\) with \(i = 1, 2, \ldots, N\) is obtained, where \(N\) is a function of the position of the features in the environment.

To estimate the best angles \(\alpha^*\) and \(\beta^*\), we build a Hough Space (Figure 3) which bins the values of \(\{\alpha_i, \beta_i\}\) into a grid of equally spaced containers. Considering that the angle \(\beta\) is defined in the interval \([0, \pi]\) and that the angle \(\alpha\) is defined in the interval \([0, 2\pi]\), we set 300 bins for the variable \(\alpha\) and 180 bins for the variable \(\beta\). The number of bins of the Hough Space encodes the resolution of the estimation.

The angles \(\alpha^*\) and \(\beta^*\) are therefore computed as
\[
<\alpha^*, \beta^*> = \text{argmax}\{H\},
\]
where \(H\) is the Hough Space.

The factors that influence the distribution are the error on the estimation of the relative rotation, the image noise, and the percentage of outliers in the data. The closer we are to ideal conditions (no noise on the IMU measurements), the narrower will be the distribution. The wider is the distribution, the more uncertain is the motion estimate.

To detect the outliers, we calculate the reprojection error relative to the estimated motion model.

The camera motion estimation can be then refined processing the remaining subset of inliers with standard algorithms [13], [20].

B. 2-point RANSAC

Using (6) we compute the motion hypothesis that consists of the translation vector \(T\) and the rotation matrix \(R = I_3\) by randomly selecting two features from the correspondence set. To have a good estimation, we check that the distance between the selected features is below a defined threshold (see VI). If it is not the case, we randomly select another pair of features. Constraints on the motion of the camera can be exploited to discard wrong estimations (see Section VI). The inliers are then computed using the reprojection error. The hypothesis that shows the highest consensus is considered to be the solution.
V. QUADROTOR MOTION MODEL

We consider a quadrotor equipped with a monocular camera and an IMU.

The vehicle body-fixed coordinate frame \( \{B\} \) has its \( Z_B \)-axis pointing downward (following aerospace conventions [22]). The \( X_B \)-axis defines the forward direction and the \( Y_B \)-axis follows the right-hand rule.

Without loss of generality we can consider the IMU reference frame \( \{I\} \) coinciding with the vehicle body frame \( \{B\} \).

The modelization of the vehicle rotation in the World frame \( \{W\} \) follows the \( Z-Y-X \) Euler angles convention: being \( \phi \), \( \theta \), \( \psi \) respectively the Roll, Pitch and Yaw angles of the vehicle, to go from the World frame to the Body frame, we first rotate about \( z_W \) axis by the angle \( \psi \), then rotate about the intermediate \( y \)-axis by the angle \( \theta \), and finally rotate about the \( X_B \)-axis by the angle \( \phi \).

The transformation between the camera reference frame \( \{C\} \) and the IMU frame \( \{I\} \) can be computed using [21]. Without loss of generality, we can therefore assume that also these two frames are coincident (\( \{I\} \equiv \{C\} \equiv \{B\} \)).

A quadrotor has 6DoF, but its translational and angular velocity are strongly coupled to its attitude due to dynamic constraints. If we consider a coordinate frame \( \{B_0\} \) with the origin coincident with the one of the vehicle’s body frame \( \{B\} \) and the \( X_{B_0} \) and \( Y_{B_0} \) axes parallel to the ground, we observe that, in order to move in the \( X_{B_0} \) direction, the vehicle must rotate about the \( y \)-axes axis (Pitch angle), while, in order to move in the \( Y_{B_0} \) direction, it must rotate about the \( x \)-axis (Roll angle) (Figure 5).

These motion constraints allow us to discard wrong estimations in a RANSAC based outlier detection approach. By looking at the relation between the \( \delta x \) and \( \delta y \) component of the estimated translation vector and the \( \Delta \phi \), \( \Delta \theta \) angles provided by the IMU measurements (the same used in [4]), we are able to check the consistency of the motion hypothesis. If the estimated motion satisfies the condition

\[
\begin{align*}
&((\Delta \phi > \epsilon) \& (\Delta \phi \cdot T_y > 0)) \quad \| \\
&((\Delta \theta > \epsilon) \& (\Delta \theta \cdot T_x < 0)) \quad \| \\
&((\Delta \phi < \epsilon) \& (\Delta \theta < \epsilon)),
\end{align*}
\]

we count the number of inliers (the number of correspondences that satisfy the motion hypothesis according to a predefined threshold) by using the reprojection error, otherwise we select another feature pair. The condition in (11) is satisfied if the \( \delta x \) and \( \delta y \) components of the motion hypothesis are coherent with the orientation of the vehicle. If both the angles \( \Delta \phi \) and \( \Delta \theta \) are below the threshold \( \epsilon \), we cannot infer nothing about the motion and we proceed in the evaluation of the model hypothesis using the reprojection error.

The value of the threshold \( \epsilon \) (see VI) is a function of the vehicle dynamics and of the controller used.

Using (1) and considering \( p = 0.99 \) and \( \epsilon = 0.5 \), we calculate the minimum number of iterations necessary to guarantee a good performance to our algorithm and we set it to 16.

VI. RESULTS

To evaluate the performance of our algorithms, we run simulations and experiments on real data. We compared the proposed approaches with the 5-point RANSAC [12] on synthetic data, and with the 5-point RANSAC [12] and the 8-point RANSAC [23] on real data.

A. Experiments on synthetic data

We built a synthetic scenario for our simulation by using the Robotics and Machine Vision Toolbox for Matlab [22]. We simulated a quadrotor equipped with a downlooking monocular camera and an IMU, moving in an indoor environment (Figure 6). Random features were generated without any assumption on the structure of the environment.

The onboard downlooking monocular camera was simulated as a perspective camera with the same intrinsic parameters of the camera that we used in the experiments. A white gaussian noise with a standard deviation of 0.5 pixels was added to each extracted feature.

We generated a trajectory consisting of a take-off and of a constant-height maneuver. The camera framerate is \( 15 Hz \), its resolution is \( 752 \times 480 \). For the reprojection error in the 2-point RANSAC and in the Hough algorithm, we set a threshold of 0.5 pixels. For the 5-point RANSAC, we set the minimum number to trials to 145 iterations, and the threshold to 0.5 pixels for the reprojection error.

Figure 7 shows the results of a simulation run along the trajectory depicted in Figure 6 in the ideal case of no noisy IMU measurements. The helicopter takes off and performs a constant height maneuver.

In Figure 8 we present the results related to simulations where the quantities \( \Delta \phi \), \( \Delta \theta \) and \( \Delta \psi \) are affected by a Gaussian noise with standard deviation of 0.3 degrees. Those errors do not affect the performance of the 5-point
algorithm (that does not use IMU readings to compute the motion hypothesis). In this case, the Hough and the 2-point RANSAC approaches can still detect more than half of the inliers. The motion hypothesis can then be computed on the obtained set of correspondences by using standard approaches [13], [20].

In Figure 9, we present the results related to simulations where the quantities $\Delta \phi$ and $\Delta \theta$ are affected by a Gaussian noise with standard deviation of 0.3 degrees and in Figure 10 only the angle $\Delta \psi$ is affected by a Gaussian noise with standard deviation of 0.3 degrees. These two plots show that errors on rotations about the camera optical axis (that in our case coincides with rotations about the vehicle $Z_B$ axis, i.e. errors on $\Delta \psi$) affects more the performances of both the algorithms than errors on $\Delta \phi$ and $\Delta \theta$.

B. Experiments on real data

The proposed approaches are tested on our nano quadrotor (Figure 11) equipped with a MicroStrain 3DM-GX3 IMU (250 Hz) and a Matrix Vision mvBlueFOX-MLC200w camera (FOV: 112 deg and a resolution of 752 x 480).

The monocular camera calibration has been performed using the Camera Calibration Toolbox for Matlab [24].

To extrinsically calibrate the IMU and the camera, we used the Inertial Measurement Unit and Camera Calibration Toolbox [21].

To validate the performance of our methods, we flew the quadrotor in our flying arena, equipped with an Optitrack motion capture system with sub-millimeter accuracy. The trajectory consisted of a take-off and a constant-height maneuver above the ground, as shown in Figure 12 and was generated using the TeleKyb Framework [25]. We recorded a dataset composed of camera images, IMU measurements and ground truth data provided by the Optitrack.

We processed our dataset with SURF features, matching them in consecutive camera frames. We run the 8-point RANSAC method on each correspondences set to have an additional term of comparison.

To evaluate the performance of our methods, we compared the number of inliers detected using the Hough and the 2-point RANSAC methods with 5-point and an 8-point RANSAC. For the 2-point RANSAC we set $\epsilon = 0.1$ deg.

The results of this comparison are shown in Figure 13.

Figure 14 shows the error characterizing the estimated relative rotation between two consecutive camera frames obtained by IMU measurements and the ground truth values.
Looking at both Figure 13 and Figure 14 we can notice that the smaller are the errors on the angles estimations, the higher is the number of inliers detected by the Hough and the 2-point RANSAC method.

Our algorithms and the algorithms that we used for the comparison, are implemented in Matlab and run on an Intel Core i7-3740QM Processor. We summarize their computation time in Table II. We can notice that the computation time of the 5-point RANSAC is almost 67 times the computation time of the 8-point RANSAC. This is due to the fact that the 5-points returns up to 10 motion solutions for each candidate set. Singular Value Decomposition (SVD) and Groebner-basis decompositions are involved and this explains the high computation time.

The computation time of the Hough algorithm is function of the number of feature pairs used to compute the distribution in Figure (3). Smart policies for the choice of the pairs of features to use (based for example on the feature positions in the image plane and not only on their relative position) can be used in order to reduce the computational complexity of the approach.

Experimental results show that the 2-point RANSAC algorithm can be a good replacement of the 5-point RANSAC. The motion hypothesis can always be refined by processing the found inliers with classic methods [13], [20].

### VII. CONCLUSIONS

In this paper, we proposed two algorithms (Hough and 2-point RANSAC) to address the outlier rejection task systems equipped with a monocular camera rigidly attached to an IMU. We used a quadrotor micro aerial vehicle as platform to demonstrate the validity of our results. We show that the relations between the vehicle’s translational and angular velocity and its attitude can be exploited in order to discard wrong estimations in the framework of a RANSAC-based approach.

Both methods rely on onboard IMU measurements to calculate the relative rotation between two consecutive camera frames and to the reprojection error to detect the inliers. The two algorithms differ in the way to compute the motion hypothesis.

The computation time of the Hough algorithm (Table II) is function of the number of feature pairs used to compute the distribution in Figure (3). Smart policies for the choice of the pairs of features to use (based for example on the feature positions in the image plane and not only on their relative position) can be used in order to reduce the computational complexity of the approach.

Experimental results show that the 2-point RANSAC algorithm can be a good replacement of the 5-point RANSAC. The motion hypothesis can always be refined by processing the found inliers with classic methods [13], [20].

### TABLE II: Computation time

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Hough</th>
<th>2-point</th>
<th>5-points</th>
<th>8-points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time [s]</td>
<td>0.498</td>
<td>0.048</td>
<td>2.6869</td>
<td>0.0396</td>
</tr>
</tbody>
</table>
REFERENCES


