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Quantitative DEM of dense granular packings with a multiple contacts force model

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Modeling Granular Media across scales, Montpellier, July 2014

MOTIVATION

1. 3D Experiments

Access to the micro-structure, 3D force vectors

Used as a reference for a comparison with DEM

2. DEM

Usual DEM + Hertz interactions not good enough

Interactions between multiple contacts

ACCESSING THE MICRO-STRUCTURE

X-rays / micro-ct

Fine resolution

Most materials

Costly

Confocal: emulsions

Microscopic

Costly

Difficult to control applied stresses

This work: refractive index matching

Macroscopic grains

Easy to control, tri-axial shearing

Cheap

Submersed

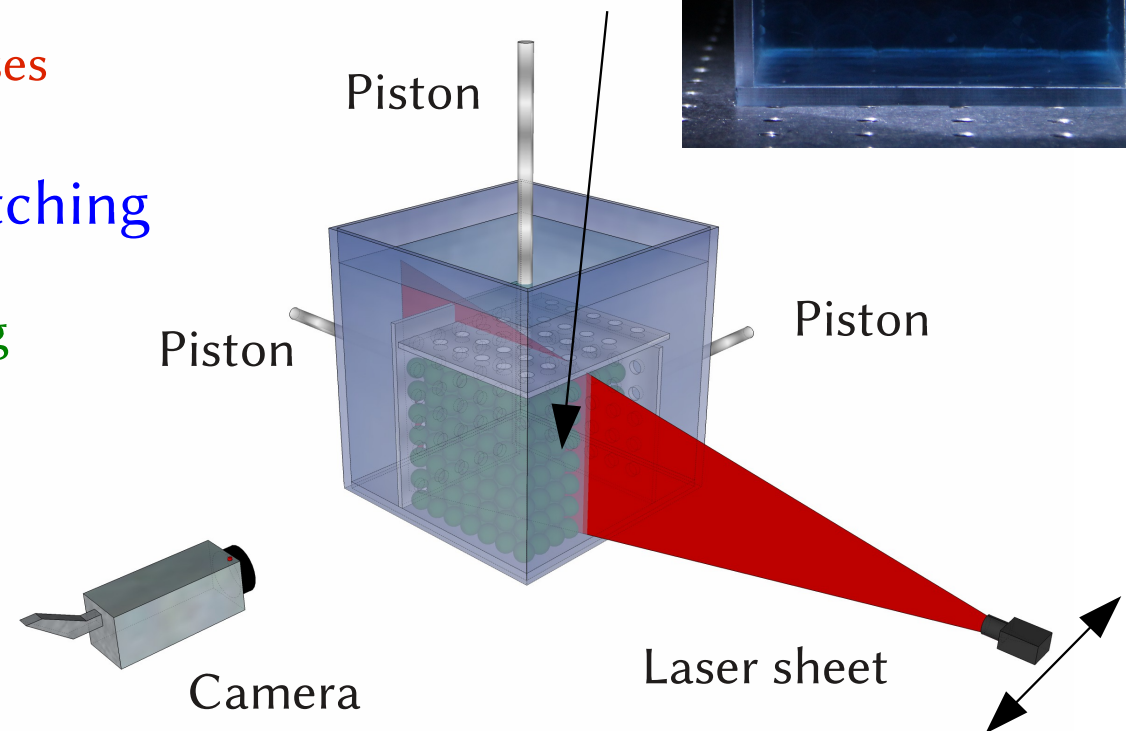
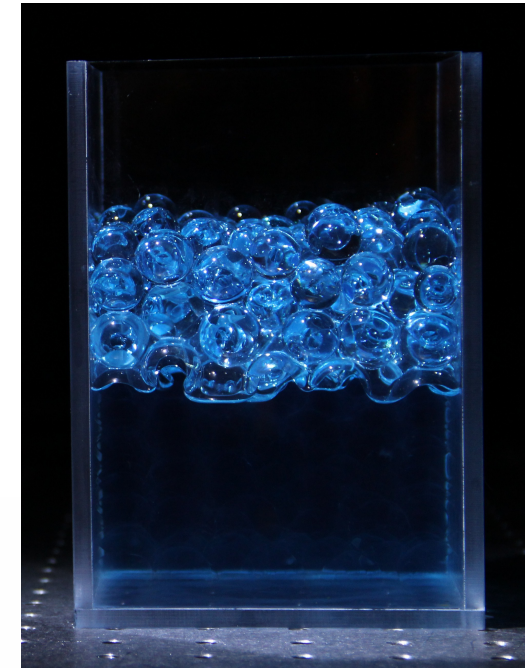
Next slides on:

1. Structure
2. Forces in 3D

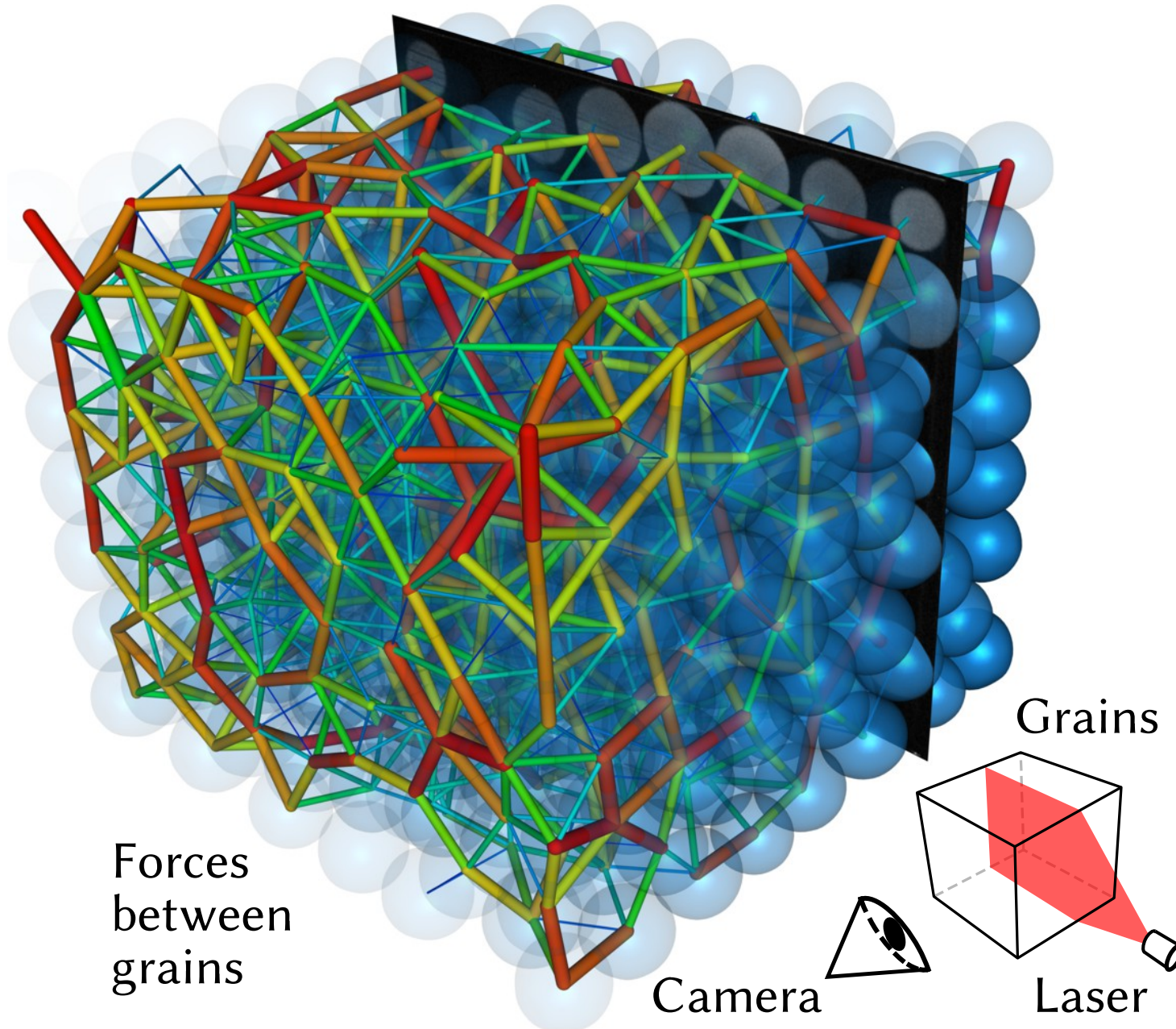
Mukhopadhyay *et al.*
Phys. Rev. E 84, 011302, 2011

Dijksman *et al.*
Rev. Sci. Instrum. 2012

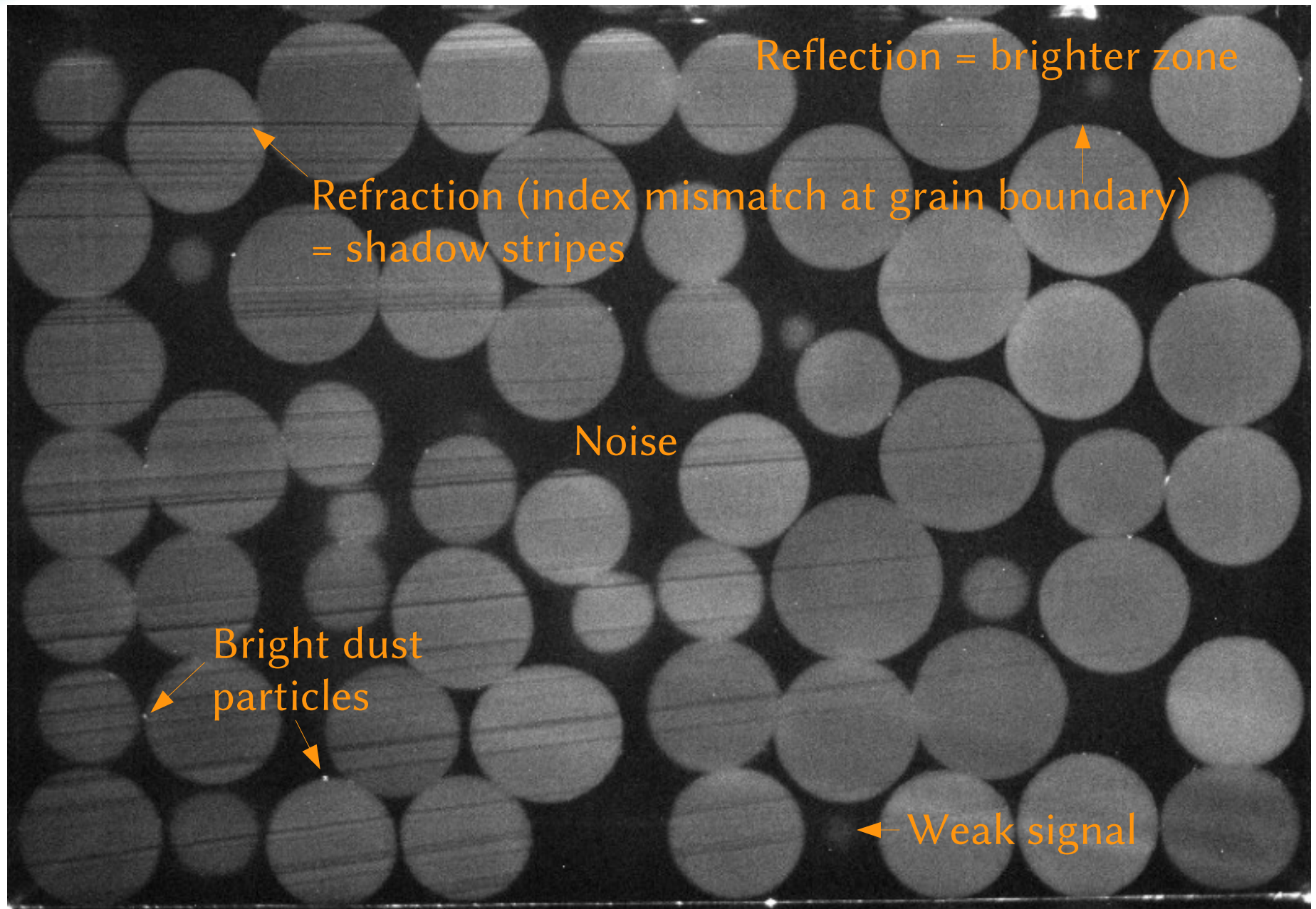
Hydrogel grains
index-matched
+ fluorescent dye



WHAT WE GET



TYPICAL IMAGE



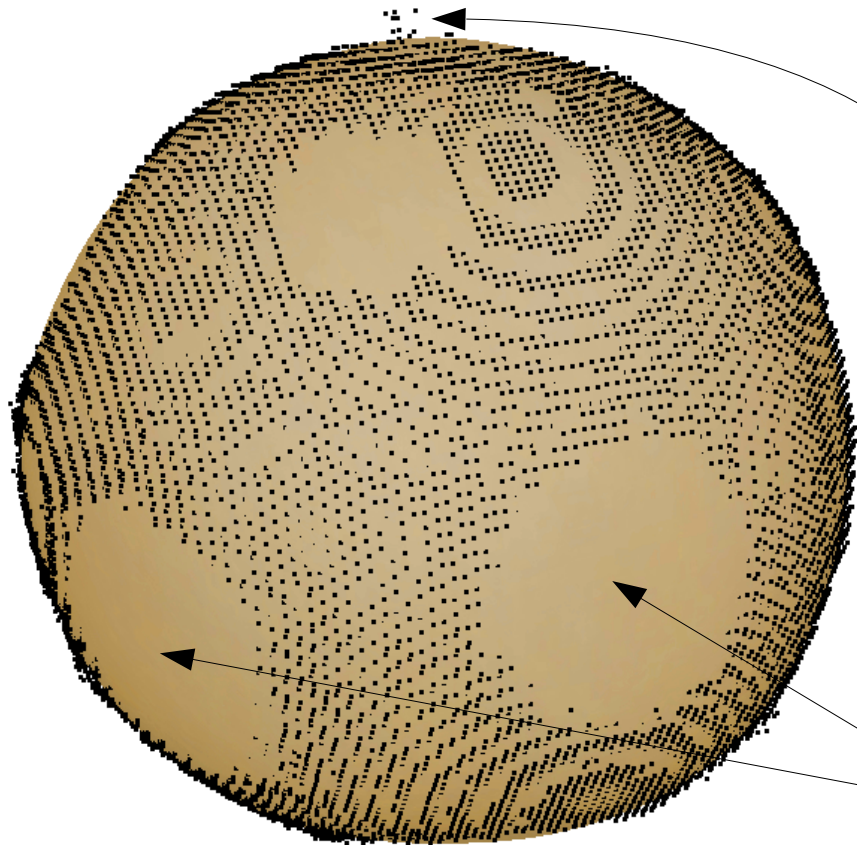
FROM 2D IMAGES TO 3D GRAINS

Step 1: Stack the images into 3D voxels

Step 2: Detect border voxels and assign them to unique grains

Step 3: Fit an analytic surface to these borders

Step 4: Use these surfaces to get accurate forces

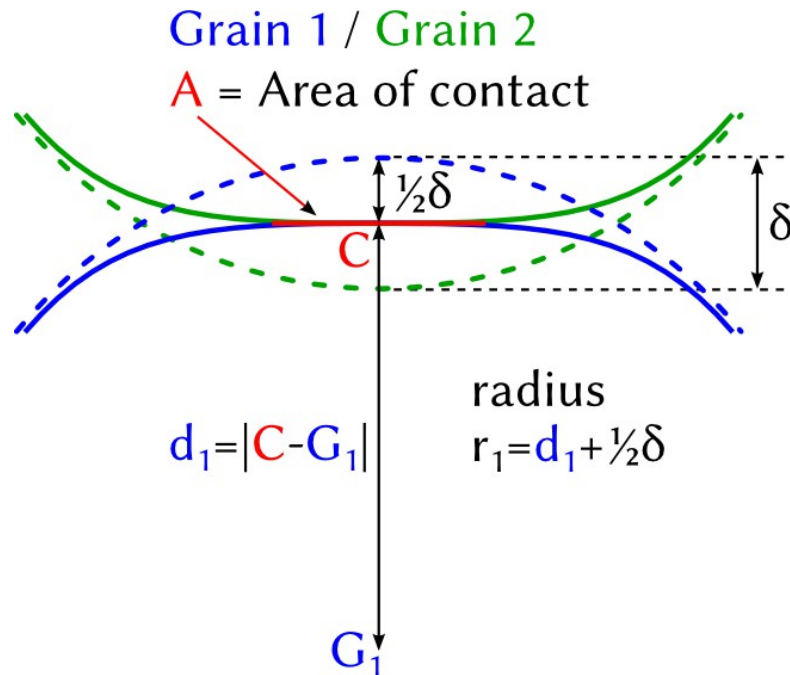


Outliers completely eliminated

Contacts = no border between grains
BUT surface area is well defined

INFERRING FORCES IN FULL 3D

Analytic shape descriptions \Rightarrow contact properties



Measured here: A, C, G_1, G_2, d_1, d_2 .

Unknown: δ

Contact properties \Rightarrow forces

$1/r = 1/r_1 + 1/r_2$ radius of curvature at contact

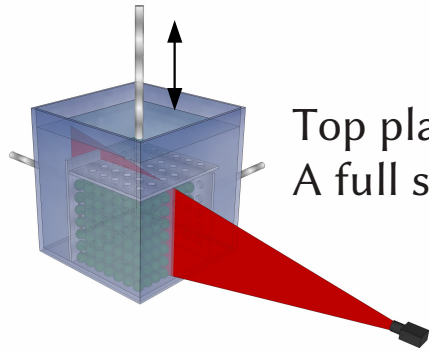
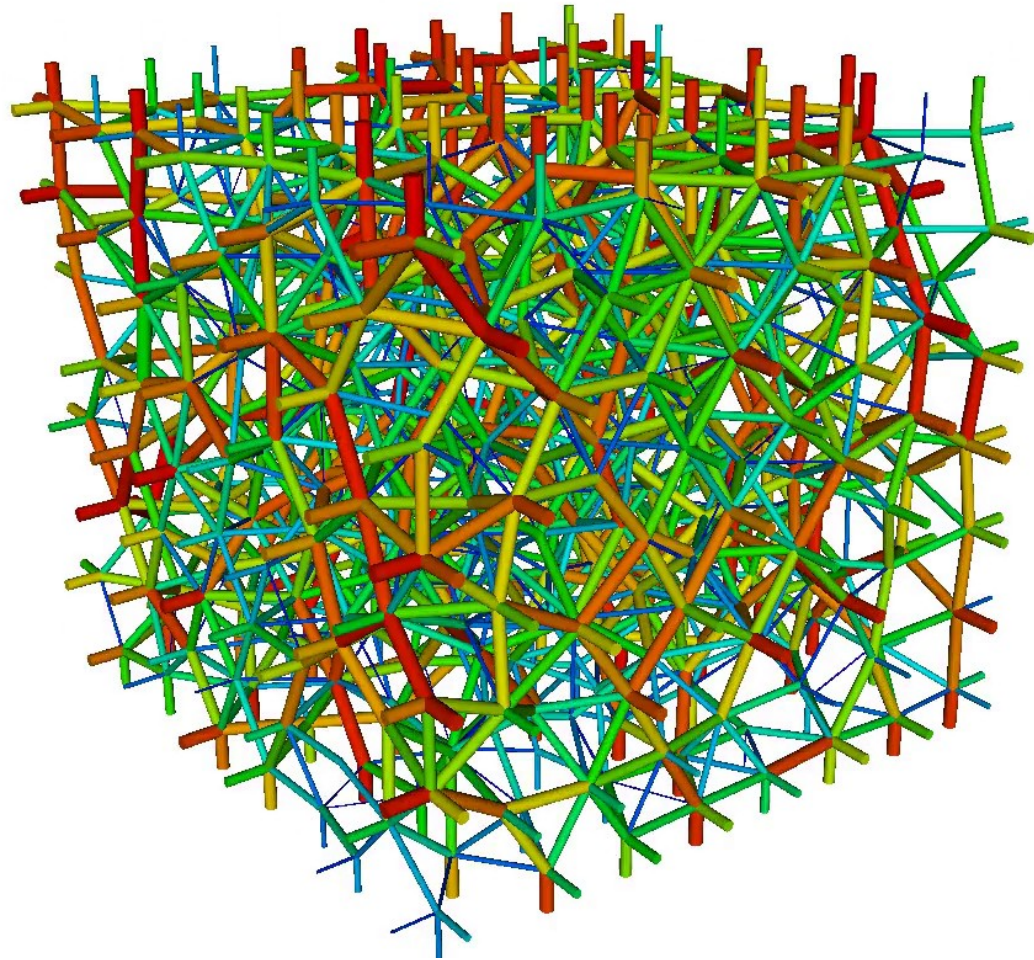
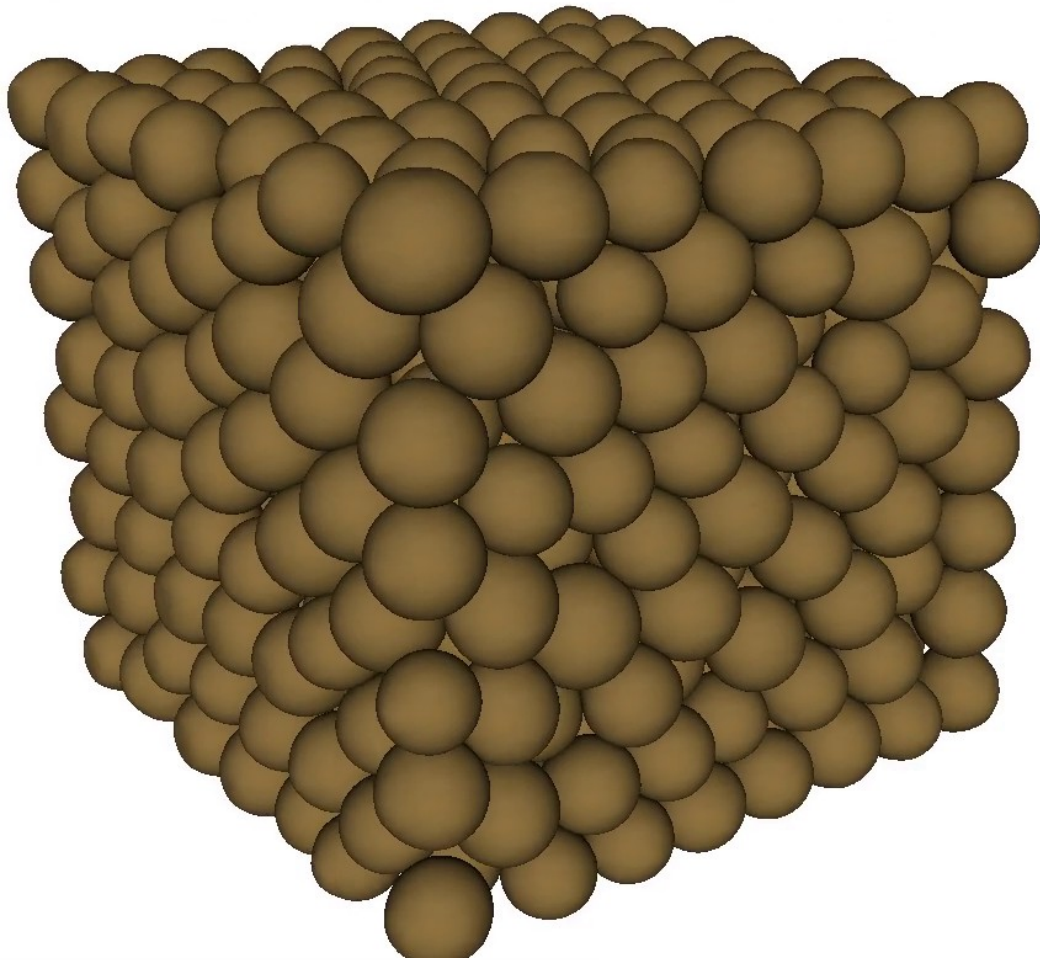
$F = E r^{1/2} \delta^{3/2}$ $E = \text{effective Young modulus}$

$F = E \delta a$ $a = \sqrt{A/\pi}$ radius of the contact

Hence $r \delta = A/\pi \Rightarrow \delta \Rightarrow F$ (with given E)

\Rightarrow Vector forces in full 3D, with orientation, position, norm
 + grain centers of mass, stress tensor, etc.

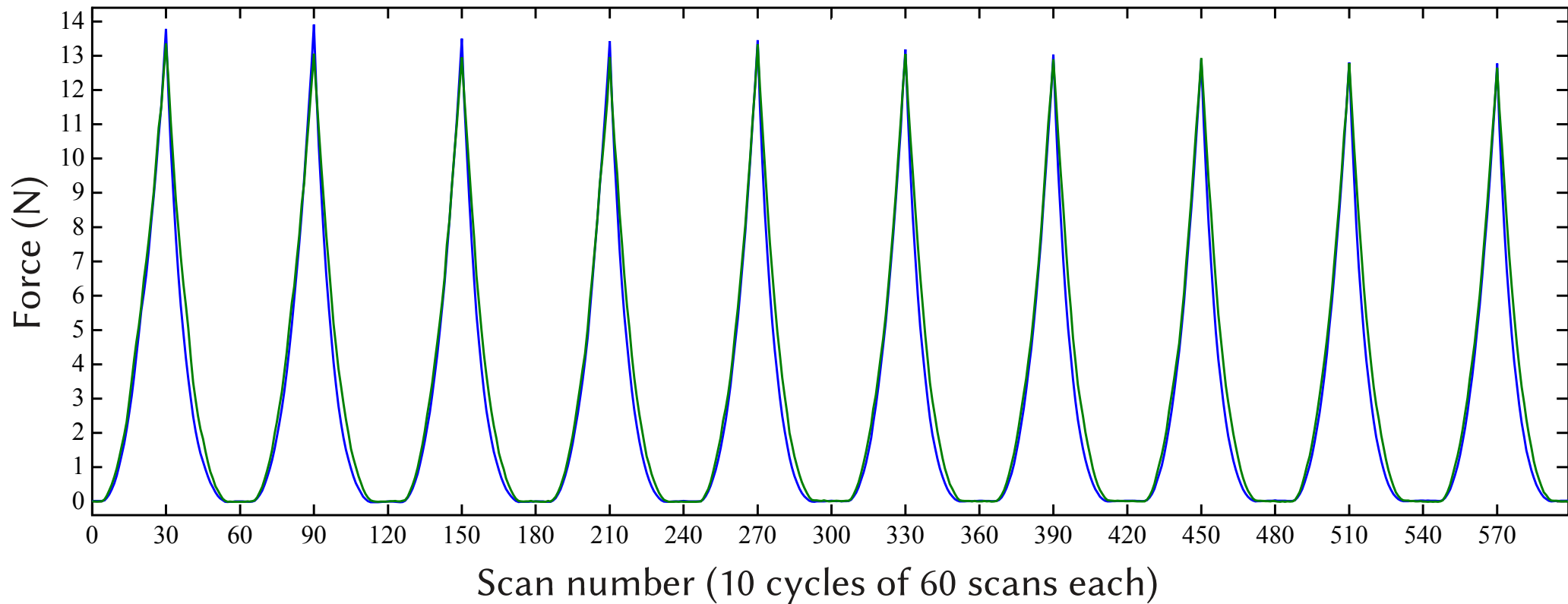
UNI-AXIAL COMPRESSION CYCLES



Top plate moves by 1mm increments
A full scan is taken between increments

Forces = struts joining the grain centers
Blue = weakest, Red = strongest

VALIDATION ON COMPRESSION CYCLES



Blue = force measured on the top plate sensor

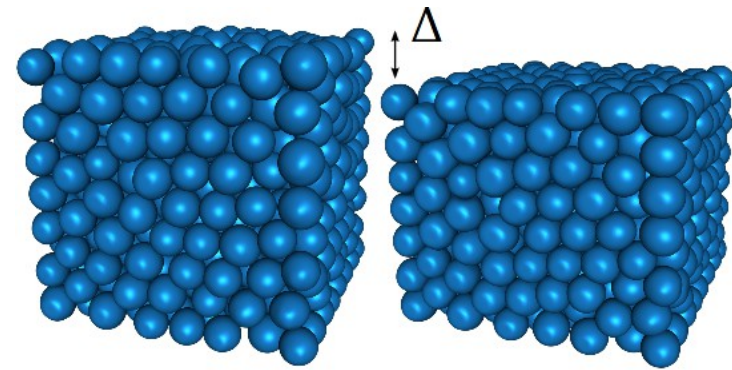
Green = force inferred from the images + measure of $E \approx 23$ kPa

Full experiment:

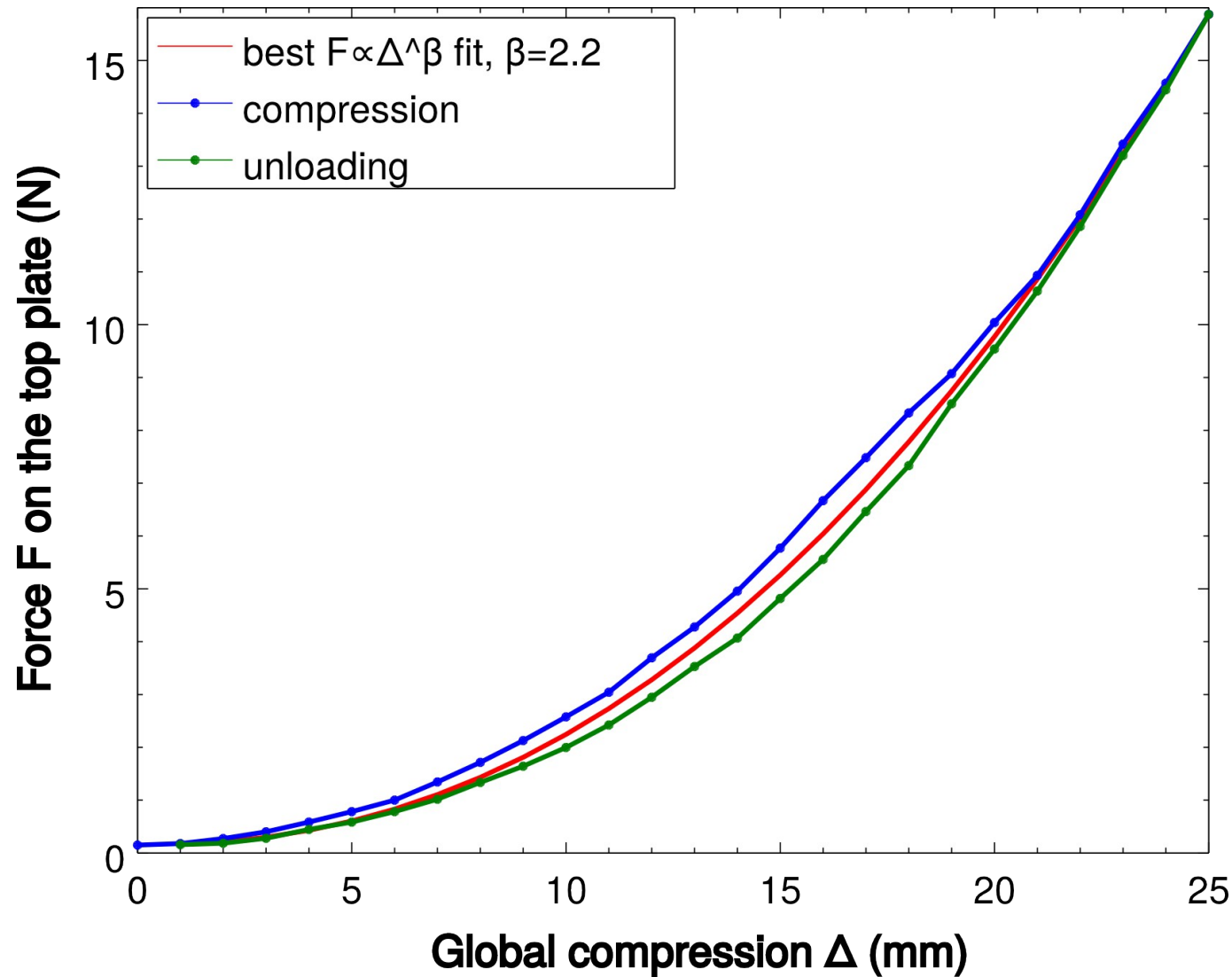
Maximal compression of $\Delta \approx 13\%$ the initial height

Average force at min compression $\approx 10^{-2}$ N

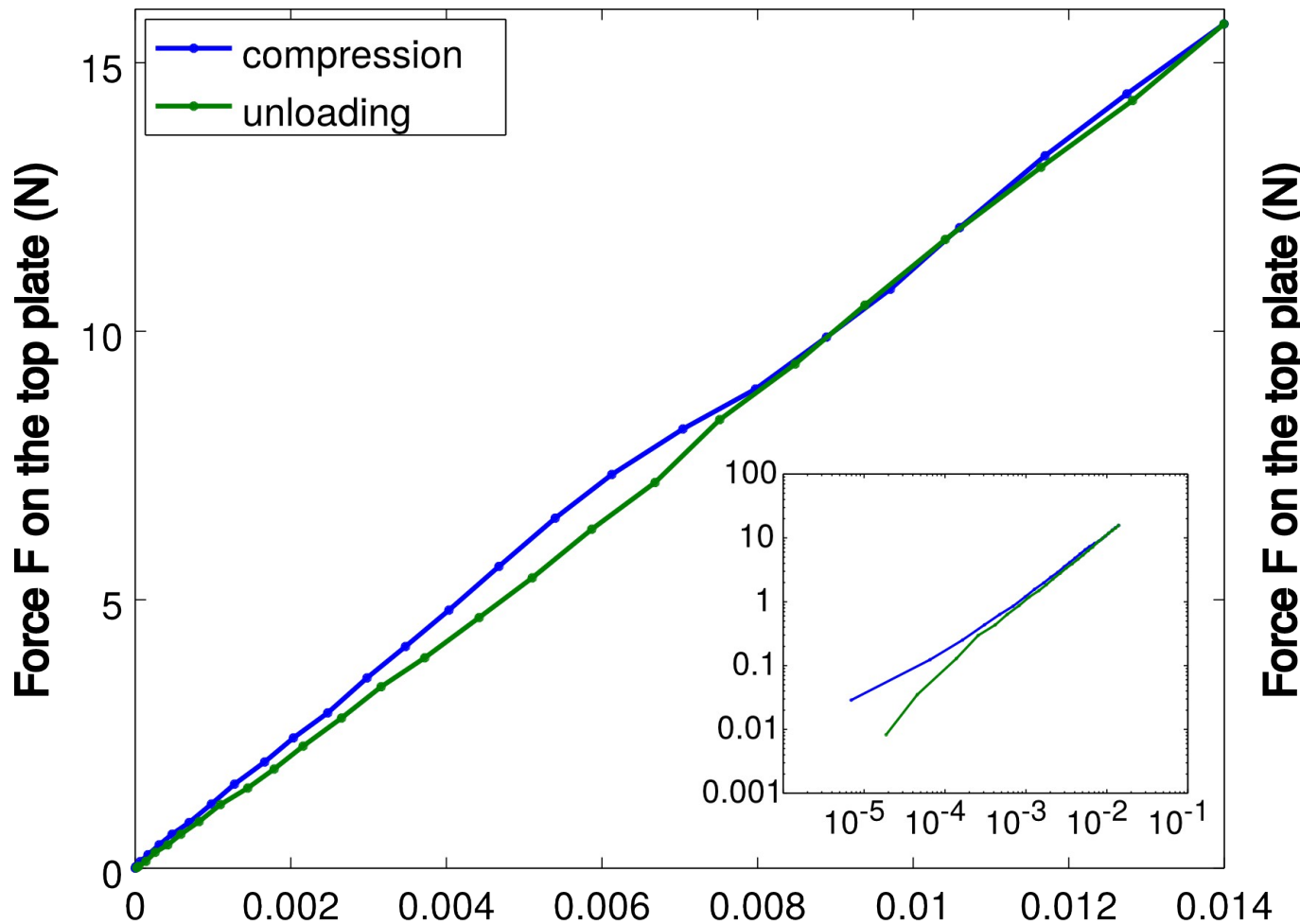
20 cycles = 1200 scans, >1M contacts (=good stats!).



NON-HERTZIAN PACKING RESPONSE



A SCALING HOLDS

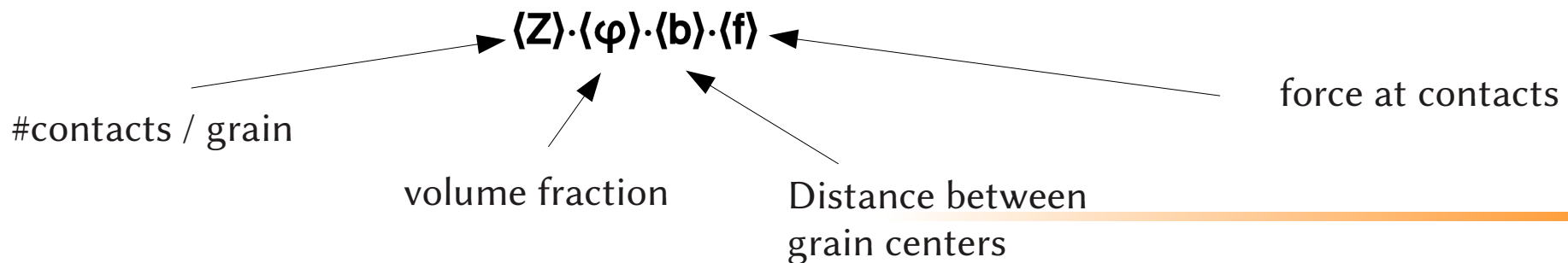


$$F \propto \Delta^\beta$$

$$f \propto \delta^{3/2}$$

Links Micro & Macro
 $\Rightarrow \Delta(\delta)$ relation

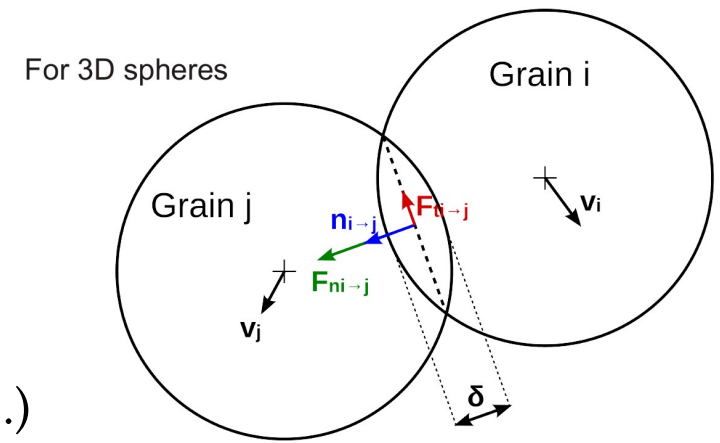
Source of the scaling
 = stress tensor
 (details on demand)



DEM OF THE “SAME” SYSTEM

Standard DEM simulation

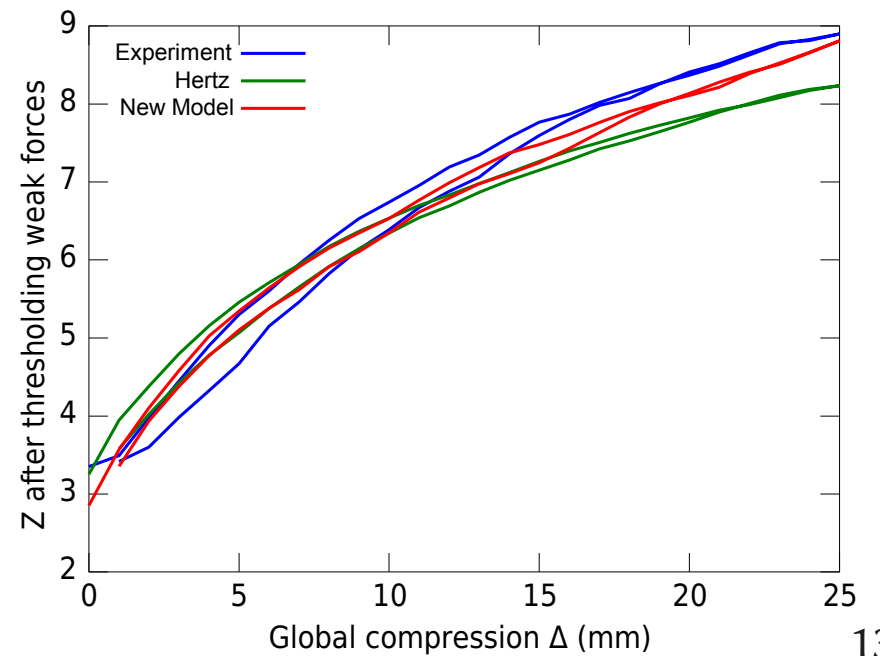
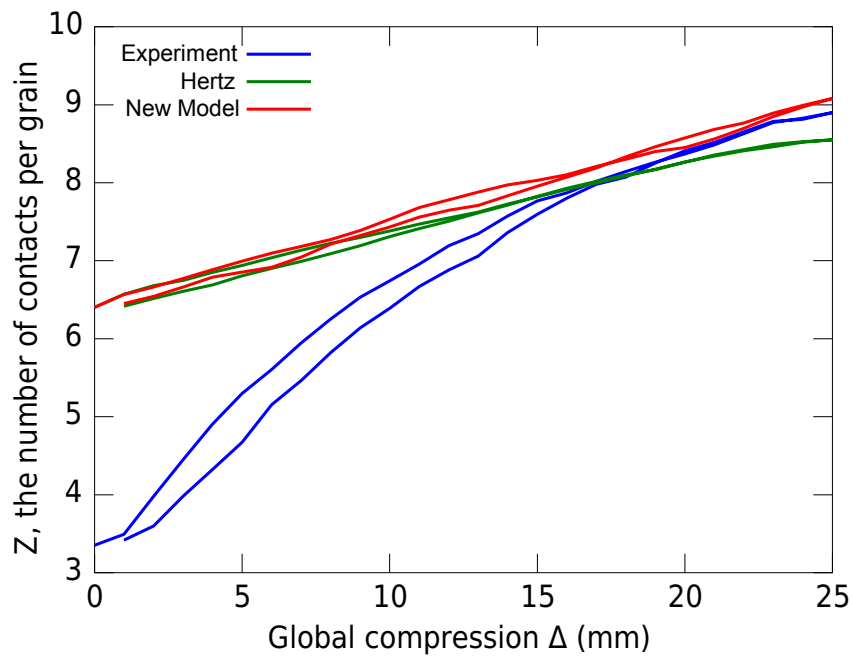
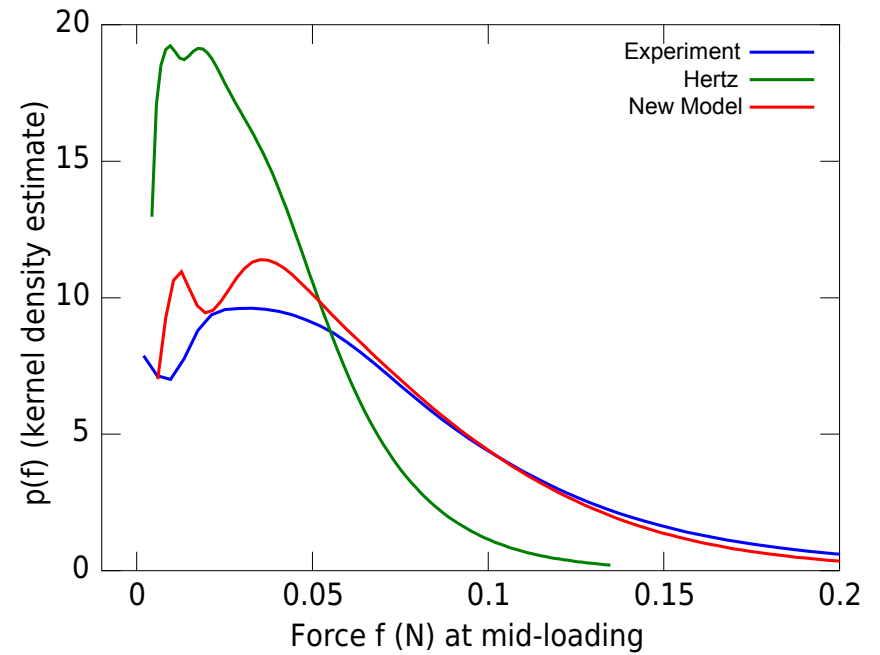
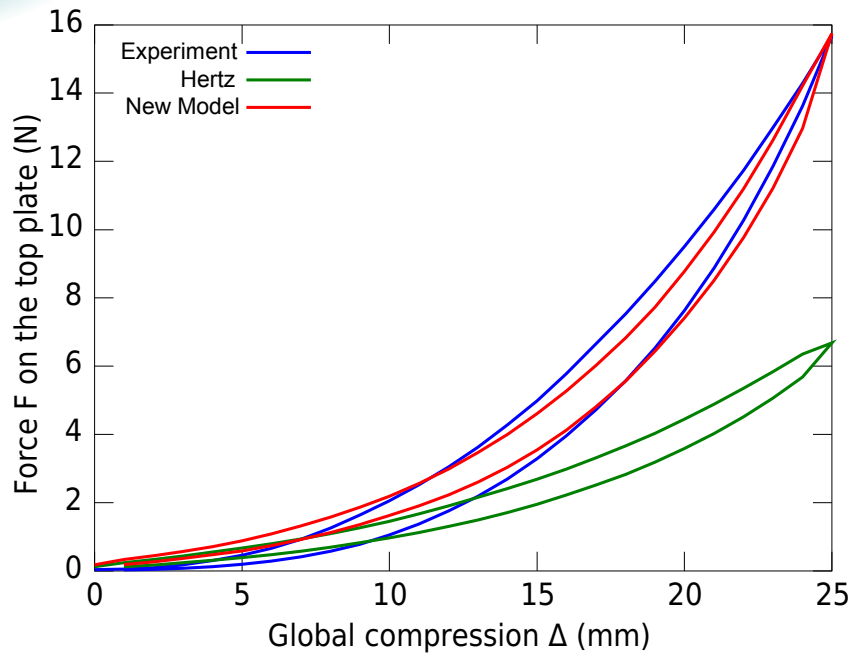
- Overlapping non-deformable spheres, overlap = δ
- + Force model $F = F(\delta)$ (either linear, Hertz, hysteretic...)
- + Solid Dynamics (friction, solid bodies)
- + Newton's law for integrating the new positions after an elementary dt
- = Trajectories of all particles + all contact properties



As close as we can get to the experimental setup

- Hertz + same material properties
- Grains are replaced by spheres of the same volume with the same center of mass
- Same box size and piston motion

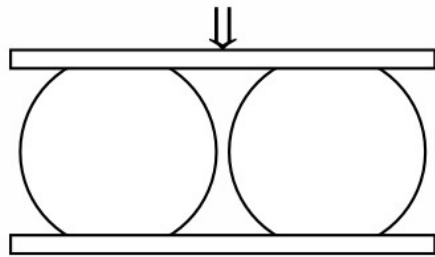
COMPARISON WITH DATA



DEALING WITH MULTIPLE CONTACTS

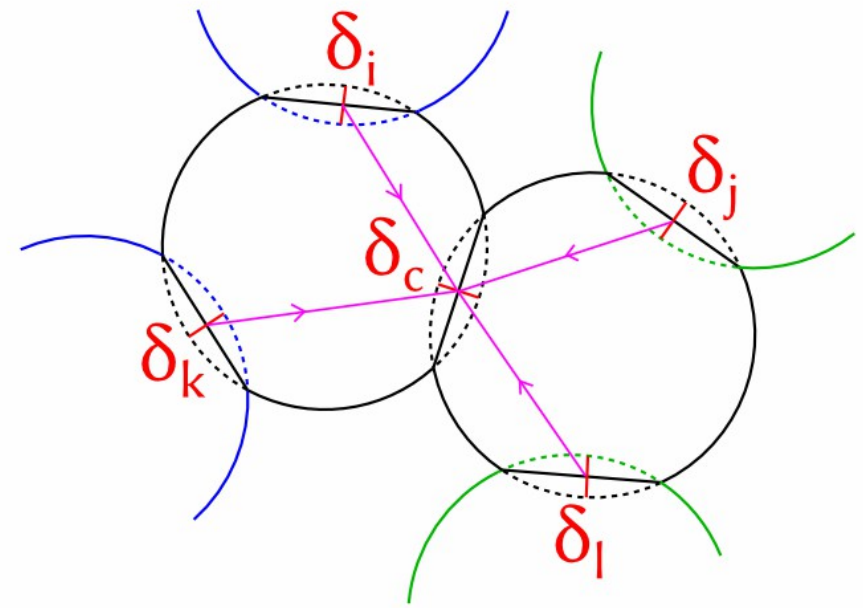
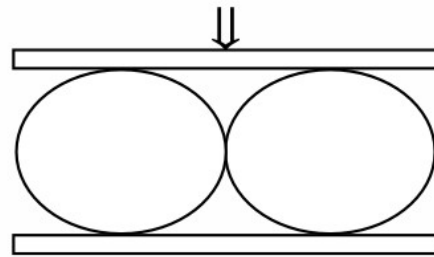
Usual DEM model

- Non-deformable spheres
- Independent contacts $F_c = f(\delta_c)$



The new model

- Surface deformations can create new contacts
- Multiple contacts $F_c = f(\delta_c, \delta_i, \delta_j, \delta_k, \delta_l)$



Correlation between contacts (ex [Gonzalez/Cuitiño])

- δ_c = the deformation at contact c.
- Other contacts i,j,k,l have an extra influence on c

New model

- A different expression for the correlations
- Surfaces are deformed **before** the grains touch: new contacts can be created
- **Backed up by data!**

INDUCED DISPLACEMENTS

Sphere-based ideas in the literature

- Point force on full sphere solution [Bondareva 1969]
- Gonzales / Cuitiño, approximating Zhapanska's solution for two spheres in contact

Problem:

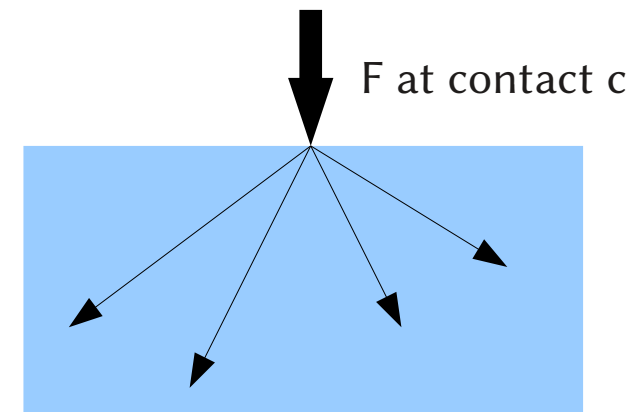
Dense granular material is more like an interconnected medium with pores

Very complex boundary (+inner) conditions.

DEM \Rightarrow Decoupling of the grains. Contact model = recoupling.

Here: Linear Elasticity on infinite half space

- Point force applied on elastic material
- Half-plane approximation = consistent with Hertz
- Geometric factor introduced in the model to compensate for the holes



Displacement δ induced by F at distance d in the continuous elastic medium

THE MODEL

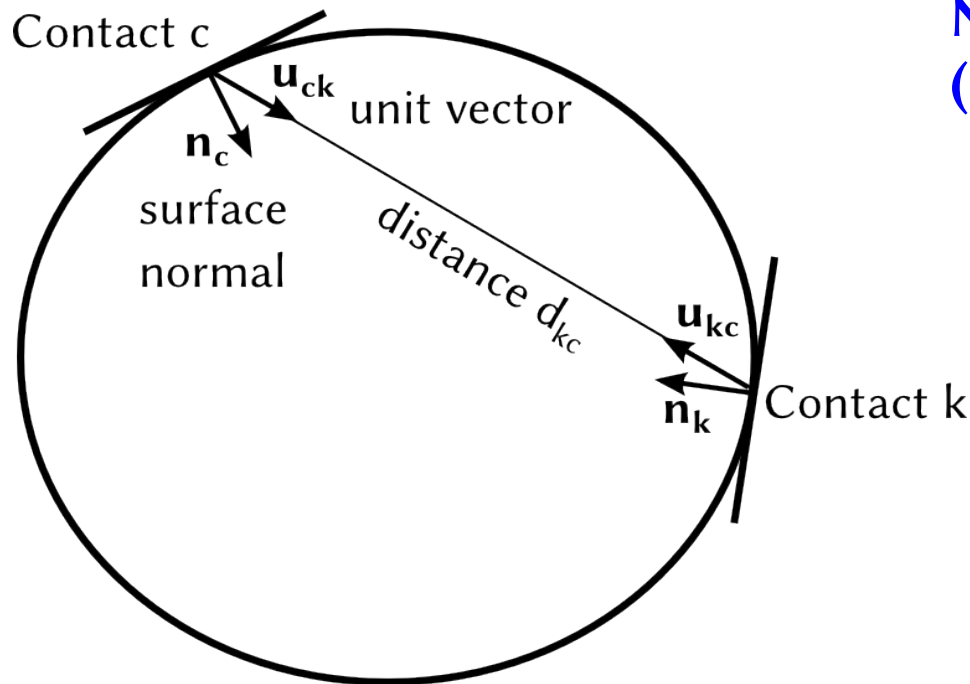
$$\delta_{k \rightarrow c} = -\frac{(1 + \nu)\delta_k \sqrt{\delta_k r_k}}{2\pi d_{kc}} \left\{ (\mathbf{n}_k \cdot \mathbf{u}_{kc}) (\mathbf{n}_c \cdot \mathbf{u}_{ck}) + (3 - 4\nu)\mathbf{n}_k \cdot \mathbf{n}_c - (1 - 2\nu) \frac{(\mathbf{n}_k + \mathbf{u}_{kc}) \cdot \mathbf{n}_c}{1 + \mathbf{n}_k \cdot \mathbf{u}_{kc}} \right\} \times \gamma$$

Linear Elasticity

$$F_c = E^* \sqrt{r_c^*} \left(\delta_L + \sum_{k \neq c} \delta_{k \rightarrow c} \right)^{3/2}$$

Geometric factor (to fit)
 $\gamma \approx 1.19$

Non-local contributions
 (same as in [Gonzales/Cuitiño])



Notes

- $\delta_{k \rightarrow c}$ has no dependency on E
 \Rightarrow hard grains OK close to jamming
- γ may change with the packing fraction... not done yet
- Only validated for $\nu = 0.5$

QUESTIONS?

