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ABSTRACT

This paper proposes a new method to detect damages in structures vibrating with nonlinear behavior. The approach proposed is based on Volterra series and can separate the linear and nonlinear contributions of the responses. An alert of damage is given based on level of contribution of the difference between the linear and nonlinear behavior identified using experimental time-series. Tests are performed involving a buckled clamped beam to illustrate the steps and the advantages of the approach to detect damages using vibrating in nonlinear regime of motion.

KEYWORDS : *Volterra series, nonlinear dynamics, damage detection, buckled beam.*

INTRODUCTION

Normally, the occurrence of structural damage is well associated with the nonlinear behavior, for instance, cracks, delamination, post-buckling, etc. [1, 2]. If the system is monitored for structural health using some linear metric, the detection of damage is obtained only by identifying nonlinear behavior in the response. However, nonlinearities in health conditions is a common practical situation due to the inherent nonlinear effects, as jump, gaps, super harmonics, cycle limit, discontinuities, beyond others that appear frequently in the responses of structures [3]. This behavior is caused mainly due to excitation condition, large displacement, geometric effects, nonlinear constitutive equations of the stress-strain, etc [1, 3, 4]. Thus, the conventional linear procedures for structural health monitoring can fail when the system monitored is highly nonlinear in the healthy state [5, 6].

In this context, the present paper propose a contribution to detect and quantify damages in nonlinear systems using Volterra series. Volterra series is a nonlinear representation using multiple convolutions that can separate the linear and nonlinear contributions [7–11]. To illustrate the results, an experimental application is performed in a buckled clamped beam with a thin beam connected. After identifying the discrete-time Volterra kernels in the reference condition (healthy state), the prediction errors from linear and nonlinear contributions are monitored. Damages are simulated by load applied in the joint point of the beam to change the stress. The structural integrity of the system, considering the inherent nonlinear behavior associated, is monitored using damage-sensitive metrics based on prediction errors computed by Volterra series. These indexes are also used to quantify the severity of the damage in the structure and to decide if the changes are caused by damage or nonlinear regime.

DAMAGE DETECTION USING DISCRETE-TIME VOLTERRA SERIES

The discrete-time Volterra series can represent appropriately the output $y(k)$ of nonlinear systems using a generalization of convolution [7, 12]:

$$y(k) = \sum_{\eta=1}^{\infty} \sum_{n_1=0}^{N_1} \ldots \sum_{n_\eta=0}^{N_\eta} \mathscr{H}_\eta(n_1,\ldots,n_\eta) \prod_{i=1}^{\eta} u(k-n_i) = y_1(k) + y_2(k) + y_3(k) + \cdots \qquad (1)$$

where $u(k)$ is the excitation signal, $\mathscr{H}_\eta(n_1,\ldots,n_\eta)$ are the Volterra kernel of η-th order, $y_1(k)$, $y_2(k)$ and $y_3(k)$ are the linear, quadratic and cubic components of $y(k)$, and so on. The most part of nonlinear systems with polynomials nonlinearities can been described with order $\eta = 3$.

A great drawback of Volterra series is the difficult to obtain convergence using a large number of terms $N_1,...,N_\eta$. Fortunately, the Volterra kernels expansion in any orthonormal basis can reduce this problem. In particular, to describe vibrating systems, the orthonormal basis $\psi_{i_j}(n_j)$ with Kautz functions can be useful [13–15]. Thus, the Volterra kernel can be given by:

$$\mathscr{H}_\eta(n_1,\ldots,n_\eta) \approx \sum_{i_1=1}^{J_1} \ldots \sum_{i_\eta=1}^{J_\eta} \mathscr{B}_\eta(i_1,\ldots,i_\eta) \prod_{j=1}^{\eta} \psi_{i_j}(n_j) \tag{2}$$

where $\mathscr{B}_\eta(i_1,\ldots,i_\eta)$ is the projection of Volterra kernels in the basis $\psi_{i_j}(n_j)$ of Kautz functions and $J_1,...,J_\eta$ is the number of samples used in the Volterra kernel projection. With this approach, Volterra series can be rewritten by multiple convolutions between orthonormal kernels $\mathscr{B}_\eta(i_1,\ldots,i_\eta)$ and the signal $l_{i_j}(k)$:

$$y(k) \approx \sum_{\eta=1}^{\infty} \sum_{i_1=1}^{J_1} \ldots \sum_{i_\eta=1}^{J_\eta} \mathscr{B}_\eta(i_1,\ldots,i_\eta) \prod_{j=1}^{\eta} l_{i_j}(k) \tag{3}$$

where $l_{i_j}(k)$ is the input signal $u(k)$ filtered by Kautz functions $\psi_{i_j}(n_j)$:

$$l_{i_j}(k) = \sum_{n_i=0}^{V-1} \psi_{i_j}(n_i) u(k-n_i) \tag{4}$$

where $V = \max\{J_1,\ldots,J_\eta\}$.

The terms of orthonormal kernels $\mathscr{B}_\eta(i_1,\ldots,i_\eta)$ can be now arranged in a vector $\boldsymbol{\Phi}$ and estimated by:

$$\boldsymbol{\Phi} = (\boldsymbol{\Gamma}^T\boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}^T\mathbf{y} \tag{5}$$

where the matrix $\boldsymbol{\Gamma}$ contains the input signal filtered $l_{i_j}(k)$ and the input $\mathbf{y} = [y(1)\cdots y(K)]$ with K the number of samples used. More details can be found in [8,9].

If a healthy state is known, the Volterra kernels can be identified and used as reference based on the output measured in this condition. So, the reference state can be estimated by:

$$y_{ref} \approx \sum_{\eta=1}^{3} \mathscr{B}_\eta(k) = \underbrace{y_{1,ref}}_{\text{linear}} + \underbrace{y_{2,ref} + y_{3,ref}}_{\text{nonlinear}} \tag{6}$$

where 3 kernels were considered. If we compare the prediction error between the experimental output y_{exp} with the reference estimated by Equation (6), given by:

$$e_{ref} = y_{exp} - y_{ref} \tag{7}$$

is expected that the statistical difference should be not significant. Now, if an unknown structural condition is measured, given by y_{exp}, the same Volterra model can be used to try to estimate it:

$$y_{unk} \approx \sum_{\eta=1}^{3} \mathscr{B}_\eta(k) = y_{1,unk} + y_{2,unk} + y_{3,unk} \tag{8}$$

If there is no damage in the system, the reference Volterra model can be able to estimate correctly the behavior. So, the prediction error:

$$e_{unk} = y_{exp} - y_{unk} \tag{9}$$

should be low and close to the reference error given by Equation (7). A feature index for damage detection can be extracted through [11,16]:

$$\lambda_\eta = \frac{\sigma(e_{\eta,unk})}{\sigma(e_{\eta,ref})}, \qquad \eta = 1,2,3 \tag{10}$$

where $\sigma(\circ)$ is the standard derivation operator. The prediction errors at reference condition, $e_{\eta,ref}$, and in unknown conditions, $e_{\eta,unk}$, are given by:

$$e_{\eta,ref} = y_{ref}^{exp} - \sum_{m=1}^{\eta} y_{m,ref}, \qquad \eta = 1,2,3 \tag{11}$$

$$e_{\eta,unk} = y_{unk}^{exp} - \sum_{m=1}^{\eta} y_{m,unk}, \qquad \eta = 1,2,3 \tag{12}$$

The prediction errors are residues that can consider the nonlinearities contained in the system depending on the η considered. This represents an evolution when compared as others methods based on residues using linear models, for example used by [17, 18].

APPLICATION IN A NONLINEAR HARDENING BEAM

Figure 1 shows the experimental setup used to illustrate the approach of damage detection proposed based on Volterra series. An aluminum beam with $300 \times 18 \times 3$ mm of length, width and thickness, respectively, is connected to a steel beam with $120 \times 18 \times 0.5$ mm of length, width and thickness, respectively. The excitation was applied using a shaker attached to the aluminum beam at 50 mm from clamped end. A preload is applied in the connection between the beams to include geometrical effects that cause nonlinear vibrating regime with hardening stiffness effect. The structure is monitored using six accelerometers and a laser vibrometer to measure the velocity in the joint between the beams. A force sensor is also used to measure the excitation force applied. All signals measured were sampled with 1024 Hz and 8192 samples.
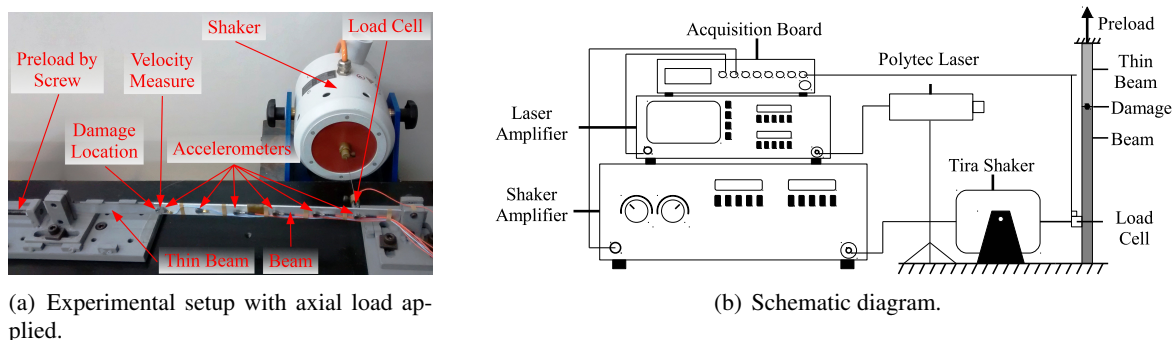

(a) Experimental setup with axial load applied.

(b) Schematic diagram.

Figure 1 : View of the experimental setup.

Detection of the nonlinearities

The first test was done using a chirp input applied during 4 seconds with three different levels of amplitude to excite the first mode range from 20 to 100 Hz. Figure 2(a) shows the frequency response function (FRF) plots. Clearly the FRF presents some changes caused by nonlinear effects and drop out in function of level of force applied by the shaker. A controlled stepped sine test was also performed to show the jump of the frequency response when the level of force amplitude is high, Figure 2(b). These results show clearly that the structure contains nonlinear regime associated with hardening stiffness.

Identification of the Volterra kernels

The identification of the Volterra kernels is performed using the chirp input signal in two steps [10]. In the first step is used the low amplitude to identify the first Volterra kernel $\mathscr{H}_1(n_1)$ that corresponds to linear contribution. In the next step is applied a high level of force amplitude to excite nonlinearities
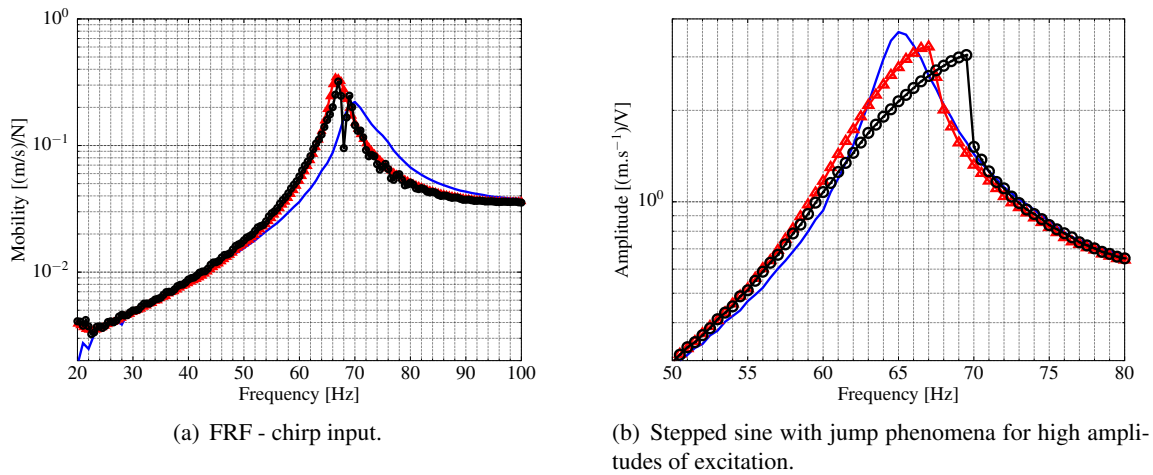
(a) FRF - chirp input.

(b) Stepped sine with jump phenomena for high amplitudes of excitation.

Figure 2 : The continuous line $-$ represents an amplitude of 0.01 V applied in the signal generator, the line with $\triangle$ is the medium amplitude (0.10 V), and the line with $\circ$ is the high level of amplitude (0.20 V).

and to extract the high order Volterra kernels $\mathscr{H}_2(n_1, n_2)$ and $\mathscr{H}_3(n_1, n_2, n_3)$. All parameters in the Kautz functions are used based on optimization procedure [8, 10].

The Volterra model identified is validated using a sinusoidal excitation with frequency close to the first natural. Figure 3(a) shows a good concordance between the power spectral density (PSD) of the output experimental $y(k)$ and the output estimated by the Volterra models. Figure 3(b) shows the spectral contribution of each kernel in the total frequency response. The contribution of the second kernel is essentially caused by shaker-structure interaction.
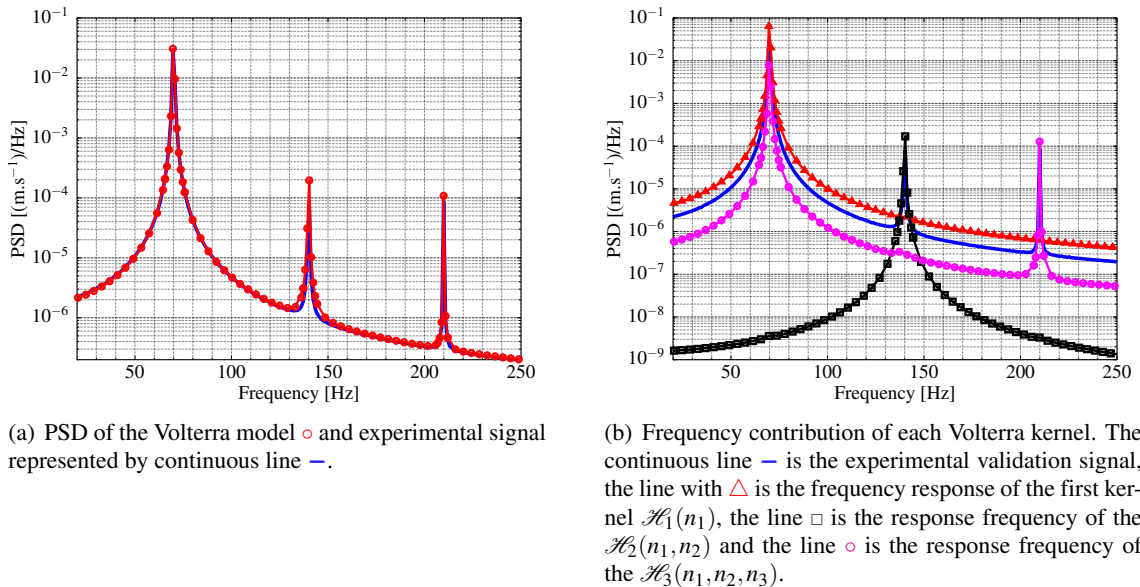
(a) PSD of the Volterra model $\circ$ and experimental signal represented by continuous line $-$.

(b) Frequency contribution of each Volterra kernel. The continuous line $-$ is the experimental validation signal, the line with $\triangle$ is the frequency response of the first kernel $\mathscr{H}_1(n_1)$, the line $\square$ is the response frequency of the $\mathscr{H}_2(n_1, n_2)$ and the line $\circ$ is the response frequency of the $\mathscr{H}_3(n_1, n_2, n_3)$.

Figure 3 : PSD of the Volterra model and experimental validation output when is applied a high amplitude level (0.20 V).

Figure 4(a) shows the output obtained by the multiple convolution computed using the Volterra kernels $\mathscr{H}_1(n_1)$ and $\mathscr{H}_3(n_1, n_2, n_3)$. The second kernel $\mathscr{H}_2(n_1, n_2)$ is not used because the experimental response is perfectly symmetric and the importance of these kernel is due to shaker-structure interaction. The prediction error computed by Equation (7) is shown in Figure 4(b).

Figure 5 shows the linear contribution $y_1(k)$ obtained by the convolution using the first kernel
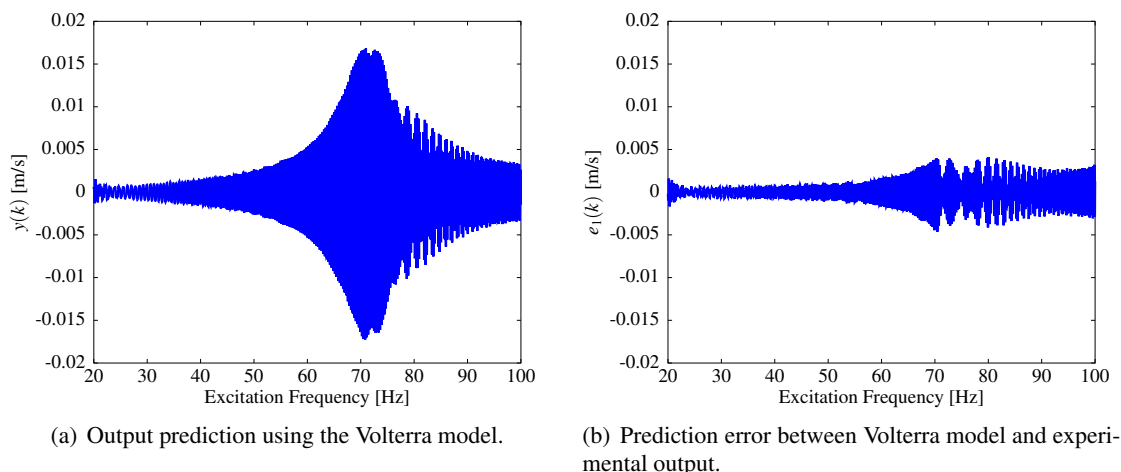
(a) Output prediction using the Volterra model.

(b) Prediction error between Volterra model and experimental output.

Figure 4 : Healthy condition with low excitation amplitude (0.01 V) - linear vibration regime.

$\mathscr{H}_1(n_1)$ and the cubic contribution $y_3(k)$ computed by the third kernel $\mathscr{H}_3(n_1, n_2, n_3)$. It is worth noting that the nonlinear contribution is low due to the low level of amplitude that well characterize linear vibrating regime.

(a) Linear contribution obtained by convolution with $\mathscr{H}_1(n_1)$.

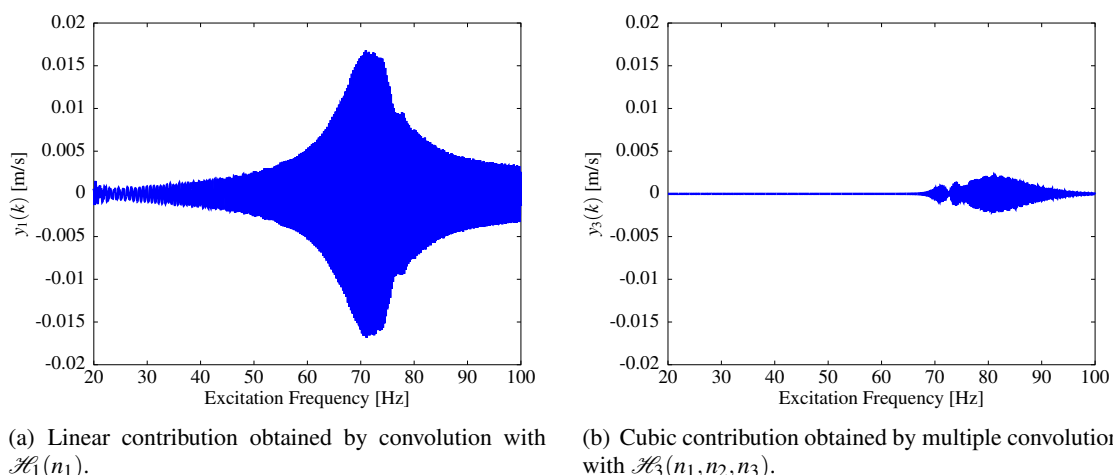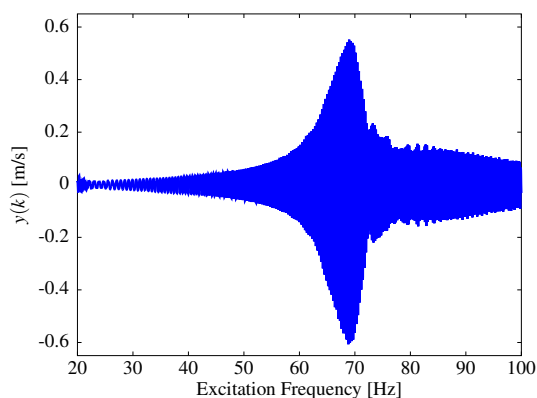(b) Cubic contribution obtained by multiple convolution with $\mathscr{H}_3(n_1, n_2, n_3)$.

Figure 5 : Linear and nonlinear (cubic) contribution of the response in healthy condition considering the low level of amplitude (0.01 V).

When the amplitude applied in the signal generator is increased, 0.20 V, the nonlinear vibrating regime is reached and the nonlinear contribution $y_3(k)$ has importance in the total response $y(k)$ of the Figure 6. Figure 7 presents the contribution of $y_1(k)$ and $y_3(k)$ for this condition. Now the cubic contribution is relevant comparing the total response $y(k)$ because the nonlinear behavior is mandatory.

Damage detection

If we compare the healthy condition in linear vibration regime, Figure 4(a), with the healthy condition in nonlinear vibration regime, Figure 6, we can observe a difference that can be associated wrongly with damage. The procedure proposed in the present paper can help to overcome this drawback.

Table 1 presents the structural changes simulated from load applied in the joint between the thin beam with the main beam. The load applied was measured using a torque wrench. For each case of Table 1 the data were measured using the chirp excitation with three levels of amplitude excitation in the same conditions obtained in the healthy condition. The estimated output obtained by Equation (8)
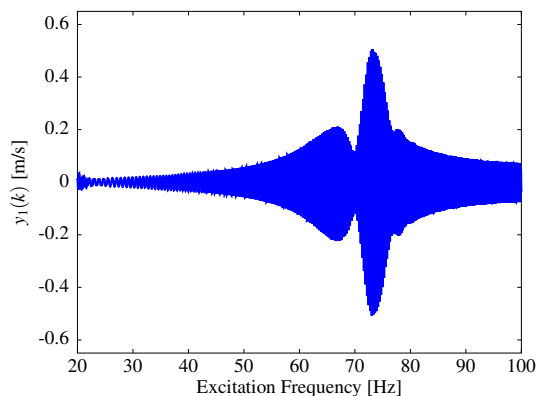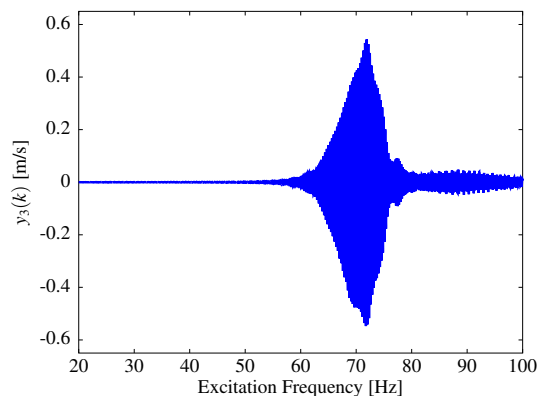
(a) Output prediction using the Volterra model.
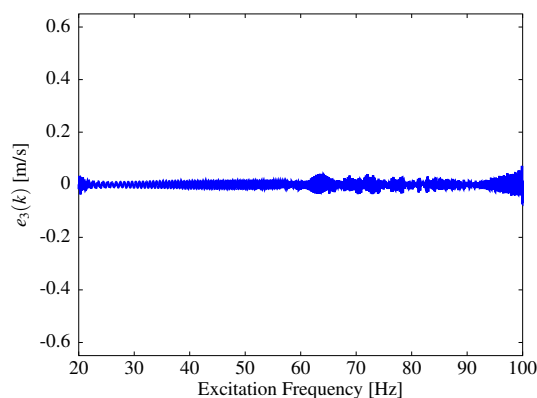
(b) Prediction error between Volterra model and experimental output.

Figure 6 : Healthy condition with high excitation amplitude (0.20 V) - nonlinear vibration regime.

(a) Linear contribution obtained by convolution with $\mathscr{H}_1(n_1)$.

(b) Cubic contribution obtained by multiple convolution with $\mathscr{H}_3(n_1, n_2, n_3)$.

Figure 7 : Linear and nonlinear (cubic) contribution of the response in healthy condition considering the high level of amplitude (0.20 V).

were computed and the indexes λ_η calculated by Equation (10) were evaluated using the prediction errors from Equation (9).

Table 1 : Structural condition simulated.

Structural Condition	Load [N]	Structural Condition	Load [N]
0	0	IV	4
I	1	V	5
II	2	R	0
III	3	-	-

Figure 8 shows the indexes to three amplitude levels. Figure 8(a) shows the results when $\eta = 1$ in Equation (10). If the level of amplitude is low, only the linear contribution $y_1(k)$ is relevant and the linear index can associate the change in the output with structural change. However, if the level of

amplitude is high, the nonlinear contribution $y_3(k)$ is relevant and does not appear in the linear index λ_1. Thus, if the level of amplitude is high the index λ_1 will fail. Figure 8(b) presents when $\eta = 2$ in Equation (10) and the results is similar to the case when $\eta = 1$ because the quadratic contribution is low and the index λ_2 considers the levels $y_1(k)$ and $y_2(k)$.

The unique index that is able to detect structural changes in all conditions is the λ_3 because when $\eta = 3$ we can consider the prediction error considering the sum of all linear and nonlinear contributions in the total response. Figure 8(c) shows clearly this condition when the structural changes are detected when the system presents nonlinear regime of vibration motion. Figure 8(d) illustrates a zoom to see that when the level of amplitude is low, the cubic index can also detect the structural changes because the third Volterra kernel has contribution of the first harmonic, as seen in Figure 3(b).
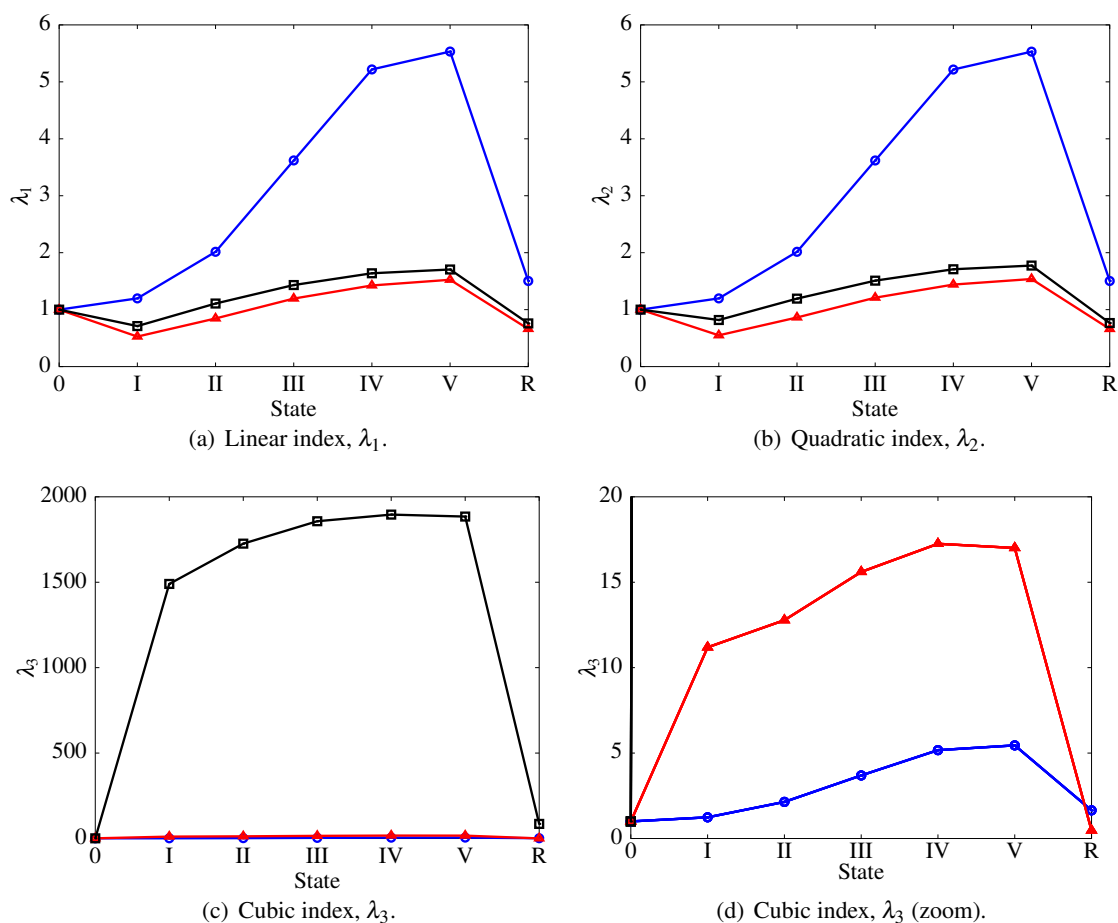

(a) Linear index, λ_1.

(b) Quadratic index, λ_2.

(c) Cubic index, λ_3.

(d) Cubic index, λ_3 (zoom).

Figure 8 : Damage indexes for three different amplitude levels. The symbol $\circ$ corresponds to the low amplitude (0.01 V), $\triangle$ corresponds to medium amplitude (0.10 V) and $\square$ corresponds to high amplitude (0.20 V).

FINAL REMARKS

The results obtained have shown that the use of discrete-time Volterra series can be useful to separate the linear and nonlinear contributions of the output. Additionally, the damage sensitive-index proposed, λ_3, considering the high order Volterra kernels contribution is much more efficient to detect structural changes when the structure vibrates in nonlinear regime.

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