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## ROBUST VIRTUAL DYNAMIC STRAIN SENSORS FROM ACCELERATION MEASUREMENTS

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### ABSTRACT

The possibility to deduce dynamic strains from acceleration measurements is investigated in this paper. The classic technique applied on beam-like structures which consists in using vertical accelerations to estimate longitudinal strains with a second order central finite difference is compared with two methods based on horizontal accelerations. Time domain responses of accelerometers and strain sensors installed on a simply supported beam are simulated, and the strains deduced from accelerations are compared with the real strains. The possibility of locating damage with the mode shape obtained with estimated strains is also addressed.

**KEYWORDS :** *Dynamic strain measurements, finite difference, acceleration sensors, damage localization, strain mode shapes.*

### INTRODUCTION

The use of dynamic strain measurements has been increasing in the last decades, especially in the structural health monitoring community since these quantities have been demonstrated to be locally sensitive to damage [1]. Different types of dynamic strain sensors are available on the market such as strain gages, fiber optics [2] or piezoelectric transducers. While very useful in many applications, these technologies still suffer from several drawbacks such as high cost of interrogation units (fiber optics), insufficient resolution for ambient vibration measurements (typically 1 microstrain for strain gages and fiber optics), possible need of charge amplifiers (piezoelectric sensors), as well as the difficulty to install these sensors and to cover continuously the structures. If dynamic strain sensors are not available, a classic approach consists in measuring vertical accelerations on beam-like structures from which longitudinal strains or curvatures can be deduced by applying a second order finite difference [3, 4]. This method has been widely applied because accelerometers are very easy to install and relatively cheap, but this approach is very sensitive to the spacing between the accelerometers, as well as to the noise on measurements [5]. In this paper, we investigate the use of horizontal accelerations to deduce dynamic strains. Two very simple techniques which can be applied on any kind of structure are proposed and tested on a simply supported beam. Based on time domain simulated sensor responses, we show that strains deduced from horizontal accelerations are very close to the real strain sensor responses, and that they are much more robust with respect to noise measurements than strains deduced from vertical accelerations. We also investigate the possibility to locate damage based on the mode shapes identified from the strains deduced from horizontal accelerations.

### STRAIN DEDUCED FROM ACCELERATIONS

In the present section, we present three techniques that will be compared to estimate strains from displacements (double integrated accelerations). For simplicity, we will assume a regular grid of horizontal and vertical displacement sensors as shown in Figure 1.

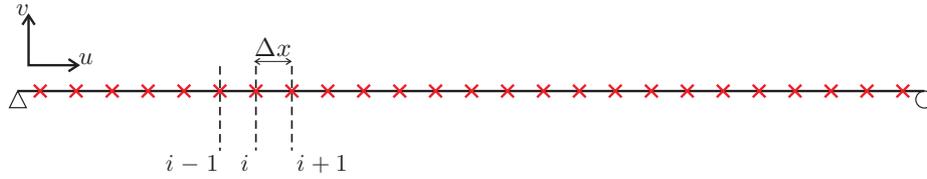


Figure 1: Regular grid of horizontal and vertical displacement sensors installed on a simply supported beam.

**Finite difference**

A classic method which has been applied in many experimental research studies such as in [4] consists in estimating the strains from vertical displacements. Indeed, in beam-like structures, the strain  $\epsilon$  at height  $y$  can be derived from vertical displacements  $v$  (double integrated vertical accelerations) as follows:

$$\epsilon(x)|_y = y \frac{d^2v}{dx^2}, \tag{1}$$

Therefore, a central difference scheme (usually second order central finite difference) can be used to estimate the strains. If we assume a regular spacing between measurements sites,  $\epsilon(x)|_y$  can be approximated as follows:

$$\epsilon(x)|_y \approx y \frac{v_{i+1} - 2v_i + v_{i-1}}{\Delta x^2} = \epsilon_{FDv} \tag{2}$$

This approximation of  $\epsilon(x)|_y$  is obtained from a Taylor series expansion of  $v_i$ , and it can be shown that the truncation error is  $O(\Delta x^2)$  [6]. Note that it is possible to decrease the truncation error by involving more sensors in the estimate of  $\epsilon(x)|_y$ . For instance, Chapra and Canale consider in [7] sensors from  $i - 2$  to  $i + 2$ , leading to a truncation error of  $O(\Delta x^4)$ . While this technique can be extended to plate-like structures such as in [8], it cannot be applied with complex structures for which the geometry differs strongly from a beam or a plate.

An alternative which has been less often investigated is based directly on the definition of strains. By definition, the strain in direction  $x_i$  is the first derivative of displacement in direction  $x_i$  with respect to the same direction. Anew, a Taylor series expansion allows to approximate the strain between measurement sites  $i$  and  $i + 1$ :

$$\epsilon(x)|_y \approx \frac{u_{i+1}|_y - u_i}{\Delta x} = \epsilon_{FDu}, \tag{3}$$

and the truncation error is  $O(\Delta x^3)$ . As for the estimate of strains based on vertical displacements, it is possible to take into account more sensors in order to decrease the truncation error.

**Virtual dynamic strain sensors**

In the new technique proposed in this paper to estimate strains from horizontal displacements, we propose to decrease exponentially the influence of a sensor as its distance with the location of the strain of interest increases. This is motivated by the fact that the displacements and strains in the same direction are only dependent in the close vicinity of the strain of interest. As Peerlings which proposes a nonlocal and gradient-enhanced damage in [9], the idea is to take into account more sensors in the vicinity of the area of interest. For a constant spacing  $\Delta x$  between  $N$  horizontal displacement sensors, the virtual dynamic strain sensor is therefore defined as follows:

$$\varepsilon_x \approx \frac{\sum_{j=1}^i -e^{j-i} u_j + \sum_{j=i+1}^N e^{i+1-j} u_j}{\underbrace{\left( \sum_{j=1}^i e^{j-i} + \sum_{j=i+1}^N e^{i+1-j} - 1 \right) \Delta x}_{\text{scaling factor}}} = \varepsilon_V \quad (4)$$

It can be seen that a scaling factor has been introduced at the denominator in order to adjust the level of strains computed. It is worth mentioning that if only two displacement sensors  $i$  and  $i + 1$  are taken into account, the virtual strain reduces to the first order finite difference approximation given by Equation (3). Finally, the definition of virtual dynamic strains given by Equation (4) can be very easily extended to a network of sensors with an arbitrary (non-constant) spacing  $\Delta x$ , but it is not discussed in the present paper.

It can be interesting to compare the coefficients weighting the horizontal displacement sensor responses computed with the virtual strains technique and with a generalized finite difference that can be built from a Taylor series development (not detailed here) as in [10]. Figure 2 plots these coefficients for two different positions of the strain that has to be estimated for a unitary spacing between the measurement sites ( $\Delta x = 1m$ ):

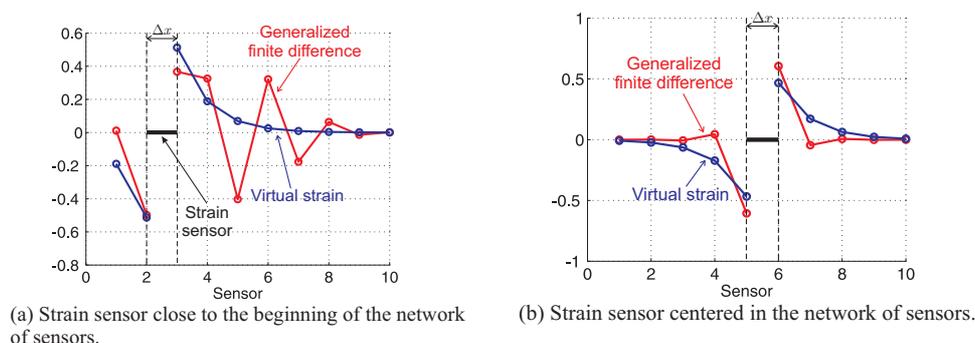


Figure 2: Comparison of generalized finite difference and virtual strains coefficients ( $\Delta x = 1m$ ).

While the coefficients computed with the virtual strains are all negative on the left of the strain to be estimated and all positive on its right, the generalized finite difference coefficients alternate in sign. The coefficients of the generalized finite difference depend also on the position of the strain to be estimated in the network of sensors. In particular, Figure 2(a) show that sensors far from the location of the strain can have a strong contribution, which is physically not relevant.

Note that the three techniques which have been described in the present section can be used directly with accelerations. In this case, one has to consider the second derivative with respect to time of  $u$ ,  $v$  and  $\varepsilon$  in the previous definitions. This allows to introduce a new type of sensors which is the “acceleration strains”  $\ddot{\varepsilon} = \frac{d^2\varepsilon}{dt^2}$ . That kind of sensors has not yet been investigated in the literature since such sensors cannot be found on the market, but they are very easy to obtain numerically.

## NUMERICAL CASE STUDY

### Case study description

In order to investigate the performances of the different techniques described in the previous section for inferring strains from accelerations, we consider a  $1m \times 0.05m \times 0.05m$  simply supported beam made of steel and modeled with 102 Euler-Bernoulli beam elements using the *Structural Dynamic Toolbox* under *Matlab* [11]. A vertical force at  $x = 0.235m$  with band-limited white noise between 0Hz and 4000Hz is applied. Using an in-house numerical simulator which is applying a time integration scheme

based on the Duhamel’s formula [12], and which has already been used in [13], we compute the time histories of a network of 100 vertical and horizontal acceleration sensors placed on the top surface of the beam as illustrated in Figure 3. The exact strains are also computed based on the rotational degrees of freedom, and each measurement lasts for 5 seconds with a sampling rate of 8000Hz.

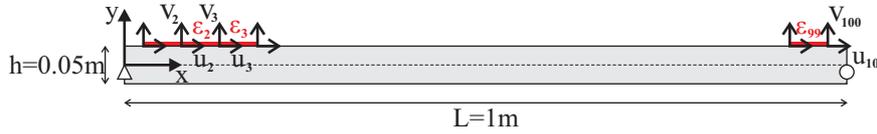


Figure 3: Network of sensors: 100 horizontal acceleration sensors, 100 vertical acceleration sensors and 99 longitudinal strain sensors.

Noise based on Equation (5) will be introduced in order to assess the robustness of each technique described in the previous section:

$$y_n(t) = y_n^0(t) + \underbrace{\beta \lambda \max_{[0s,5s]}(y_n^0(t))}_{noise}, \tag{5}$$

where  $y_n(t)$  and  $y_n^0(t)$  are the noisy and non-noisy responses of sensor  $n$ .  $\lambda$  is the random parameter, with its continuous distribution  $f(\lambda)$  following a Gaussian distribution with zero mean and unitary standard deviation as it is usually assumed [14].

**Comparison of time histories and PSDs**

Before applying the techniques to estimate the strains, we verify the levels of sensor responses (standard deviation  $\sigma$  of the 5 seconds non-noisy time histories) for the three types of sensors that will be used: horizontal accelerations ( $\ddot{u}^0$  in  $m/s^2$ ), vertical accelerations ( $\ddot{v}^0$  in  $m/s^2$ ) and the longitudinal (exact) acceleration strains ( $\ddot{\epsilon}^0$  in  $s^{-2}$ ). Figure 4 shows the evolution of the level of sensor responses for an increasing thickness  $h$  of the beam (the curves corresponding to the 0.05m thick beam studied in this paper are highlighted).

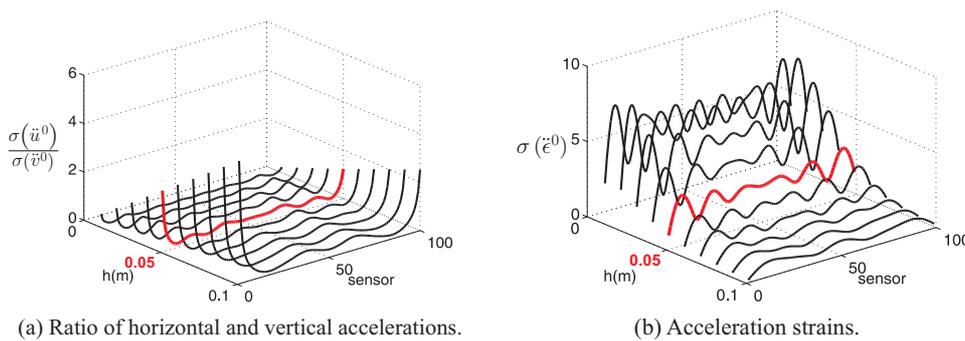


Figure 4: Levels of sensor responses.

While the vertical accelerations are always bigger than horizontal accelerations, the relative level of the later with respect to the former increases when the beam becomes thicker. On the other hand, the level of real acceleration strains decreases as  $h$  increases. In the present application, the use of horizontal accelerations is justified since horizontal accelerations have a comparable levels of response than vertical accelerations. It is also worth mentioning that horizontal accelerations are preferable close to the boundaries, at which the vertical accelerations tend to zero.

We plot in Figures 5 and 6 the acceleration strains estimated from accelerations with the three techniques previously described and the real acceleration strains for two different networks of accelerometers. The level of noise on the accelerometers responses is fixed to  $\beta = 1\%$ , and the results are compared with non-noisy real acceleration strains  $\ddot{\epsilon}^0$ . The acceleration measurement sites and the position of strains of interested are given above the time histories.

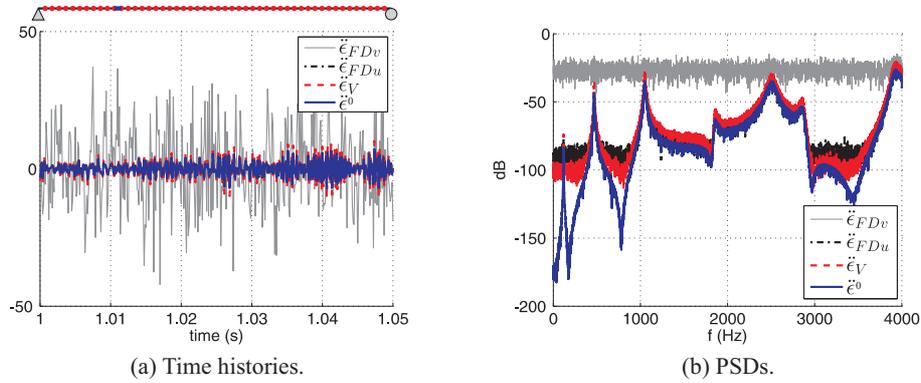


Figure 5: Noisy estimated acceleration strains and non-noisy acceleration strains ( $\Delta x = 0.0392m$ ).

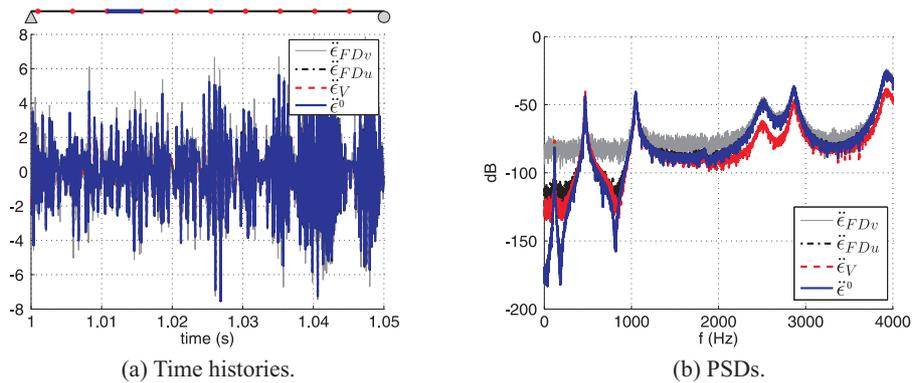


Figure 6: Noisy estimated acceleration strains and non-noisy acceleration strains ( $\Delta x = 0.098m$ ).

From these figures, we observe that the strains deduced from vertical accelerations are very sensitive to measurement noise as it was discussed previously. In particular, for a very dense network of vertical acceleration sensors, the very flat PSD of  $\ddot{\epsilon}_{FDv}$  shown in Figure 5(b) illustrates that the response obtained is completely masked by the noise. The strains deduced from horizontal accelerations are much closer to real strains, and much more robust with respect to noise measurement. Based on the PSDs, we can see that  $\ddot{\epsilon}_{FDu}$  matches very well the real strain  $\ddot{\epsilon}^0$  in all the frequency band, whatever the density of accelerometers network. On the other hand,  $\ddot{\epsilon}_V$  deviates from  $\ddot{\epsilon}^0$  at high frequencies as the distance between acceleration measurements sites increases. However, it seems that virtual strains are more interesting at low frequencies for which the PSDs are closer to the PSDs of  $\ddot{\epsilon}^0$ .

Because it is particularly difficult to assess the actual level of noise on real measurements, we have investigated the sensitivity of each method with respect to an increasing level of noise, from  $\beta = 0\%$  to  $\beta = 20\%$ . In order to quantify the quality of estimated strains, we compute the RMS value over the 5 seconds of measurements of the error  $e = \ddot{\epsilon} - \ddot{\epsilon}^0$  between the estimated strains and the real non-noisy strains.

Figure 7 shows the evolution of the RMS value of the error made on the estimate of acceleration strain with the three techniques for the same sensors than Figure 5 to 6.

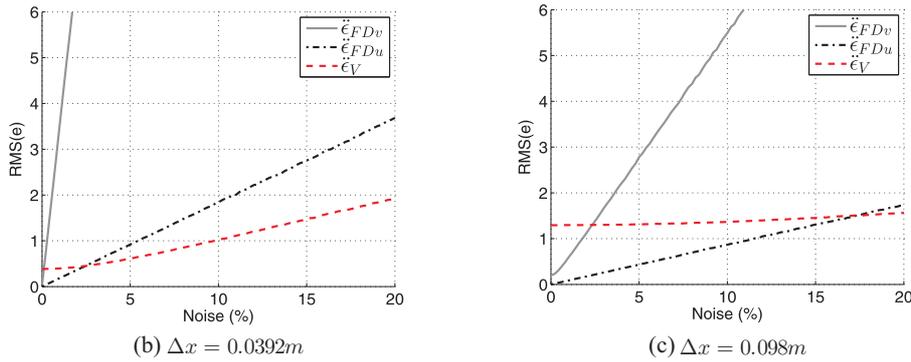


Figure 7: Sensitivity of acceleration strain estimate with respect to noise.

As it could be expected,  $\ddot{\epsilon}_{FDv}$  is very sensitive to noise measurements, and the error between estimated strains and real strains become unacceptable from very small levels of noise. The estimate of acceleration strains from horizontal accelerations is more robust with respect to noise measurement, and the technique minimizing the error depends on the level of noise. Indeed,  $\ddot{\epsilon}_{FDu}$  is more accurate for low levels of noise, while  $\ddot{\epsilon}_V$  shows better performances for higher levels of noise, respectively from 2.5% and 17.4%. Despite the fact that the original estimate of acceleration strains (for  $\beta = 0\%$ ) with virtual strains is the worst, this technique is the more robust with respect to noise measurement. Indeed, the slope of the corresponding  $RMS(e)$  increases from 0 at  $\beta = 0\%$ , but is always smaller than the slopes of  $RMS(e)$  for the two other methods. Finally, the robustness of the three methods with respect to noise increases as the distance between measurement sites  $\Delta x$  increases.

### Identification of mode shapes

As explained in the introduction, one of the interest of dynamic strain measurements in the context of SHM lies in the fact that the effect of damage on the low order strain mode shapes is localized in the close vicinity of the damaged area, which can be used to locate the damage. The mode shapes obtained with the estimated strains are therefore of particular interest, and should be affected similarly to mode shapes identified with real strain measurements when damage appears in the structure. Figure 8 depicts the first two mode shapes that have been identified with real strains and strains deduced from horizontal accelerations. These mode shapes have been identified from the time histories with the *Macec Toolbox* in *Matlab* which applies the stochastic subspace identification method [15].

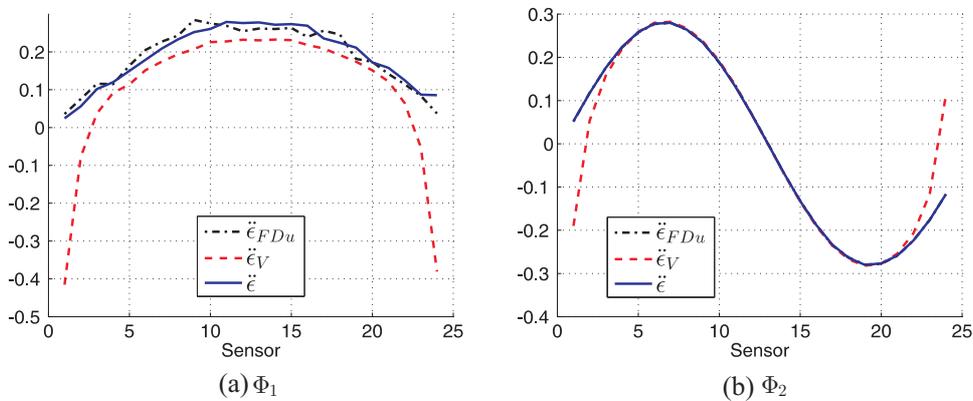


Figure 8: Mode shapes identified with real strains, and strains deduced from horizontal accelerations ( $\beta = 1\%$ ).

Figure 8(a) shows that the identification of the first mode shape  $\Phi_1$  is difficult since the obtained mode shape is quite irregular. On the other hand, the second mode shape is easily identified with the three types of measurements. Note however that the virtual strains  $\ddot{\epsilon}_V$  overestimate the mode shape components at the edges of the beam. This is due to the fact that sensors distant of the edges are taken into account to compute the virtual strains as it can be seen in Figure 2.

In order to verify if the strains  $\ddot{\epsilon}_{FDu}$  and  $\ddot{\epsilon}_V$  are able to reproduce a local change of mode shape due to damage, we have made the same simulations as previously when a damage of 10% of stiffness decrease between  $x = 0.392m$  and  $x = 0.451m$  is considered. From the time histories, we have identified the second mode shape, and we present in Figure 9 the difference between the undamaged and damaged mode shape.

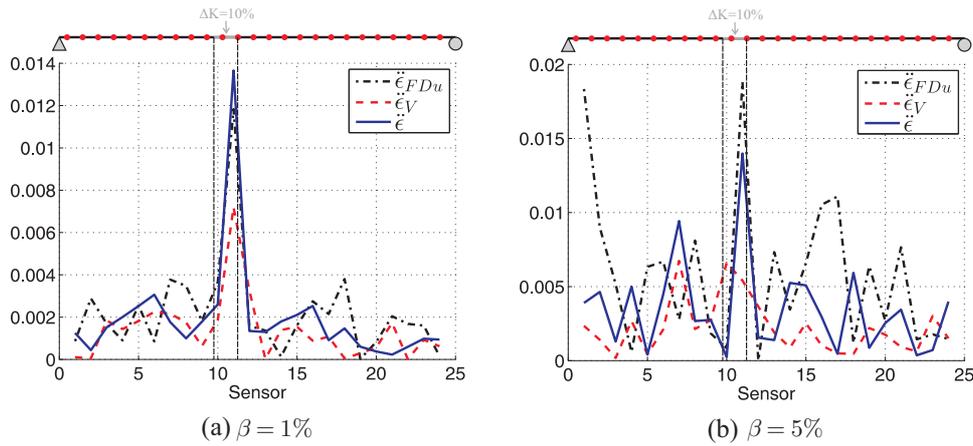


Figure 9: Effect of damage on the second identified mode shape.

For a small level of damage ( $\beta = 1\%$ ), the local effect of the damage is better reproduced with  $\ddot{\epsilon}_{FDu}$  than with the virtual strains  $\ddot{\epsilon}_V$ , but both techniques allows to locate the damage based on the change of the second mode shape. If the noise measurement is increased up to  $\beta = 5\%$ , the local effect of damage on the second mode shape is less obvious. Indeed, it can be observed in Figure 9(b) that the position of the damage still corresponds to a maximum change of  $\Phi_2$  identified from real strains  $\ddot{\epsilon}$ , but this maximum is not easily identified.  $\ddot{\epsilon}_{FDu}$  provides a more sharp change of mode shape at the location of the damage, but there is also a noticeable change of  $\Phi_2$  in the first components of the mode shape which could suggest that a damage occurred there as well. The virtual strains are unfortunately not able to reproduce the local change of  $\Phi_2$  for such a level of noise measurement. This is due to the fact the exponentially decreasing law of the virtual strains smooth the mode shapes as it can be seen in Figure 8, so that local changes of mode shapes are reduced. Considering a more sudden exponential decrease in the definition of virtual sensors could enhance the local sensitivity of virtual strain mode shapes, which is one of the perspectives of this work.

**CONCLUSION**

In this paper, we have investigated the possibility to deduce strains from horizontal accelerations. Two techniques based on horizontal accelerations have been proposed and tested numerically on a simply supported beam. The first one is based on a first order finite difference, while the second one involves more measurement sites in the estimate of strains and is referred to as virtual strains. The strains estimated from horizontal accelerations have been compared with those deduced from vertical accelerations and the real strains. The results obtained show that strains deduced from horizontal accelerations are much more robust with respect to noise than strains deduced from vertical accelerations, and provide very good estimates of dynamic strains for different sparsity of acceleration measurement sites.

The main advantages of the two robust techniques based on horizontal accelerations to deduce dynamic strain measurements are that (i) accelerations are very easy to measure, (ii) accelerometers are very easy to install and can be removed, (iii) they can be used on any type of structure. The possibility to locate damage based on the change of the mode shapes obtained with the estimated strains has been illustrated, and the concept of acceleration strains avoiding double time integration of time histories has been proposed. Future research studies will focus on network of accelerometers with non-constant measurement sites, more complex structures and a possible improvement of the law used to define the virtual strains before experimental tests.

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