Damage Detection and Localisation Using Mode-Based Method and Perturbation Theory

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ABSTRACT

In this paper, the detection and the localization of a local perturbation are assessed by analysing the frequency changes only (fundamental mode and overtones). After describing the method used herein applied to the bending beam and based on the perturbation theory, experimental application to a 1D plexiglas beam is shown using frequency and modal analysis technique. The damage is considered as a local perturbation of Young's modulus. Finally, the localisation of damage is done using classical modal-based methods and perturbation theory. The frequency values are caught by the Random Decrement Technique applied to the time history vibrations for one sensor at the free extremity of the beam. Detection and localization are successful, even for small and transient changes of the structure properties.

KEYWORDS : *perturbation, detection, localisation.*

1 INTRODUCTION

During the last two decades, a significant amount of research has been conducted in the area of non-destructive damage evaluation (NDE) via changes in the dynamic modal responses of a structure. Rytter [1] classified the NDE methods into 4 levels following their objectives: level 1 is the detection of damage, level 2 corresponds to the identification of the damage location, level 3 is the quantification of the damage severity, and level 4 is the prognosis of the integrity of the structure.
The first level LV1 is to detect if changes has occurred. The analysis of the damage related to the variation of the fundamental frequency of the buildings was a common practice in earthquake engineering. As supported by Farrar et al. [2], frequencies are certainly the modal parameter the most sensitive to changes, especially because the loss of stiffness directly impacts the frequency values. Data were collected and processed for understanding the variations of the modal parameters during and after earthquakes, related with the shaking level and the damage observed [3] [4] [5] [6] [7]. The second level (LV2) is to simultaneously detect and determine the location of damage. For example, variation of frequencies may only reflect a global change of the system properties and it is not often sufficient to locate within the structure the origin of the perturbation. For that reason, damage detection methods were developed based on the mode shape analysis. The experimental assessment of mode shapes is sometimes insufficiently accurate to detect and locate small variations, when compared to the sensitivity of the modal frequency analysis. The third level (LV3) is to detect, locate and estimate the severity of damage. Few applications of LV3 are available in practice. However, the estimate of the severity may contribute significantly to the action of the decision-makers in case of emergency after an extreme event.

In most cases, structural damage detection is based on a comparison between response from a "pre-damage" state and a "post-damage" state. Several methods have been described in the literature to identify and locate damages, most of them based on comparisons between a damaged and an undamaged condition and efficiency for severe damage. For Structural Health Monitoring application using vibration techniques, these methods focused on mode shape derivatives as indications of damage, such as the mode flexibility method [8], the curvature flexibility method [9], the mode shape curvature method [10] or a combination of these methods. The experimental

assessment of mode shapes is sometimes insufficiently accurate to detect and locate small variations, compared to the sensitivity of the modal frequency analysis. Some example in the literature concluded on the efficiency of these methods. Nevertheless, this last procedure requires recordings at several places of the building for defining the mode shapes and the frequencies of the structure, both modal parameters being used to identify the origin of the variations observed. For example, variation of frequencies may only reflect a global change of the system properties and it may not be sufficient to locate within the structure the origin of changing.

The objective of this study is to propose a natural-frequency-based method for structural damage detection and localization through perturbation theory, as associated with a high-resolution deconvolution method to treat the inverse problem. The second objective is to apply this method to experimental conditions, when ambient vibrations excite a structure. After presenting the theoretical approach for the one-dimensional (1-D) clamped–free beam with bending and the detection of weak changes in elastic properties based on perturbation theory, experimental tests are then performed, considering different perturbations applied to the beam, which are localized through a linear inversion approach. An adaptive technique is then further applied that optimizes the perturbation localization.

2   THEORETICAL APPROACH FOR 1D BENDING BEAM

Neglecting shear deformation, the 1-D equation governing the transverse vibration of a beam of length L is the Bernoulli-Euler equation:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2\psi(x,t)}{\partial x^2}\right)+m(x)\frac{\partial^2\psi(x,t)}{\partial t^2}=0 \tag{1}$$

where $\psi(x,t)$ is the spatial-temporal beam deformation at position x and time t, $m(x)$ is the mass per unit length, and $EI(x)$ is the product of the elastic Young's modulus and the inertial moment at each point on the beam. Since the goal of this approach is to evaluate, to first order, the change in frequency ω_n when a slight perturbation is made to the elastic properties of the beam, we may assume that the stiffness coefficient $EI(x)$ is perturbed at position x such that:

$$EI(x)\rightarrow \widetilde{EI}(x)=EI(x)+\delta EI(x) \tag{2}$$

This perturbation naturally induces a relative change in the modal deformation $\varphi_n(x)$ and the frequency ω_n of the beam and the perturbed system now satisfies the modified wave equation:

$$\frac{\partial^2}{\partial x^2}\left(\widetilde{EI}(x)\frac{\partial^2\widetilde{\varphi_n}(x)}{\partial x^2}\right)-\widetilde{\omega}_n^{\,2}m(x)\widetilde{\varphi_n}(x)=0 \tag{3}$$

In perturbation theory for linear operators, the change in the modal deformation $\varphi_n(x)$ is classically projected over the set of the unperturbed modes that lead us to consider the perturbation as:

$$\delta\omega_n=\frac{1}{2\omega_n}\int_0^L\delta EI(x)\left[\frac{\partial^2\varphi_n(x)}{\partial x^2}\right]^2 dx \tag{4}$$

This equation can be used to invert for the stiffness perturbation $\delta EI(x)$ at each point x on the beam from the measurement of the frequency change $\delta\omega_n$ observed for each mode n. In practice, both ω_n

and $\varphi_n(x)$ are obtained either (1) from a numerical simulation using a model of the beam for a set of modes or (2) from the data acquired on the beam by one or a set of sensors.

In a second step, Eq. (5) is used to invert for the stiffness perturbation $\delta EI(x)$ at each point x on the beam from the measurement of the frequency change $\delta\omega_n$ observed for each mode n. In this inversion process, we assume that ω_n and $\varphi_n(x)$ are known for the unperturbed problem. The inversion algorithm is built from the discretization of the integral in Eq. (4) over a finite number of segments of height Δx on the beam. Linear inversion processes as the Fréchet kernel associated to mode n and a perturbation Δx located at x_k on the beam is used. From the linear discretization, the inversion problem can be rewritten using a matrix formulation involving p modes along the beam:

$$\begin{bmatrix} \delta\omega_1 \\ \delta\omega_2 \\ ... \\ \delta\omega_p \end{bmatrix} = G \begin{bmatrix} \delta EI(x_1) \\ \delta EI(x_2) \\ ... \\ \delta EI(x_M) \end{bmatrix} \tag{5}$$

where the kernel matrix G is such that:

$$G = \begin{bmatrix} \dfrac{1}{2\omega_1}\dfrac{L}{M}\left(\dfrac{\partial^2\varphi_1(x_1)}{\partial x^2}\right)^2 & ... & \dfrac{1}{2\omega_1}\dfrac{L}{M}\left(\dfrac{\partial^2\varphi_1(x_M)}{\partial x^2}\right)^2 \\ ... & ... & ... \\ \dfrac{1}{2\omega_p}\dfrac{L}{M}\left(\dfrac{\partial^2\varphi_p(x_1)}{\partial x^2}\right)^2 & ... & \dfrac{1}{2\omega_p}\dfrac{L}{M}\left(\dfrac{\partial^2\varphi_p(x_M)}{\partial x^2}\right)^2 \end{bmatrix} \tag{6}$$

Note that following the same methodology the perturbation theory can be applied to the case of a 1-D shear beam. From Eq. (6), the solution of the inverse problem requires the estimation of G^{-1}. Assuming Gaussian-distributed uncertainties, the estimated inverse of G is obtained as in Tarantola [11]:

$$\widetilde{G^{-1}} = (G^T C_d^{-1} G + r * C_m)^{-1} G^T C_d^{-1} \tag{7}$$

where the *a-priori* model covariance matrix C_m resembles a diagonal matrix, but with Gaussian decrease outside the diagonal. The spatial correlation length l is assumed to be stationary along the beam, and should be of the order of the smallest wavelength associated with the highest-order mode taken into account in the inversion. The data covariance matrix C_d is diagonal, with diagonal elements that correspond to the uncertainty on the measured frequency fluctuations $\delta\omega_n$. From the estimate of $\widetilde{G^{-1}}$, we then obtain an estimation of the stiffness perturbation as:

$$\begin{bmatrix} \widetilde{\delta EI}(x_1) \\ \widetilde{\delta EI}(x_2) \\ ... \\ \widetilde{\delta EI}(x_M) \end{bmatrix} = \widetilde{G^{-1}} \begin{bmatrix} \delta\omega_0 \\ \delta\omega_1 \\ ... \\ \delta\omega_p \end{bmatrix} \tag{8}$$

3 DESCRIPTION OF THE EXPERIMENT

The structure used in this study is made of a continuous Plexiglas beam anchored at the bottom and free at the extremity (i.e. clamped-free beam). Figure 1 shows the experimental setup used to study the bending vibration in the (xy) plane in a clamped-free configuration. The data acquisition is

performed through 29 accelerometers powered by two conditioning amplifiers. The accelerometers are mono axial (Bruel and Kjaer, Type 4344) with 1.45 g of mass and a frequency band between 1-20 kHz. All the sensors were oriented in the horizontal direction, on the widest face of the beam in order to record the beam vibration in the y direction. The amplifiers are connected to a data acquisition unit by an RS232 port that allows to define and to control some parameters of the acquisition such as the gain of each accelerometer. A maximum gain of 40 dB was selected to amplify the signal recorded by each sensor. The beam vibration was gathered by these 29 accelerometers and transmitted directly to the computer through the data acquisition unit.
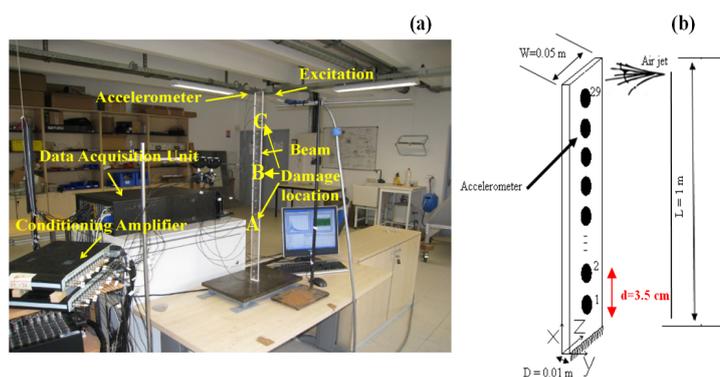

Figure 1. Experimental bed test for detection and localization of changes used in this work (a) Clamped-free beam set up tested in the laboratory. (b) Sketch diagram with geometric dimensions and accelerometer layout

The 29 accelerometers were spread along the height of the beam with a 3.5 cm interval. The beam was continuously excited by air jet assuming to ba a white noise low-amplitude excitation. Data was collected by windows of 10 seconds at a sampling frequency of 5000 Hz (Fig. 2). The transfer of the data to the data acquisition unit requires about 20 seconds, that is to say the recording obtained for the experiments was a not totally continuous recording. The local perturbation was produced by 30 seconds of heating the continuous Plexiglas beam in one positions corresponding to the position A, B and C in Fig. 1.
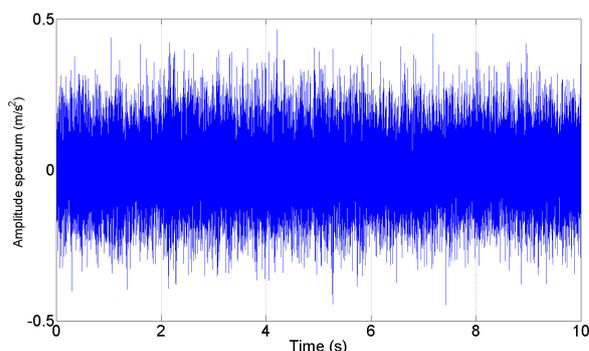

Figure 2. Example of 10 seconds window recorded at the top of the beam.

A first analysis of the beam response can be easily obtained by computing the Fourier spectrum (Fast Fourier Transform) at the top or at each accelerometer. Figure 3 shows the variation of the Fourier amplitude along the height of the beam. The theoretical behaviour of continuous beam is observed, with nodes and anti-nodes corresponding to the shape of the modes. The frequencies ratio obtained by the experimental tests $f_2/f_1=6.3$, $f_3/f_1=18.44$ and $f_4/f_1=35.79$, let us assume in the rest of the document an experimental bending beam behaviour.
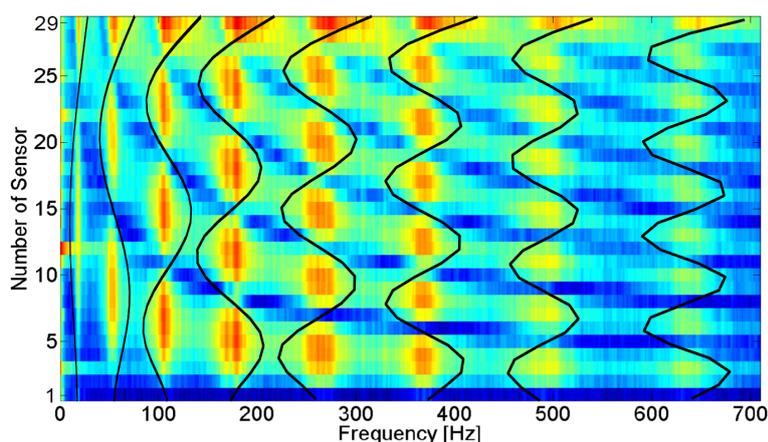
Figure 3. Amplitude of the Fourier spectra computed at the 29 sensors and representing the shapes of the 9 first modes of the undamaged beam. The black line is the numerical mode shapes.

4 RESULTS

The variation of the frequency values are first tracked by the Random Decrement Technique [12][13][14] and applied to the recordings provided by the sensor located at the top of the beam. In this case, RDT is computed mode by mode, using a sliding window of 110 seconds with 10% of overlapping. For each mode, the signal is filtered by a band-pass filtered using a second-order Butterworth filter, centred on ±10 % of the frequency. The length of the sliding window, the bandwidth of the filter and number of periods are chosen so as to reduce the error to estimate the frequency.

Figure 4 shows the variations of the frequency obtained by RDT for modes 4 to 9 using data recorded at the top. As shown by Mikael et al. [15], the RDT method provides a very accurate estimate of the frequency of the system, detecting clearly the local perturbation effects.
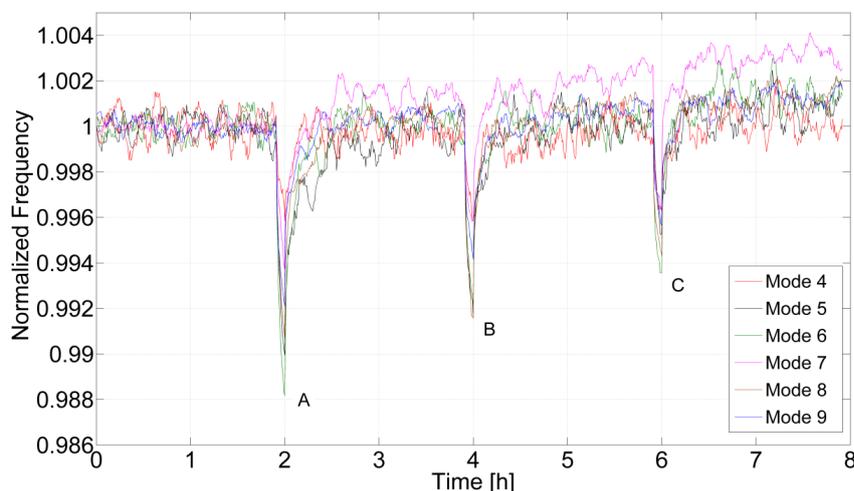

Figure 4. Time variation of the normalized frequencies of modes 4 to 9 computed by RDT using sensor located at the top of the beam (free condition) for experiment E1. A, B and C mark the time when the heat flow was applied for 30 seconds at positions A,B and C respectively.

Figure 5 gives the experimental mode shapes extracted using the FDD method applied to the experimental data. The modal analysis method used is the Frequency Domain Decomposition, a non-parametric frequency domain method [16].
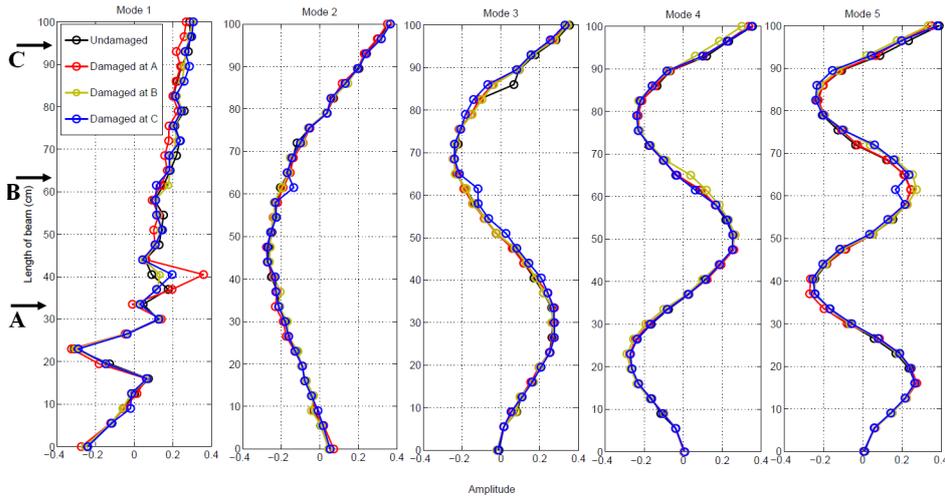
Figure 5. Mode shapes extracted by the FDD method for undamaged and damaged beam at positions A,B and C.

More details and theoretical background of this method can be found in [16]. FDD was applied to the data recorded by the 29 accelerometers located along the beam. Two analyses were done, considering the undamaged and the damage states for detecting and localizing the damage using the Uniform Load Surface method proposed by [9]. This method combines certain aspects of the MSC method [10] and flexibility method [8] to develop the change in uniform load surface curvature (ULS). The basic idea is that a localized loss of stiffness will produce a curvature increase at the same location.

The flexibility matrices can be approximated by the modal parameters of damaged and undamaged beam and at the end of the process the curvature change at location i is evaluated as follows:

$$\Delta F_i^{"} = \sum_{i=1}^{N} \left| \left\{ F_i^{"*} \right\} - \left\{ F_i^{"} \right\} \right| \tag{9}$$

where $\left\{ F_i^{"*} \right\}$ and $\left\{ F_i^{"} \right\}$ are the damaged and undamaged curvature of the uniform load surface at the ith degree of freedom, respectively. $\Delta F_i^{"}$ represents the absolute curvature change, and N represent the number of the degree of freedom (or identified number of mode shapes). The elements of $\Delta F_i''$ that have comparatively large values correspond to damage location. Note that the absolute change in curvature is first evaluated for each unit load flexibility shape and then summed.
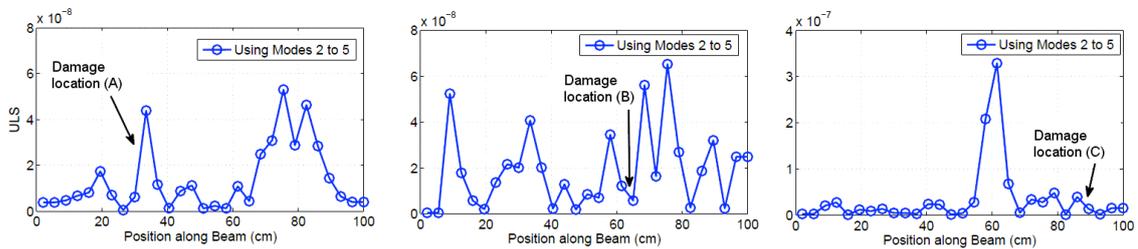

Figure 6. Localization of the damage at position A, B and C using the ULS method and considering modes 2 to 5 for experiment E1.

Considering damaged and undamaged states, we can apply the ULS method for detecting and localizing the damage. We observe Fig. 6 the ULS estimate for the A, B and C positions of the damage. The curve represents the value of the ULS function along the beam. In this case, the localization of the damage position is bad, showing some information for A localization, a lot of fluctuation along the beam for case B and a ghost of the damage not defined at the good position for

the case C. Because of the small and transient nature of the change, mode shapes used for the ULS method are not modified enough for being accounted for the application of the ULS.

The second method used for detecting the damage is based on the perturbation theory was presented previously. Only one recording at the top of the beam is required and it is not necessary to define the mode shapes. Moreover, for small and fast changes, the frequencies react immediately while the mode shapes remain unchanged or at least with changes that could not be detected. Figure 7 shows the three steps of the localization process based only on the variation of the modal frequencies obtained with the RDT method. The three steps of the localization method are described for the three cases of damage, i.e. at the positions A, B and C. Because of the theoretical model of the beam (bending beam) and the bad resolution of the first modes, the inversion scheme has difficulties to distinguish the real to the ghost localization. Similar difficulties are also observed for the case B, considering modes 1 to 9, the position of the damage being localized close to the symmetry axis of the beam, and then of the modes. For cases A and B, even if the symmetrical ghost of localization is not completely removed, the amplitude of the inversion permits us to discriminate the position of the damage, which could be more difficult in case of blind analysis. For the last case, position C, the method is very efficient for localizing the changes, considering modes 1 to 9. Even after the second step, because of the distance of the damage to the symmetry axis of the beam, the localization is quite effective.
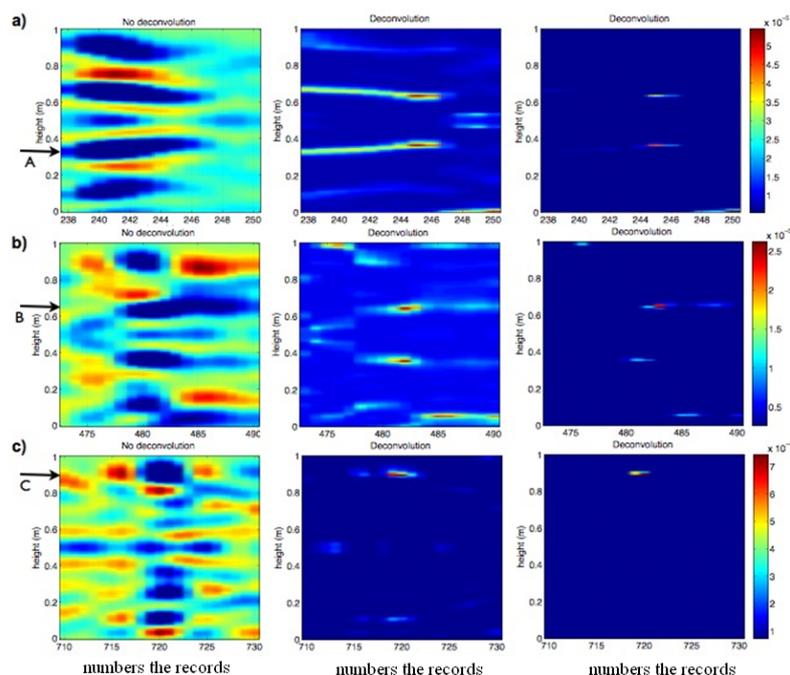

Figure 7. Results of the damage localization method applied to the experimental data for the three cases of perturbation (a: A; b: B; c: C).

5   CONCLUSIONS

This study has discussed the non-invasive evaluation of damage via changes in the dynamic modal response of a structure. The basic idea is that the modification of the stiffness, mass or energy dissipation characteristics of a system can alter its dynamic response. Unlike conventional methods of detection, localization and quantification of changes, this study is devoted to the development of a novel approach of localization based only on variations of the modal frequencies tracked using a time-domain method (the RDT) and on perturbation theory. A new inversion method is proposed, based on the sensitivity kernel approach. Finally, to be placed under equivalent conditions as the

experiment, the beam is excited with equivalent white noise and its motion is recorded at the top with time-varying localization of the perturbation.

The effectiveness and robustness of the method is tested in a configuration that is equivalent to that used experimentally. The instantaneous variations in the frequencies caused by the change in the position of the perturbation along the beam are tracked by applying the RDT to the top motion. Even for small frequency, this new approach improves the accuracy of the damage localization, which suggests temporal and geometrical monitoring of the perturbation. In conclusion, the combination of the linear inversion, the high-resolution deconvolution, and the RDT provide effective tools to detect and localize damage in a beam-type structure.

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