# Wavelet Based Characterization of Acoustic Attenuation in Polymers Using Lamb Wave Modes

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# WAVELET BASED CHARACTERIZATION OF ACOUSTIC ATTENUATION IN POLYMERS USING LAMB WAVE MODES

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## ABSTRACT

Polymers have been used in a wide range of applications ranging from fabrication of sophisticated medical equipment to manufacturing aircrafts. The design advantages of using polymers are its high strength-to-weight ratio, resilience, and compatibility with net-shape processes. In recent years, researchers have been trying to ascertain the mechanical as well as acoustical properties of polymers. Acoustical properties like attenuation of propagating ultrasonic waves through polymers vary in a broad spectrum depending on their chemical structure and stoichiometry. Guided wave techniques are widely used for nondestructive evaluation and inspection as well as examining the integrity of structures. This study demonstrates that guided wave techniques can be effectively utilized for material characterization, where efficient characterization depends on identification and selection of appropriate propagating wave modes and suitable signal processing techniques. The focus of this investigation is to estimate acoustic attenuation of acrylic (PMMA, polymethyl methacrylate), thermoplastic, using guided Lamb wave. Lamb waves are generated and received by piezo-electric transducers in a pitch-catch configuration. The received signals are first isolated from the inherent white noise using db4 based wavelet algorithm. The de-noised signals are then processed using Gabor Transform, which provides information about the group velocities of the propagating modes. The experimentally determined group velocities are compared with theoretical group velocities of the investigated polymers to identify the propagating Lamb wave modes. An effort has been made to estimate the attenuative properties of the thermoplastic from selective propagating Lamb wave modes.

***KEYWORDS :*** *Lamb wave, Gabor Transform, Polymethyl methacrylate (PMMA), thermoplastic, wavelet transform.*

## 1.0 INTRODUCTION

Ultrasonic non-destructive testing (NDT) has been practiced for the last several decades. NDT ultrasonic testing uses high frequency acoustic energy to conduct inspections and measurements. This type of inspection can be used for flaw detection, dimensional measurements as well as material characterization. In industrial applications, ultrasonic testing is commonly used on metals, plastics, composites, and ceramics. Acrylic PMMA plastics are among the most common thermoplastics used in industrial application and typically exhibit good mechanical properties. In thermoplastics, changes in orientation or length of polymer chains tend to result in subsequent variations in acoustic attenuation and ultrasonic wave velocity. This study investigates the propagation of guided acoustic waves through plastic medium and how they are affected by the attenuation of the medium.

Over the last few decades investigators have studied and developed different ultrasonic techniques for understanding the basic physics of material response to external loading. Lord Rayleigh [1] first studied the propagation of elastic waves on the free surface of a semi-infinite solid

where the amplitude of the wave decays rapidly with depth. Lamb [2] studied the more complicated problem of the propagation of free waves in the layered medium corresponding to the coupled longitudinal and transverse motion and identified two possible types of wave modes, namely 'symmetric' and 'anti-symmetric' modes. Achenbach and Keshava [3], Kundu and Mal [4], Evans [5], Levesque and Piche [6] and Castings and Hosten [7] studied the application of matrix transfer method for solving propagation of inhomogeneous waves in layered mediums. Na et al. [8, 9] generated guided waves for detecting delamination between steel bars and concrete interface

Successful assessment of acoustic properties of a medium involves in proper use of signal processing techniques or tools. In recent years wavelet analysis has become a popular technique for processing received signals with time-varying spectra. Many investigators have used the wavelet analysis to characterize damages in materials. Cho et al. [10] discussed the detection of subsurface lateral defects using wavelet transform on propagating Lamb waves. Rioul and Vitterli [11] and Abbate et al. [12] used wavelet transform for processing signals with non-stationary spectral contents. Kaya et al. [13] used wavelet decomposition to detect flaws in stainless steel samples. Ahmad et al [14, 15] used Daubechies wavelet functions in detecting defects for free and embedded pipes.

Gabor transform can be used as another form of wavelet analysis. Gabor [16] adapted the Fourier transform to analyze only a small section of the signal at a time – a technique called windowing of the signal. Gabor Transform (also known as 'Short Time Fourier Transform', STFT), maps a signal into a two-dimensional space of time and frequency. Gabor transform represents a compromise between the time and frequency based views of a signal. It provides information about both when and at what frequency a signal event occurs. 'Elemental signals' occupy the smallest possible area in the information diagram. Any signal can be expanded in terms of these elemental signals by a process that includes time analysis and frequency analysis. Gabor's work was not widely known until 1980 when Bastiaans et al. [17-19] related the Gabor expansion and the short term Fourier transform. Bastiaans introduced the sampled short time Fourier transform to compute the Gabor coefficients and successfully derived a closed form Gaussian function.

Murase and Kawashima [20] tried out different wavelet transforms and showed that when Gabor functions are used as mother wavelets then one can plot group velocity curves for a thin aluminum plate. Ahmad and Kundu [21] also used the Gabor wavelet transform to plot group velocities for defective and defect-free cylindrical pipes from experimental data.

## 2.0 THEORY

For the analysis of non-stationary or transient signals, Gabor analysis transforms a signal into a joint time-frequency domain. If $s(t)$ is the signal and it is windowed by a function $w(t)$ around time $\tau$ or $w(t-\tau)$ then the Fourier transform (FT) is given by

$$[FTs](\omega, \tau) = \int s(t) w(t - \tau) \exp(i \omega t) dt \qquad (1)$$

In Gabor transform, the window function is taken as the Gaussian function

$$w(t - \tau) = \exp\left[-(t - \tau)^2\right] / \sigma^2 \qquad (2)$$

Where, σ is a constant. In this work, Gabor wavelet based on the Gaussian function has been used. The mother wavelet and its Fourier transform are given by

$$\psi(t) = \pi^{-1/4} \left(\frac{\omega_p}{\gamma}\right)^{1/2} \exp\left[-\frac{t^2}{2}\left(\frac{\omega_p}{\gamma}\right)^2 + i\omega_P t\right] \qquad (3)$$

$$\hat{\psi}(\omega) = (2\pi)^{1/2} \pi^{-1/4} \left(\frac{\omega_p}{\gamma}\right)^{1/2} \exp\left[-\frac{t^2}{2}\left(\frac{\omega_p}{\gamma}\right)^2 (\omega - \omega_P)\right] \qquad (4)$$

Where, $\omega_p$ is the center frequency and $\gamma$ is a constant taken as $\gamma = \pi(2/\ln 2)^{1/2} = 5.336$.

### 3.0 EXPERIMENT

The primary objective of this research is to investigate how Lamb wave propagates through the thickness of a PMMA (polymethyl methacrylate) thermoplastic plate. Experiments were carried out to determine which Lamb wave modes are likely to propagate. Our second objective is to quantify how these Lamb wave modes attenuate while propagating through the plastic medium. In the process it will be explored how different signal processing techniques like Fourier transform and Gabor Transform (Short Time Fourier Transform, STFT) influence the assessment process. Both Fourier transform and Gabor Transform will endow with identifying the possible propagating modes. The plastic layer was kept under traction free boundary condition. Figure 1 shows the schematic diagrams for the boundary conditions as well as the instrumental arrangements. Relatively high frequency (5 MHz) Piezoelectric transducers were used as transmitters as well as receivers. Table 1 shows the acoustic properties of PMMA plastic and the thickness of the layer.

Table 1. Acoustic properties of Plastic (PMMA)

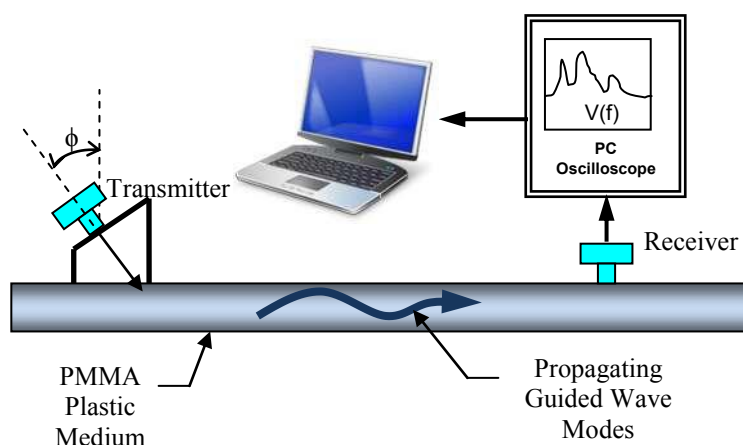| P-wave speed (m/s) | S-wave speed (m/s) | Density (kg/m$^3$) | Plate Thickness (mm) |
|---|---|---|---|
| 2996 | 1050 | 1100 | 25.17 |



Figure1. Experimental setup

### 4.0 RESULTS

Experiments were carried out to identify the propagating Lamb wave modes through the plastic medium for traction free boundary condition. During the experiment, Lamb waves or guided waves were generated by piezo-electric transducers. Guided waves were generated at one end of the plate and received at the other end by another receiving transducer in a pitch-catch configuration. Propagating guided waves are generated by carefully selecting the incident angle of the incident acoustic rays from the piezo-electric transducers. Plexi-glass wedges were used to facilitate the inclination of the incident rays. The received signals were in the form of time series curves (amplitude vs time) and then converted to V(f) curves (i.e. amplitude vs frequency) by Fourier Transform. The received signal is then processed in two phases. The first segment used experimental V(f) curves to identify the propagating modes through the plate. The next segment applied Gabor Transform on experimental time series signals to locate the time instance at which the propagating modes are generated and obtain signal strengths. From the acquired signal strengths of the propagating modes attenuative properties of the plastic medium are calculated.

### 4.1. Experimental V(f) Curves and Identification of Propagating Modes

Generation of guided waves in a plastic medium is very sensitive to the incident angle of the transducers. The first challenge is to find the appropriate incident angles for which strong guided

waves can be generated. The incident angles of the transmitter were adjusted experimentally to obtain strong signals. The incident angle (Φ) of 30° was found to produce strong guided wave signals. Figures 2(a), (b) and (c) show the experimental V(f) curves for an incident angle of 30° when the receiving transducer is kept at 25mm, 50mm and 75mm from the transmitter respectively, in open air or traction-free boundary condition. The receiver is kept at different locations to investigate how the strength of the propagating modes decays when the guided wave modes travel through the plastic medium. As expected, when the receiver is close to the transmitter, the strength of the propagating modes are the strongest and vice-versa. Theoretical Phase velocity dispersion curves for a plastic plate were plotted using the geometric and material properties of the plastic plate given in Table 1 using DISPERSE software. In order to identify the propagating guided wave modes through the medium, experimental phase velocities are calculated from the peaks obtained in the experimental V(f) curves using Snell's law $[v_{ph} = v_c/Sin\theta_c]$.



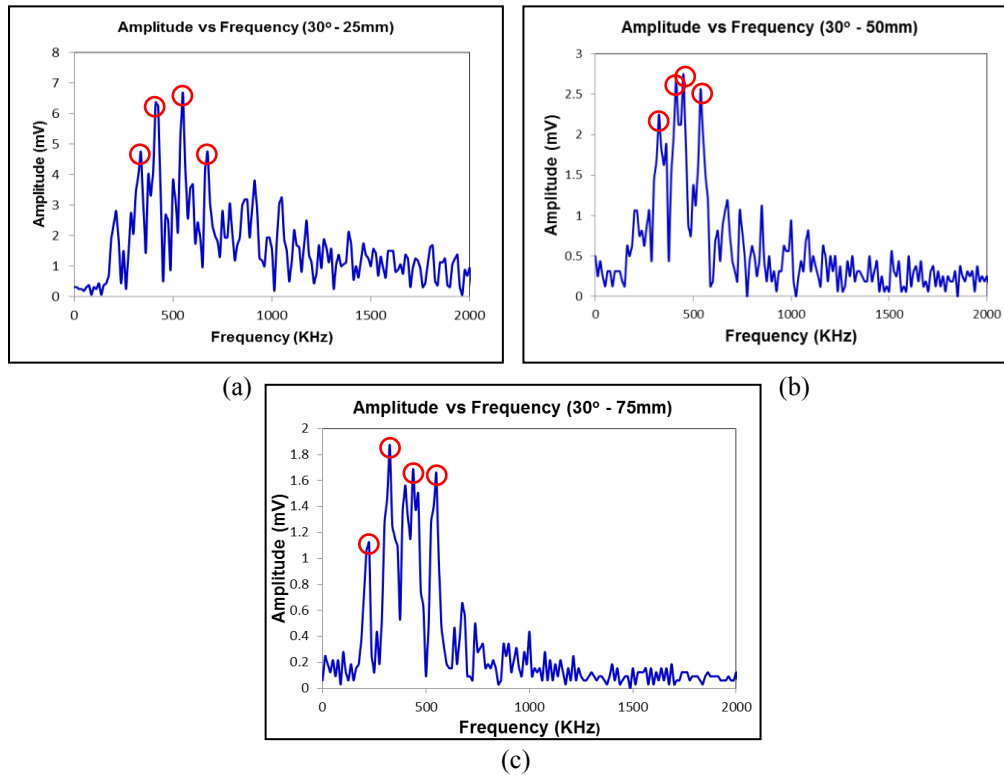(a)                                                                                     (b)



(c)

Figure 2. Experimental V(f) curves.

Figures 2(a), (b) and (c) show the experimental V(f) curves for the received signals when the transmitter is kept at 30° angle and the receiver is kept at 25mm, 50mm and 75mm from the transmitter respectively. In Figure 2(a), major peaks are developed at frequencies 337.5, 412.5, 550 and 675 KHz. Major peaks in a V(f) curve represents generation and propagation of a Lamb wave mode which may be symmetric or anti-symmetric. Figures 3(a) and (b) show theoretical phase velocity dispersion curves for a 25.17 mm thick plastic plate where both symmetrical [Figure 3(a)] and anti-symmetric modes [Figure 3(b)] are plotted. The experimental phase velocities ($V_{ph}$), calculated from the peak frequencies at 337.5, 412.5, 550 and 675 kHz, obtained from Figure 2(a), are plotted in Fig. 3(a) (the red colored square shapes). The phase velocities corresponding to the peaks at 337.5 and 412.5 kHz matches exactly with the theoretical anti-symmetric modes A7 and A9 respectively [Figure 3(b)]. For symmetric modes, phase velocities corresponding to frequencies 412.5, 550 and 650 KHz match relatively closely with S10, S13 and S15 modes while for anti-symmetric modes, they match closely with A12 and A15 modes.

Figure 2(b) shows the experimental V(f) curves when the receiver is placed at a distance of 50 mm from the transmitter. The peaks are generated at frequencies 325, 412.5, 450 and 537 KHz. The calculated phase velocities are plotted on the theoretical phase velocity curves (the green colored diamond shapes). It can be observed from Figure 3(b) that the experimental phase velocity ($V_{ph}$) curves corresponding to the peaks at frequencies 412.5 and 450 KHz exactly match with A9 and A10 modes respectively. Overall the peaks also show close match with symmetric modes S9, S10, S11 and S12, as well as anti-symmetric modes A7 and A12. When the receiver is placed 75mm from the transmitter, peaks occur at 325, 400, 437.5 and 550 KHz. The calculated phase velocity ($V_{ph}$) corresponding to 437.5 KHz frequency (yellow triangles) matches exactly with anti-symmetric mode A10. The other close matching modes are S9, S10, S11, S13, A7, A9 and A12. Table 2 shows the probable Lamb wave modes that are likely to propagate through the plastic medium. From the experimental results, it can be said the most likely propagating modes are A9 and A10 even though there are chances that other modes, listed in Table 2, may propagate.

Table 2. Experimental mode generating frequencies and matching modes.

| Transmitter - Receiver Distance (mm) | Symmetric Modes | | | Anti- Symmetric Modes | | |
|---|---|---|---|---|---|---|
| | Frequency (KHz) | Matching Modes | Matching Remarks | Frequency (KHz) | Matching Modes | Matching Remarks |
| | | | | | | |
| 25 | 412.5 | S10 | Close match | 337.5 | A7 | Exact Match |
| | 550 | S13 | Close match | 412.5 | A9 | Exact Match |
| | 675 | S15 | Close match | 550 | A12 | Close match |
| | | | | 675 | A15 | Close match |
| | | | | | | |
| 50 | 312 | S9 | Close match | 312 | A7 | Close match |
| | 412.5 | S10 | Close match | 412.5 | A9 | Exact Match |
| | 450 | S11 | Close match | 450 | A10 | Exact Match |
| | 537.5 | S12 | Close match | 537.5 | A12 | Close match |
| | | | | | | |
| 75 | 325 | S9 | Close match | 325 | A7 | Close match |
| | 400 | S10 | Exact Match | 400 | A9 | Close match |
| | 437.5 | S11 | Close match | 437.5 | A10 | Exact Match |
| | 550 | S13 | Close match | 550 | A12 | Exact Match |

## 4.2. Attenuation of Propagating Modes

In this section we investigated how the generated modes attenuate while propagating through the plastic medium. Attenuation is the property of the material which decays the strength of the acoustic waves while propagating through the medium. In the previous section we identified the propagating modes. In this section, we investigated how these modes attenuate while propagating through the medium. The attenuation can be calculated by comparing the amplitude (obtained from the time-series curves) of the propagating modes at different distances. As described earlier, we identified the propagating modes by comparing the experimental phase velocities ($V_{ph}$) with the theoretical phase velocities using peak frequencies obtained from the experimental V(f) curves. In order to establish the attenuation, we need to compare the strengths of these propagating modes at different distances along their travel path. To obtain the strength of the signals, we need to use the amplitudes of the experimental time series curves. The challenge is to correlate the frequencies with the time frame at which the modes are generated, in other words identify the time of generation of these modes. After identifying the time, the strength of the received signal can be assessed from the time series curves. Strengths are finally compared to quantify the attenuation of the propagating modes.

Gabor Transform can convert a time series signal into a time-frequency signal by plotting the group velocities of the propagating modes. We used AGU Vallen software to plot the group velocity curves of the received signals using 2-D Gabor Transform. The experimental signals were first de-noised by wavelet analysis using Daubechies 'db4' function. Figures 4(a), (b) and (c) show

the Gabor Transforms of the received time-series signals for the conditions when the receiver is kept at 25mm, 50mm and 75mm from the transmitter respectively. Theoretical group velocities are obtained using DISPERSE software. These group velocities are converted into frequency-time series (t = L/$V_g$, where, L is the length of the distance traveled by the propagating modes and $V_g$ is the group velocity). These converted theoretical group velocities are superimposed over the 2-D experimental group velocities obtained from Gabor Transform (Figure 4) in order to determine at which time instances they are generated. After identifying the time instances when the modes are formed, the maximum signal strength is calculated by obtaining the amplitude of the received signal at that corresponding time from the time-series signal [upper part of Figures 4(a), (b) and (c)].



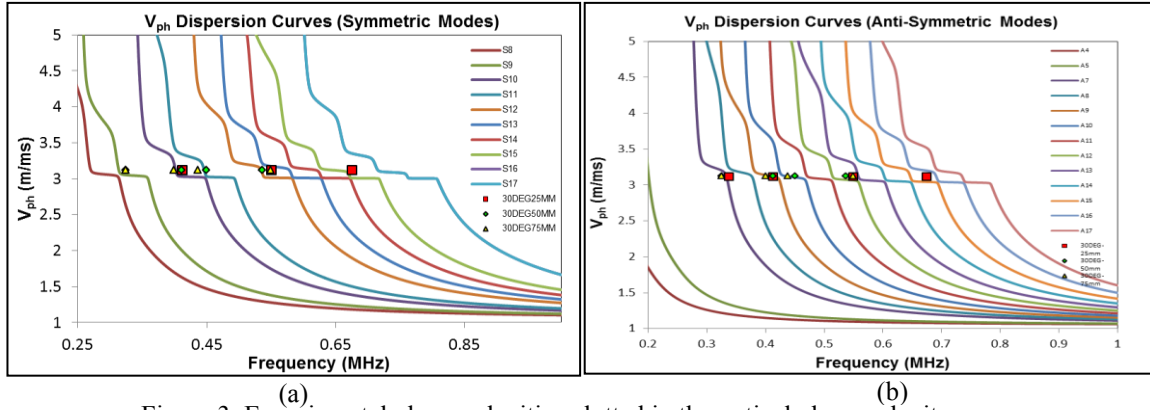(a)                                                                              (b)
Figure 3. Experimental phase velocities plotted in theoretical phase velocity curve.

For this investigation, we selected the modes S10, A7, A9 and A10 – which has at least one exact match with the experimental obtained modes for the three length conditions (Table 2). Figure 4(a) shows that when theoretical S10, A7 and A9 modes are superimposed over experimental group velocity plots, obtained by 2-D Gabor Transform, they match well with the experimental group velocity contours. It can also be observed that all the modes generate at a time frame between 40 ~ 45 μ-sec. For this time duration, the corresponding absolute magnitude of the signal strength is 120 mV which is obtained from the time-series curve [upper portion of Figure 4(a)]. Figure 4(b) shows that the experimental group velocities match with theoretical S10, A7, A9 and A10 modes. The time frame is in between 60 ~ 70 μ-sec which corresponds to a maximum absolute amplitude of 44 mV. For Figure 4(c), the experimental group velocities match well with theoretical S10, A7, A9 and A10 modes. It can be observed from Table 2 and also from Figures 3(b) and 4(a) that anti-symmetric mode A10 is not generated when the distance between the receiving transducer and the transmitting transducer is kept at 25mm apart. Modes S10, A7 and A9 are generated for all three cases, i.e. when the transducers are kept at 25mm, 50mm and 75mm apart.

In order to calculate the attenuation of the propagating modes we isolated the modes into two groups. Group 1 consists of S10, A7 and A9 modes as they are generated for all three length conditions. A10 mode is placed in another group as it is generated only for 50mm and 75mm case. Trendline curve fitting technique is used to fit approximate curves for assessing the equation for attenuation. Figure 5(a) shows the attenuation trend for S10, A7 and A9 modes. Figure 5(b) shows the attenuation for A10 mode. It can be observed from both the cases that strength of the signals decays exponentially with the distance. The attenuation equations obtained from Figures 5(a) and 5(b) are given in Equation (5) and (6) respectively.

$$A = 251.14\, e^{-0.032\,d}$$  -    for S10, A7 & A9 modes    (5)

$$A = 147.89\, e^{-0.024\,d}$$       for A10 mode                     (6)

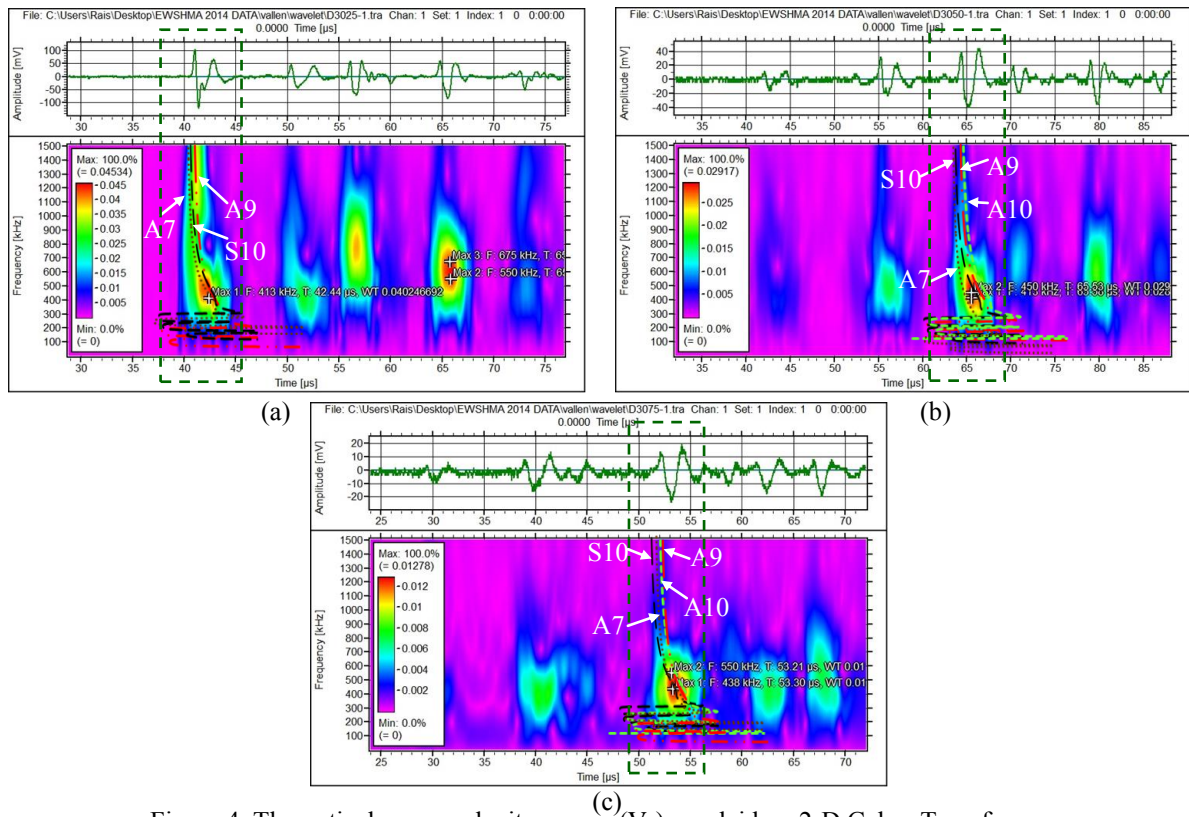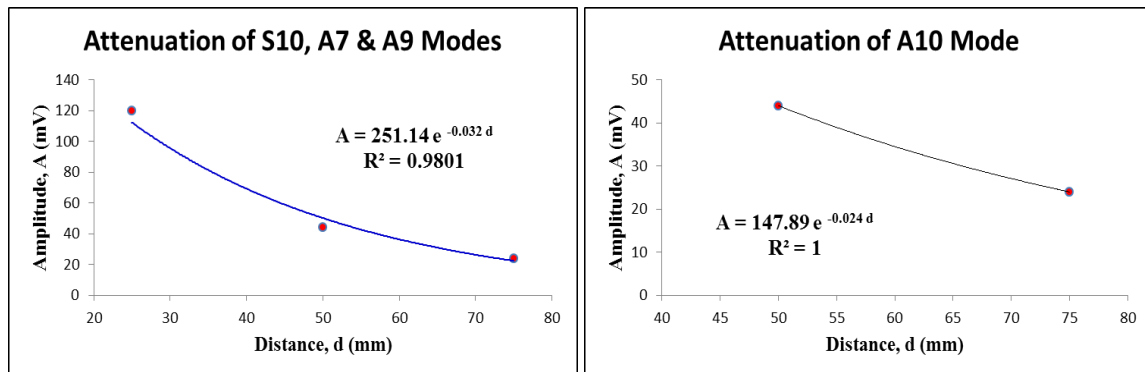where, A is the amplitude in mV and d is the distance in mm.

Figure 4. Theoretical group velocity curves ($V_g$) overlaid on 2-D Gabor Transform.



Figure 5. Attenuation of the propagating modes.

## 5.0 CONCLUSION

This study outlines a technique to identify the propagating Lamb wave modes in a PMMA polymer medium and assess the attenuation of these propagating modes using Gabor Transforms. In this research, conventional V(f) curves and Gabor Transforms are used to identify the modes. Comparisons between the theoretical and experimental group velocity plots have been carried out by superimposing the theoretical group velocity dispersion curves over the experimental group velocity plots obtained from Gabor transformation. It was observed that the theoretical and experimental group velocities match very well. From the group velocity curves, information about the time instances for generation of the modes are obtained, which helps in finding the decaying strength of the modes and successively leads to finding the attenuation characteristics of the received signal.

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