Sequential Importance Sampling Based on a Committee of Artificial Neural Networks for Posterior Health Condition Estimation

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Abstract

The output of real-time diagnostic systems based on the interpretation of signals from a sensor network is often affected by very large uncertainties if compared with local non-destructive testing methods. Sequential Importance Resampling (SIR) is used in this study to filter the output distribution from a committee of Artificial Neural Networks. The methodology is applied to a helicopter panel subject to fatigue crack propagation. Strain signals are acquired during crack evolution and a diagnostic unit trained on simulated experience provides damage assessment in real-time. This information is filtered through a SIR routine, providing model identification, model parameter estimation and crack length probability density function updating, conditioned on the observations at discrete time steps.

Keywords : *model identification, artificial neural network, committee, sequential importance resampling, fatigue crack.*

Introduction

Within the Structural Health Monitoring framework, the solution of the damage identification problem is itself very complex, as it requires the identification of structural anomalies and their characterisation in terms of damage type, location and extent. All these parameters are required for the correct estimation of damage evolution, thus to allow a feasible exploitation of the real-time information for the dynamic schedule of maintenance inspections. Distributed sensor networks for real-time damage identification suffer from the common drawback that the confidence on the system output is often by far lower than classical Non-Destructive Testing (NDT) methods, usually adopted for local measures, and this poses a limit on the application of new methodologies.

Machine learning techniques, Artificial Neural Networks (ANN) [1] in particular, are well established tools that can be applied for diagnosis of structures, nevertheless they require a lot of cases as example during training in order to approximate the functions that relate any monitored quantity (e.g. strain for the purposes of the present paper) to damage parameters. These examples can be experimentally collected [2] or retrieved from numerical simulation [3], depending on the application. If a model-based approach for training is applied (like in this study) it is very important to guarantee sufficient generalisation capability of the algorithm to process real data. Many regularisation techniques can be used such as cross-validation, early stopping, addition of small noise quantities during training, etc. In [2, 4] the authors grouped different models trained on *Bootstrap* datasets into one *committee* [1], and demonstrated the validity of the method to gain further regularisation of the network. Basically, the diagnostic system output is not a crisp indication but a distribution of outputs. The simplest way to treat this output is to average the prediction of the *N* models belonging to the committee, considering that the combination of *N* multiple models can reduce the output error by a factor *N* with respect to the average error of the single ANN models, provided the errors of the individual models are uncorrelated [1]. The approach followed in this paper consists in applying Sequential Monte-Carlo sampling (SMC) techniques [5,

6] in order to filter the committee output distributions sequentially available from a diagnostic system. It allows not only the reduction of the uncertainty related to the health condition (e.g. crack length), but also the selection of the proper model that better describes the state of the monitored structure. Suppose a real-time diagnostic system provides continuous estimation of damage extent on a structure. Two circumstances can be recognised: (i) stationary signal (or non-propagating damage), when measures are affected by operative changes, such as temperature and boundary loads; (ii) damage evolution, when measures are experiencing both a trend due to damage evolution and disturbances due to external influences. Selection of the best model for observation fitting is itself a fundamental task in signal processing and it can be efficiently performed through Bayesian computational approaches [7]. As it is widely known, when non-linear system and non-Gaussian noises are involved, a closed form solution of the Bayesian problem cannot be derived. An approximated solution can be obtained with SMC sampling techniques. Among the different available techniques [8][9][10], Sequential Importance Resampling (SIR) based on augmented state vector has been considered hereafter [11, 12]. The objective is to provide model selection, based on the output distribution provided by an ANN committee for damage assessment, then updating also the probability density functions (pdf) of the parameters governing the damage evolution, assumed a proper damage evolution model has been identified. This filtered information can be subsequently exploited for prognosis of residual life [12].

The method is applied to a helicopter fuselage panel with aluminium skin and riveted stringers (Figure 1) subject to Fatigue Crack Growth (FCG). Strain signals acquired from a Fibre Bragg Grating (FBG) sensor network are processed through a committee of ANNs, providing a continuous estimation of crack length. The diagnostic output is filtered in a SIR algorithm which provides model selection (propagating versus non-propagating damage), crack length filtering as well as model parameter estimation.

The paper organises as follows: section 1 and section 2 reviews the experimental FCG test and the diagnostic system output respectively; section 3 briefly summarises the theoretical aspects needed to develop a SIR algorithm for model selection, crack length filtering and FCG parameters updating; section 4 is focused on the practical implementation of the algorithm; the results after the application of the SIR algorithm are detailed in Section 5. A conclusion section is finally provided.

1   FATIGUE CRACK GROWTH TEST

The FCG test bed is shown in Figure 1-a. The panel specimen (Figure 1-b) is made of aluminium 2024-T6. The panel skin dimensions are 600mm x 500mm. Four riveted stiffeners are present, equally spaced 150mm apart. The panel has been fixed on the ground and has been connected to an actuator by means of a gripping system, designed to distribute the load constantly along the panel width. Fibre Bragg Gratings (FBG) have been selected among the available technologies for strain sensing due to their multiplexing option.
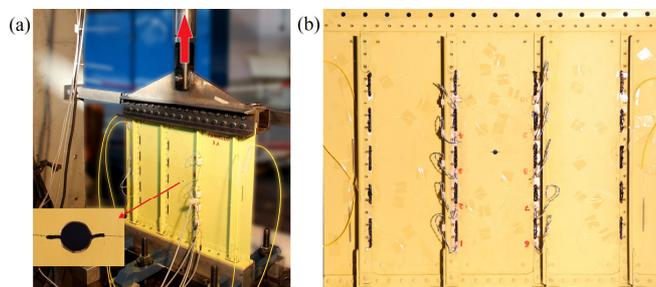

Figure 1: (a) Test rig for FCG and (b) helicopter fuselage panel specimen under monitoring.

A sinusoidal 12 Hz fatigue load has been applied (10^6 cycles) and initial data have been acquired on the undamaged panel, to be used as baseline level. The maximum load amplitude was set to 10 kN with a load ratio R=0.1. An artificial damage of 16mm width has then been initiated in the centre of

the central bay, to facilitate and control crack initiation. A 12 Hz fatigue load with 35kN amplitude and load ratio R=0.1 was applied ($4x10^5$ cycles) and a real crack propagated through the skin. Real crack propagation was measured with a calliper.

HBM-Catman Easy-AP software has been used as an interface between the desktop PC and the optical interrogator for FBG strain acquisition. The program was set to automatically acquire 2000 samples, at a sampling rate of 1 kHz, approximately every 500th load cycle. A trigger has been set relative to one sensor to extract the strain pattern relative to the instant when the peak for each cycle is reached. The files, saved during crack propagation, are processed through the diagnostic algorithm described below.

2   DIAGNOSTIC SYSTEM OUTPUT AND PROBLEM STATEMENT

A brief overview of the diagnostic system is provided hereafter. It is not the author intention to enter into the details of the diagnostic system structure optimisation as an in-depth explanation could be found in [3, 4]. Nevertheless the focus is here on the analysis of diagnostic results, aimed to appreciate the main requirements for the subsequent filtering algorithm.

As anticipated in the introduction, a model-based approach is considered. A database of strain measures numerically simulated with a validated Finite Element Model (FEM) was presented in [3]. Strain patterns in correspondence of sensor positions have been stored for different crack damages varying the position and the extent of the damage. A damage index database was created extracting the relevant features from the numerical signals. This database was used to train three ANN structures for anomaly detection, localisation and quantification respectively.

Only the quantification ANN algorithm will be considered from now on and the anomaly detection will be performed based on the filtered output of the SIR algorithm, as will be explained in the next sections. The relevant parameters describing the damage quantification algorithm are reported in Table 1. The network structure was optimised according to the procedure detailed in [4].

Table 1. Parameters for damage assessment ANN design.

Diagnostic Level	ANN type	Input layer	Output layer	Hidden layer nodes	Training strategy	ANN # in committee
Quantification	Function Fitting	20 damage indices	Crack length	16	Scaled Conjugate Gradient	60

Sixty ANNs have been grouped into one committee and a distribution of ANN outputs is thus available at each diagnostic step, like show in in Figure 2. The averaged committee output and the real crack length are also indicated in the same figure.

Though damaged and undamaged states have been included in training, the algorithm is not able to describe small cracks, due to the very low sensitivity of the measured strain pattern to damage cases located in the centre of the bay (Figure 1). As a consequence, the averaged committee output presents a bias for healthy condition (up to $11x10^5$ cycles). This has to be taken into account in the model selection algorithm presented later.

The committee output dispersion is related to the generalisation capability of the diagnostic algorithm (Figure 2): (i) very high dispersion is found for smaller cracks (up to 30mm); (ii) reduced variability is encountered when the algorithm recognises damage conditions adopted during training; (iii) dispersion increases in relation to damage extent, due to the more complex behaviour of the damage index sensitivity. Nevertheless, taking advantage from the disposal of a series of real-time observations, sequential filtering of the committee output will provide a posterior estimate of the monitored variables (crack length and FCG parameters), conditioned on these observations.

Although temperature compensation was performed based on the signal from a FBG temperature sensor, many fluctuations in the diagnostic output can be seen in Figure 2. The highest spikes are due to the usage of halogen lamps during real crack length measure. The filtering capability of the algorithm will be assessed in presence of these disturbances.
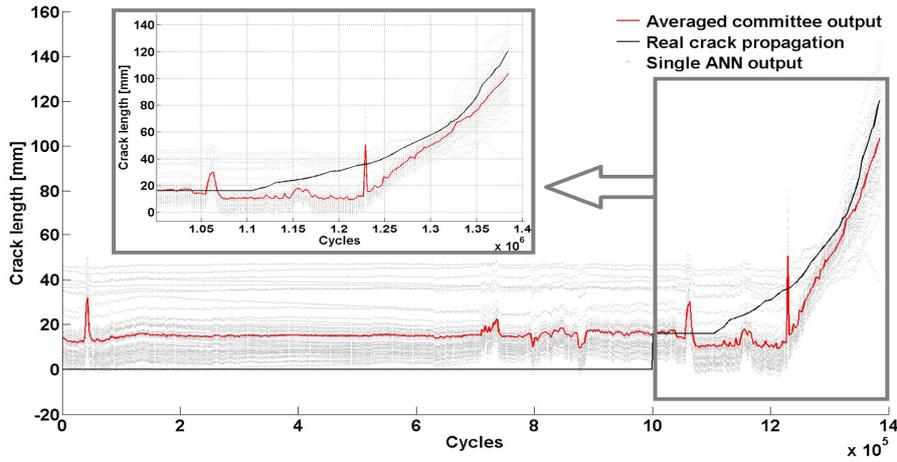
Figure 2: Output from diagnostic system. The specimen is undamaged up to 10^6 cycles. Real crack propagates 10^5 cycles after artificial notch creation (1.1×10^6 cycles).

3 THEORY FOR DIAGNOSTIC OUTPUT FILTERING

Literature about SMC sampling is vast; therefore this section mainly summarises those aspects relevant to the problem under investigation, namely (i) the adaptation of the algorithm to receive as input a committee distribution of measures, (ii) the inclusion of FCG parameters in the state vector of the system and (iii) the implementation of model selection in the SIR algorithm.

The implementation of SIR algorithm requires the definition of a Dynamic State Space (DSS) usually including the model evolution function (f) and the observation equation (h). In general terms, in a model identification framework one has two or more model evolution functions depending on the complexity of the problem, each one based upon a different parameter vector and noise quantities. The following DSS formulation applies:

$$x_k = f_M\left(x_{k-1}, \vartheta_{M,k-1}, \omega_{M,k-1}\right) \tag{1}$$

$$z_k = h\left(x_k, \eta_k\right) \tag{2}$$

Where $M = 1,2,\ldots,N_M$ with N_M being the total number of evolution models considered in the analysis, x_k is the state condition at k^{th} discrete time, ϑ is the model parameter vector, ω_{k-1} is a random noise perturbing the theoretical damage evolution and η_k is the uncertainty affecting the observations z_k, intrinsic to the diagnostic system. Subscripts associated to ϑ relates the parameter estimation to the considered model (M) at the current time step.

Focussing on the application to the damage identification problem, two models are considered in this study ($N_M=2$), namely non-propagating damage M_a (3) and propagating damage M_b (4):

$$f_{M_a} : x_k = x_0 + \omega_{M_a,k-1} \tag{3}$$

$$f_{M_b} : x_k = x_{k-1} + C\left[\Delta K\left(x_{k-1}\right)\right]^m \cdot \Delta N \cdot \omega_{M_b,k-1} \tag{4}$$

Equation (3) considers a parametric level (x_0) for the non-propagating model, in order to comply with the diagnostic system output described in Section 2. x_0 is unknown at the beginning of the filtering process and will be the only parameter of the evolution function f_{M_a}, thus $\vartheta_{M_a} = [x_0]$.

Equation (4) is the discrete formulation of the Paris-Erdogan model [13]. ΔK is the stress intensity factor variation within one load cycle, dependent on the crack length, C and m are two parameters dependent on the material, ΔN is the load cycle step between two subsequent observations. Due to the very high correlation between C and m [14], only C is considered as unknown variable of the evolution function f_{M_b}, thus $\vartheta_{M_b} = [C]$, while m is deterministically evaluated as a function of C.

SIR algorithm basically consists of a sequential procedure involving prediction based on prior knowledge, updating relying on current observations and resampling according to actual posterior distributions. It is aimed at finding at k^{th} discrete time the posterior probability of the state vector $[x_k, \vartheta_k, M_k] = [\tilde{x}, \tilde{\vartheta}, \tilde{M}]$, containing crack length x, evolution model M and model parameters ϑ, like in Eq. (5) :

$$\Pr\left\{x_k = \tilde{x}, \vartheta_k = \tilde{\vartheta}, M_k = \tilde{M} \mid z_{1:k}\right\} = \sum_{i=1}^{N_{s,\tilde{M}}} W_{\tilde{M},k}^i \cdot \delta\left(\tilde{x} - x_{\tilde{M},k}^i\right) \cdot \delta\left(\tilde{\vartheta} - \vartheta_{\tilde{M},k}^i\right) \tag{5}$$

Where $N_{s,\tilde{M}}$ is the number of ith samples $x_{\tilde{M},k}^i$ drawn at k^{th} discrete time according to the evolution model $\tilde{M}$, $\vartheta_{\tilde{M},k}^i$ is the ith sample for the parameter vector relative to the evolution model $\tilde{M}$, δ is the Delta-Dirac function and $W_{\tilde{M},k}^i$ is the importance weight associated to the ith sample, normalised with respect to all the available samples from both the considered evolution models:

$$W_{M_j,k}^i = \frac{W_{M_j,k-1}^i \cdot L\left(z_k \big| x_{M_j,k}^i, \vartheta_{M_j,k}^i\right)}{\sum_{M_j}\left(\sum_{l=1}^{N_{s,M_j,k-1}} W_{M_j,k-1}^i \cdot L\left(z_k \big| x_{M_j,k}^l, \vartheta_{M_j,k}^l\right)\right)} \tag{6}$$

$L\left(z_k \big| x_{M_j,k}^i, \vartheta_{M_j,k}^i\right)$ is the likelihood of ith sample with respect to the observation z_k. In practice, the observation equation h is intrinsic to the diagnostic system, as a distribution output p_{Z_k} is actually provided by the measuring committee at each kth discrete time (approximated through a kernel density estimator). The likelihood is then approximated as $p_{Z_k}\left(x_{M_j,k}^i\left(\vartheta_{M_j,k}^i\right)\right)$.

3.1 Parameter estimation

The proposal distribution from which to draw the samples of the parameter vector has to be considered in the prediction step of the SIR algorithm. Considering constant parameters have to be identified, the first attempt is to select a DSS equation for the constant parameters on the form $\vartheta_k^i = \vartheta_{k-1}^i$. However, it leads to the well-known problem of *sampling impoverishment* or *sample degeneracy*. Then, while different methods are available in literature [11], an approach based on *artificial dynamics* is used here, due to its relative simplicity. The *sample degeneracy* can be overcome by the addition of a small change in the sample values at each step of the algorithm. This small change is a random noise added to each sample, as in Eq. (7).

$$\vartheta_k^i = \vartheta_{k-1}^i + \xi_k^i \tag{7}$$

Where ξ_k^i is a random value with zero-mean and a variance that decreases in time. However, the simplicity of the method introduces a non-negligible drawback that is the loss of information between the time steps, due to the introduction of the mentioned artificial changing in the parameter values while they are fixed. Moreover, two questions have to be solved to maximize the performances of the algorithm: the selection of the initial covariance matrix of ξ, $\sigma_{\xi,0}{}^2$, and the decreasing function depending on the discrete time $\sigma_\xi^2 = \sigma_{\xi,0}{}^2 f(k)$, in order to reach the convergence in a relatively small number of iterations.

3.2 Model selection

As it is done for parameter estimation, a proposal distribution accounting for the model functions involved in the analysis has to be considered. Let assume a transition probability matrix is selected as in Eq. (8) for a problem involving two models M_a and M_b.

$$\bar{\bar{\pi}}_{a,b} = \begin{bmatrix} \pi_{a,a} & \pi_{a,b} \\ \pi_{b,a} & \pi_{b,b} \end{bmatrix} \tag{8}$$

Where $\pi_{a,b}$ is the rate of transition from model a to model b, defined in a way that algorithm convergence is guaranteed, nevertheless with a sufficient exploration of the DSS. It follows that prediction step for model selection can be written as the Eq. (9).

$$\Pr\left(M_k = \tilde{M} \middle| z_{1:k-1}\right) = \sum_{j=a,b} \pi_{j,\tilde{M}} \cdot \Pr\left(M_{k-1} = M_j \middle| z_{1:k-1}\right) \tag{9}$$

After the updating through Eq. (6), the posterior estimation of the model is obtained through a marginalisation of Eq. (5) over the model parameters and crack length, as it is done in Eq. (10).

$$\Pr\left(M_k = \tilde{M} \mid z_{1:k}\right) = \sum_{i=1}^{N_{s,\tilde{M}}} W_{\tilde{M},k}^i \tag{10}$$

4   ALGORITHM IMPLEMENTATION

The following points summarise the algorithm operation, while Table 2 contains the most important variables involved in model selection:

1. Initialise the algorithm

    $z_0 \sim h(x_0, \eta_0)$ (committee output at k=0)

    $N_{M_a,0} = 0.95 N_s$; $N_{M_b,0} = 0.05 N_s$

    $\forall M_j = M_a, M_b$ and $\forall i = 1,...,N_s$

    $\vartheta_{M_j,0}^i \sim p\left(\vartheta_{M_j,0}, \sigma_{\xi_0}^2\right)$

    $x_{M_j,0}^i \sim h(x_0, \eta_0)$

    $w_{M_j,0}^i = 1/N_s$

2. Perform the transition for x_k and ϑ_k (prediction step)

    $\sigma_{\xi_k}^2 = \sigma_{\xi_0}^2 \cdot f(k) = \sigma_{\xi_0}^2 /(1+0.1k)$ (update artificial

    dynamics variance)

    $\forall M_j = M_a, M_b$ and $\forall i = 1,...,N_{s,M_j,k-1}$

    $\vartheta_{M_j,k}^i \sim p\left(\vartheta_{M_j,k} \middle| \vartheta_{M_j,k-1}^i, \sigma_{\xi_k}^2\right)$

    $x_{M_j,k}^i \sim p\left(x_{M_j,k} \middle| \begin{bmatrix} x_{M_j,k-1}^i & \vartheta_{M_j,k-1}^i \end{bmatrix} \omega_{M_j,k-1}\right)$

3. Calculate the likelihood based on committee output

    Acquire z_k (committee distribution)

    $\forall M_j = M_a, M_b$ and $\forall i = 1,...,N_{s,M_j,k-1}$

    $L\left(z_k \middle| x_{M_j,k}^i, \vartheta_{M_j,k}^i\right) = p_{Z_k}\left(x_{M_j,k}^i\left(\vartheta_{M_j,k}^i\right)\right)$

4. Update the weights (updating step)

    $$W_{M_j,k}^i = \frac{W_{M_j,k-1}^i \cdot L\left(z_k \middle| x_{M_j,k}^i, \vartheta_{M_j,k}^i\right)}{\sum_{M_j}\left(\sum_{l=1}^{N_{s,M_j,k-1}} W_{M_j,k-1}^i \cdot L\left(z_k \middle| x_{M_j,k}^l, \vartheta_{M_j,k}^l\right)\right)}$$

5. Calculate the posterior probability of the model

    $$\Pr\left(M_k = M_j \middle| z_{1:k}\right) = \frac{\sum_{i=1}^{N_{s,M_j,k-1}} W_{M_j,k}^i}{\sum_{M_j} \sum_{i=1}^{N_{s,M_j,k-1}} W_{M_j,k}^i}$$

6. Update the number of samples for each model (model prediction for the next step)

    $$N_{s,M_i,k} = \sum_{j=a,b} N_s \cdot \Pr\left(M_k = M_j \middle| z_{1:k}\right) \cdot \pi_{j,i}$$

7. Perform resampling according to systematic resampling procedure [15] and based on posterior distributions.

8. Repeat 2-7 each time a new measure is available

Table 2: parameters involved in SIR algorithm

Parameter	Description
M_j	Generic model
M_a	Non-propagating model
M_b	Propagating model
N_s	Total number of samples
$N_{s,M_j,k}$	Number of samples associated to model M_j, at k^{th} discrete time
$\pi_{i,j}$	Transition probability from model i to model j

5   RESULTS

This section contains the main results of the algorithm, in terms of model selection, parameter estimation and committee output distribution filtering for crack length estimation.

Referring to the real crack propagation in Figure 2, the artificial damage was created at 10^6 cycles while the real fatigue crack propagation started at 1.1×10^6 cycles. Due to the relative position of the damage with respect to the sensor grid, a significant deviation of the diagnostic output from the baseline condition is obtained after 1.2×10^6 cycles, when the crack is approximately 35mm long. The result for model selection can be appreciated in Figure 3-a. Although some fluctuations are present in the diagnostic output, the non-propagating model is indicated as the one that best fits the

real-time diagnosis up to 1.25×10^6 cycles, when the probability associated to the propagating model becomes predominant. However, some spikes are present near 0.5×10^5, 7.5×10^5 and 10.5×10^5 cycles, due to severe and uncontrolled temperature fluctuations. Though the algorithm sensitivity can be controlled through a proper setting of the model transition matrix $\bar{\bar{\pi}}_{a,b}$ and the random process noises (ω_a and ω_b), the adoption of an additional filter (e.g. moving average) can reduce the sensitivity to environmental influences.

As explained in Section 2, model parameter estimation is a requirement for the successful application of the algorithm mainly for two reasons: (i) the bias level for the non-propagating model is often unknown and (ii) the crack growth rate for the propagating model is often different from the one predictable according to material specifications. Estimation of x_0 is shown in Figure 3-b. It reflects the average crack length predicted through the non-propagating model and reported in Figure 3-d. Estimation of C is shown in Figure 3-c and only the values predicted after the identification of the propagating model have been reported. While literature suggests a value of C equal to 2.382×10^{-12} for the aluminium considered in this study, the results in Figure 3-c are strictly related to the crack growth rate measured through the ANNs. Thus, the parameter is overestimated before 1.3×10^6 cycles as the crack growth rate associated to measures is higher than reality, while it is slightly underestimated after 1.3×10^6 cycles. Superior diagnostic performances will be reflected in an improved estimation of the model parameters.
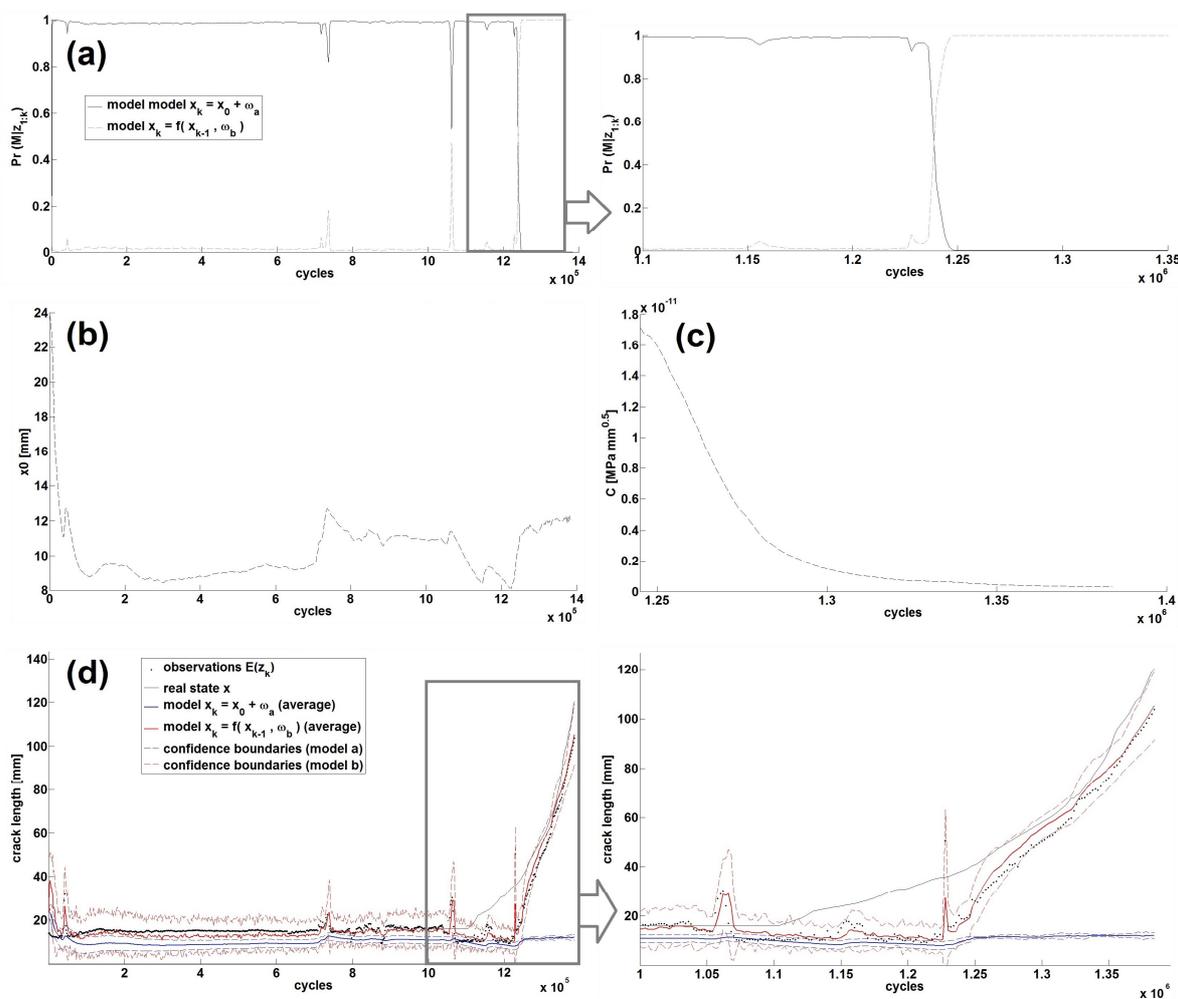

Figure 3: results of the SIR algorithm for (a) model identification, (b) x_0 parameter for the non-propagating model, (c) C parameter for propagating model and (d) crack length filtering based on the committee output.

Finally, the filtering of the committee output in terms of crack length is shown in Figure 3-d. A better filtering of baseline measures is obtained for the non-propagating model, which is also associated to a higher probability in Figure 3-a. After 1.2×10^6 cycles, only the propagating model can efficiently filter the sequence of measures. Again, a proper tuning of the random noises (ω_a and ω_b) associated to the models is necessary to guarantee a good performance of the algorithm.

CONCLUSION

A SMC sampling methodology has been applied in this study for the combined estimation of the evolution model (propagating versus non-propagating damage), model parameters and crack length, receiving as input the diagnosis from a committee of ANNs. Propagating damage was identified in correspondence of a 40mm long crack. The efficiency of the algorithm can be appreciated comparing this result with the minimum detectable crack length specified in [3] for the same structure, sensor network and damage configuration, where the alarm threshold was obtained for a 60mm long crack by a committee of ANNs for structural condition classification. Model parameter estimation and crack length filtering also showed relatively good performances, nevertheless the reliability of the system is strongly dependent on the accuracy of the diagnosis. Additionally, a proper tuning of the noise quantities involved in the algorithm and the transition matrix for model selection is required for the optimisation of the system performances.

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