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# NONLINEAR ULTRASONIC DAMAGE DETECTION FOR FATIGUE CRACK USING SUBHARMONIC COMPONENT

Zhi Wang <sup>1</sup>, Wenzhong QU <sup>2</sup>, Li XIAO <sup>3</sup>

*Department of Engineering Mechanics, Wuhan University, Wuhan 430072, China*

<sup>1</sup> wangzhi@whu.edu.cn; <sup>2</sup> qwz@whu.edu.cn; <sup>3</sup> xiaoli6401@126.com.

## ABSTRACT

Most modern structures operate under cyclic loading conditions, which may incubate fatigue cracks. In recent years, researchers have focused on more efficient procedures in SHM and damage detection using nonlinear vibration and nonlinear acoustic methods. Because superharmonics are also generated by the piezoelectric transducers and by the electronic equipment, these harmonics increase the background noise deteriorating the S/N ratio. The objective of this paper is to demonstrate the application of nonlinear ultrasonic subharmonic method for detecting fatigue cracks. A fatigue crack is qualitatively modeled as a single-degree-of-freedom (SDOF) system with non-classical hysteretic nonlinear interface force at both sides of the crack interface. The threshold of subharmonic is discussed and the influence of model parameters on the subharmonic resonance parameter condition is investigated. An aluminum beam with a fatigue crack is used to quantitatively verify the excitation voltage amplitude and frequency subharmonic resonance range. Two surface-bonded piezoelectric transducers are used to generate and receive ultrasonic wave signals. The experimental results demonstrate that the subharmonic components increase in magnitude with increasing amplitude of the input signal and can be used to accurately detect the fatigue cracks.

**KEYWORDS:** *Fatigue Crack; Structural Health Monitoring; Structural Damage Detection; Subharmonic; Excitation Amplitude and Frequency*

## 1 INTRODUCTION

Monitoring, locating and evaluating the severity of fatigue cracks in structures promptly are not only important for the structure security and the prevention of serious failure, but also provide decisions support for the maintenance of the structure. Recent studies on damage detection of cracks are mostly based on breathing crack model, simplifying the nonlinear dynamic characteristics of crack into a periodic opening and closing process<sup>[1]</sup>. While actual interfaces of fatigue cracks are tend to be closed at rest, and they only open when the external excitation reaches a certain degree of amplitude, so they are usually referred to as closed cracks. Recently, nonlinear vibration and nonlinear ultrasound method is widely used in Structural Health Monitoring and damage identification. Because superharmonics can both be generated by cracks and test equipment, these harmonics increase the background noise and deteriorate the S/N ratio<sup>[2]</sup>. Subharmonics cannot be

introduced from electronic equipment and their generation requires specific conditions, which make them more reliable for damage detection. For the threshold phenomenon of subharmonic, the excitation conditions such as excitation amplitude and excitation frequency should be confirmed before making use of subharmonic method for the detection of closed crack.

The closed crack model is usually based on breathing crack model and the crack interface force is taken into consideration. This model improved the description of the nonlinear dynamic characteristics of crack interface, and added threshold effect into the periodic opening and closing process. The crack interfaces stay closed unless the external forces reach the threshold of excitation force. Delrue<sup>[3]</sup> introduced virtual spring to simulate the interaction of two interfaces of the crack and used segment linear spring force to describe the relationship between interface force and displacement, but the hysteresis effect wasn't involved. Yamanaka team<sup>[4-5]</sup> introduced adhesion stress and atomic stress as attractive force into crack model, and accomplished the subharmonic array crack detection experiment to evaluate the effectiveness of the model, but the excitation condition was not mentioned. Johnson<sup>[6]</sup> developed a single degree of freedom dual stiffness oscillator theoretical model, finite element analysis was adopted and the result showed that subharmonic component is remarkable when the excitation frequency is close to twice the natural frequency. Naito<sup>[7]</sup> using nonlinear spring to simulate crack to study on the effects on generation of subharmonic caused by the section of excitation conditions, but wasn't experimentally verified. Van Den Abeele<sup>[8]</sup> proposed the possibility of the use of hysteresis effect in closed crack model firstly, but this idea stays in assumptions, more investigations are needed.

A SDOF qualitative model of fatigue crack is developed in this study. Segment hysteresis crack interface force is used to simulate the interaction and the opening and closing process of the crack interfaces. The excitation frequency and excitation amplitude range generating subharmonic is investigated and the influence of the model parameters is discussed. An aluminum beam with a fatigue crack is used to verify the subharmonic generation condition using two PZT transducers.

## 2 QUALITATIVE MODEL OF CLOSED CRACK

A SDOF qualitative model of fatigue crack is developed, shown in figure 1. Two crack interfaces A and B remain closed without large amplitude dynamic load. A persistent excitation with the amplitude of  $p$  and the frequency  $\omega$  is applied on side A, and transmit through the crack interface to the B side.

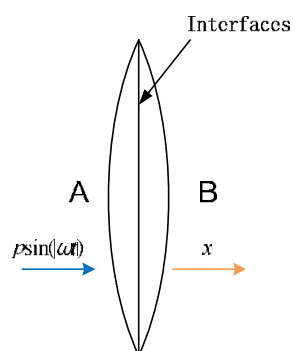


Figure 1. Closed crack model

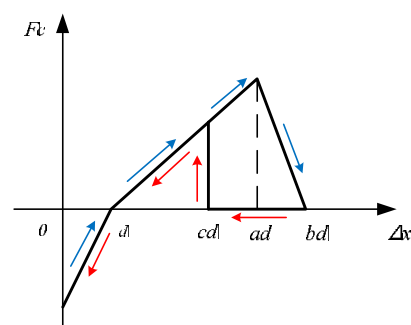


Figure 2. Relationship between crack interface force and interface distance

The differential equation of motion of the dynamic displacement  $x$  on the B side can be expressed as:

$$m\ddot{x} + \gamma\dot{x} + k\Delta x = F_c \quad (1)$$

where  $m$  is mass,  $\gamma$  is damping,  $k$  is stiffness,  $F_c$  is interface force which will be discussed in next section.  $\Delta x$  is relative displacement of two interfaces expressed as

$$\Delta x = x - p\sin(\omega t) \quad (2)$$

## 2.1 CRACK INTERFACE FORCE

The hysteresis closed crack model introduces the threshold effect into breathing crack model. If the excitation amplitude  $p$  is lower than the threshold the crack remains closed without any displacement or interaction, otherwise the crack starts to open and close. The threshold is set as 0.001 in this model. The relationships between interface force and relative displacement of the model is segment linear which made the model different from breathing crack model. Hysteresis effect is simulated through the changes of the curve of interface force-relative displacement which turned to be a closed curve as it is shown in figure2, the x-axis is relative displacement  $\Delta x$  and the y-axis is interface force  $F_c$ .

(1) When side B moves away from side A in figure 1, the interfaces force  $F_c$  changes as blue arrow shown in figure 2 and is expressed as

$$F_c = \begin{cases} k_1 \left( \frac{d}{\sigma} - \frac{\Delta x}{\sigma} \right) & \Delta x \leq d \\ k_2 \left( \frac{d}{\sigma} - \frac{\Delta x}{\sigma} \right) & d < \Delta x \leq ad \\ k_3 \left( \frac{bd}{\sigma} - \frac{\Delta x}{\sigma} \right) & ad < \Delta x \leq bd \\ 0 & \Delta x > bd \end{cases} \quad (3)$$

where  $a, b$  are coefficients,  $d$  is the equilibrium position, the interface force will be 0 when the relative displacement is equal to  $d$ ,  $\sigma$  is the fatigue crack interfacial roughness. When  $\Delta x \leq d$ , the interface force  $F_c$  performs as repulsive force and decrease with the increase of relative displacement, the crack stiffness in this stage is  $k_1$ ; When  $d < \Delta x \leq ad$ ,  $F_c$  performs as attractive force and increase with the increase of relative displacement, interface tension is maximum when relative displacement reaches  $ad$ , the crack stiffness in this stage is  $k_2$ ; When  $ad < \Delta x \leq bd$ ,  $F_c$  performs as attractive force and decrease with the increase of relative displacement, the crack stiffness in this stage is  $k_3$ ; When  $\Delta x > bd$ , the crack is completely open and  $F_c$  is 0.

(2) When the side B moves towards side A, the interface force changes as the red arrow in figure 2 and is expressed as

$$F_c = \begin{cases} k_1 \left( \frac{d}{\sigma} - \frac{\Delta x}{\sigma} \right) & \Delta x \leq d \\ k_2 \left( \frac{d}{\sigma} - \frac{\Delta x}{\sigma} \right) & d < \Delta x \leq cd \\ 0 & cd < \Delta x \leq bd \end{cases} \quad (4)$$

where  $c$  is a coefficient. When  $cd < \Delta x \leq bd$ ,  $F_c$  stays 0; when  $d < \Delta x \leq cd$ ,  $F_c$  performs as attractive force and decrease with the decrease of  $\Delta x$ , the crack stiffness is  $k_2$ ; When  $\Delta x \leq d$ ,  $F_c$  performs as repulsive force and increase with the decrease of  $\Delta x$ , the crack stiffness is  $k_1$ . As shown in figure 2, the process of the crack opening and closing from  $cd$  to  $bd$  is different, which simulated hysteresis characteristics of closed crack.

Parameters used in this model are  $m = 1, \gamma = 0.05, k = 1, k_1 = 1 \times 10^{-3}, k_2 = 0.6 \times 10^{-3}$ ,

$$k_3 = \frac{1-a}{b-a} k_2, a = 4, b = 5, c = 3, \sigma = 1 \times 10^{-3}, d = 10\sigma.$$

## 2.2 SUBHARMONICS EXCITATION CONDITIONS

MATLAB is used to calculate the closed crack model with the sampling frequency of 10 Hz and the computing time of 2000s. The natural frequency is 0.2Hz which is measured by applying a sweep frequency excitation on the model.

Base on the theory that subharmonic components appear when the excitation frequency is around twice the natural frequency<sup>[6]</sup>, simulation is taken to verify that this theory is also suitable for closed crack model. The simulation focuses on the excitation frequency around twice the natural frequency which is chosen to range from 0.38 Hz to 0.42 Hz with the step of 0.002 Hz and the excitation amplitude is from 0.0234 to 0.024 with the step of  $5 \times 10^{-5}$ . The roughness  $\sigma$  of the crack model is  $1 \times 10^{-3}$ , and the equilibrium position  $d$  is  $10\sigma$ , the region of the conditions which can produce subharmonic is shown in figure 3. Figure 3 shows that the region of conditions which can produce subharmonic looks like a V shape which means the threshold of excitation amplitude reach its minimum when the excitation frequency is around twice the natural frequency and get greater while the excitation frequency is away from twice the natural frequency.

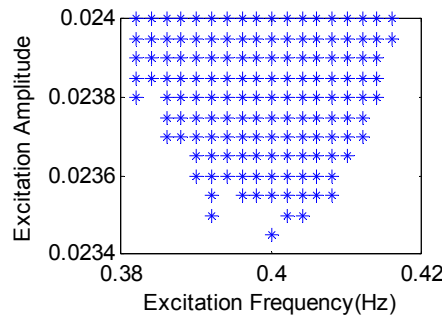


Figure 3. The region of excitation frequency and amplitude of the subharmonic threshold

A parametric study was carried out on the crack model to investigate the influence from the parameters on the region of the subharmonic generation. As previous expression,  $d$  was expressed as the equilibrium position of the interface force. When the relative displacement is less than  $d$ , then the two interface force performs as repulsive force and when the relative displacement is greater than  $d$ , then the interface force performs as attractive force. Figure 3 shows the region of the subharmonic conditions calculated under the condition that  $d = 10\sigma$ , consider about the effect caused by  $d$ , we take the  $d = \sigma$  into calculation, the excitation frequency ranges from 0.34Hz to 0.4Hz with the step of 0.004Hz and the excitation amplitude ranges from  $2.6 \times 10^{-3}$  to  $3.0 \times 10^{-3}$  with the step of  $2 \times 10^{-5}$ , the region is shown in figure 4. The region looks like a V shape but a little distortion for the reason of hysteretic characteristic. When the equilibrium  $d$  decreases, the threshold of excitation amplitude decreases with  $d$ , and the excitation frequency region moved to the left.

$\sigma$  is the crack interface roughness parameter, changing  $\sigma$  may cause the change of  $d$  and  $F_c$ . Considering with the influence caused by  $\sigma$ , we take  $\sigma = 1 \times 10^{-4}$  here which is 1/10 of the model in figure 3, the excitation frequency ranges from 0.37Hz to 0.44 Hz with the step of 0.002Hz and

the excitation amplitude ranges from  $1.65 \times 10^{-3}$  to  $1.8 \times 10^{-3}$  with the step of  $5 \times 10^{-6}$ , the region is shown in figure 5. The region also looks like a V shape as  $\sigma$  decreases, but the excitation frequency region moved to the left.

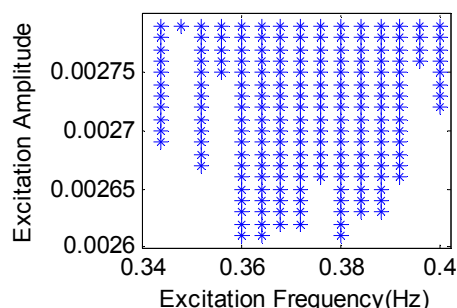


Figure 4. The region of excitation frequency and amplitude of the threshold of subharmonic ( $d = \sigma$ )

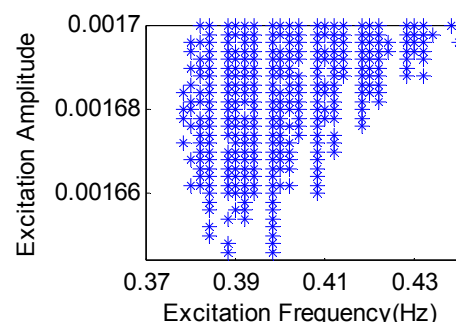


Figure 5. The region of excitation frequency and amplitude of the threshold of subharmonic ( $\sigma = 1 \times 10^{-4}$ )

Through changing parameters of the crack model, it is found that the region changes with the equilibrium position  $d$  and interface roughness  $\sigma$ , this is important for identification of fatigue cracks in different structures.

### 3 EXPERIMENTS

To verify the effectiveness of the subharmonic method, a damage detection experiment on an aluminum beam with a fatigue crack is conducted. The experiment setup and the specimen are shown in figure 6.



Figure 6. The experiment equipment (left) and specimens (right)

We used an aluminum beam with a fatigue crack in the middle of the beam and a pristine aluminum beam for the experiment. To make the fatigue crack in the aluminum beam, firstly we machined a split on the beam and applied cyclic loading by the fatigue testing machine to produce the fatigue crack, with 0.2-10.0KN/5Hz for 3600 cycles and 0.2-10.0KN/10Hz for 4300 cycles and 0.2-5.0KN/5Hz for 10000 cycles, then changed to displacement control, with 0.1mm/10Hz for 37000 cycles. The fatigue crack on the beam was 11.5 mm in length as shown in figure 6 (right). Two PZTs with diameter of 12mm and thickness of 0.6mm were surface bonded on the cracked beam for actuation and sensing separately. The PZTs were installed with the distance of 120mm away to the crack position on each side. Two foams were set under the beam to simulate free

boundary condition. A continuous sinusoidal signal generated by Function Generator (Agilent 33522A) was applied on the transmitter PZT, and an Oscilloscope (Agilent D50-X3014A) was connected to the sensor PZT to collect the response data.

### 3.1 CRACK DETECTION

By sweeping the crack beam with the excitation frequency of 25 KHZ to 35 KHZ, one of longitudinal natural frequencies is about 32.5 KHZ which is measured through finding the peak in frequency domain collected by the oscilloscope.

A sinusoid excitation with the frequency of twice the natural frequency which is 65 KHz is applied on the pristine beam and shown in figure 7. The undamaged beam is in linear condition theoretically and there will be no harmonic component in frequency domain spectra. But harmonic component is obvious in figure 7 which means that nonlinearity is not only caused by the crack in a practical testing. Harmonic methods of nonlinear ultrasonic testing are affected by the measuring equipment nonlinearity and prone to lead false alarm of damage identification which makes harmonic methods hard to recognize the crack nonlinearity correctly. While the generation of subharmonic needs some specific conditions, subharmonic method can detect cracks without the influence of equipment nonlinearity which make subharmonic method more suitable for damage detection.

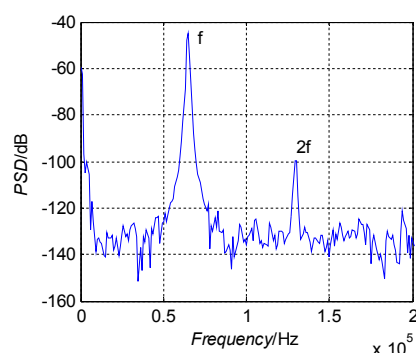


Figure 7. Spectrum of undamaged beam

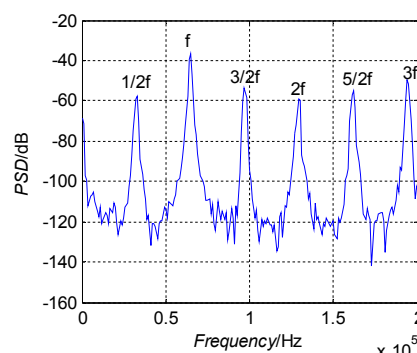


Figure 8. Spectrum of damaged beam

The same excitation is then applied on the damaged beam, and the result is shown in figure 8. Superharmonic component and subharmonic component can both be discovered in figure 8. It is found that subharmonic component is more sensitive to the crack than harmonic component, thus subharmonic method is more suitable for the nonlinear ultrasonic detection of fatigue crack.

To investigate the influence of excitation frequency on the generation of subharmonic, a sinusoidal excitation with the frequency of 44 KHz and amplitude of 10Vp-p is chosen to apply on the crack beam, and the result is shown in figure 9. Comparing figure 8, we can find that if the excitation frequency is apart from twice the natural frequency, no subharmonic would be generated with the same excitation amplitude.

To investigate the influence of excitation amplitude on subharmonic generation, we applied a sinusoidal excitation with the excitation frequency of twice the natural frequency 65 KHz and the excitation voltage of 2Vp-p on the crack beam, the response is shown in figure 10. Comparing figure 10 with figure 8, it is found that when excitation voltage is 2Vp-p there is no subharmonic

component in the spectra which means that the voltage of 10Vp-p is greater than threshold while 2Vp-p is lower than the threshold. The threshold of the generation of subharmonic will be discussed in next section.

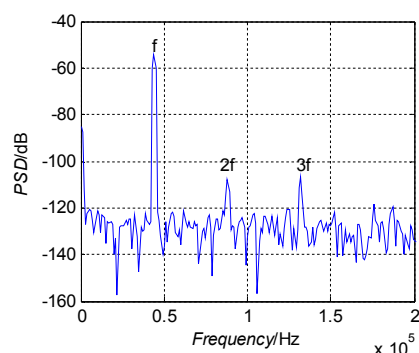


Figure 9. Spectrum of damaged beam  
(F= 44 KHz; V= 10Vp-p)

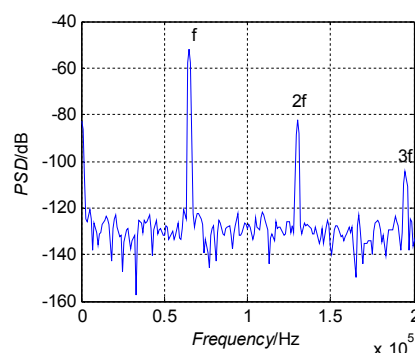


Figure 10. Spectrum of damaged beam  
(F= 65 KHz; V= 2Vp-p)

Subharmonic method is more efficient than harmonic method as shown in these experiment results. The subharmonic conditions of the experiment result shows that the threshold of excitation amplitude of the generation of subharmonic for the cracks in beams is between 2Vp-p and 10Vp-p.

### 3.2 SUBHARMONIC GENERATION CONDITION

Based on the experimental results, the excitation voltage is set to be 10Vp-p. Tuning the output signal around 64.75 KHz to obtain the frequency range which can produce subharmonic under the excitation voltage of 10Vp-p, and the range of frequency is 64.71 KHz~64.81 KHz. It is important to note that 10Vp-p is the maximum voltage signal generator can output. We tuned the excitation frequency from 64.71 KHz to 64.81 KHz with the step of 0.01 KHz and at each frequency the excitation amplitude is turned from 10Vp-p to 0Vp-p with the step of 0.1Vp-p until the subharmonic disappear. The region of excitation frequency and excitation amplitude of subharmonic is obtained as shown in figure 11.

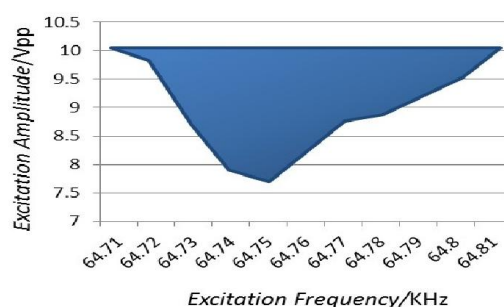


Figure 11. The region of excitation frequency and amplitude for subharmonic generation of the cracked beam

In figure 11, the generation range of subharmonic is shown as the blue area and this looks like a V shape. When the excitation frequency is 64.75 KHz, excitation amplitude reaches its minimum which is 7.8Vp-p. The experiment results show that the generation of subharmonic in real structure with fatigue cracks has specific requirements for excitation conditions. The threshold of



subharmonic is a starting point of distinguishing crack nonlinearity from equipment nonlinearity, but it also requires repeated estimation and tuning for excitation frequency and excitation amplitude. In spite of the high S/N of subharmonic component, the damage detection is only a qualitative judgment based on generation of subharmonic component in frequency domain and remains to be investigated.

## CONCLUSION

The results of simulation and experiments demonstrated that the generation of subharmonic in real structure with fatigue crack has specific requirements for excitation conditions, i.e. excitation frequency should be around twice the natural frequency and the excitation amplitude should reach the threshold. The excitation amplitude which can produce subharmonic component reaches its minimum when the excitation frequency is twice the natural frequency, and the region of excitation conditions of subharmonic looks like a V shape.

The fact that the generation of subharmonic is not affected by the nonlinearity of transducer or test equipment is an advantage of damage detection for fatigue cracks. The generation of subharmonic is related to the excitation frequency and excitation amplitude; this study can be used as reference of damage detection for practical structures to differentiate structural boundary nonlinearity.

## ACKNOWLEDGMENTS

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