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ON THE COUPLING OF GUIDED CYLINDRICAL WAVES IN THICK-WALLED STRUCTURES

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ABSTRACT

In this study, systematic analysis of the wave propagation characteristics in a thick-walled cylindrical structure was achieved with a purpose of facilitating the development of a damage detection system for train axles. The correspondence of thick hollow cylinder waves to the plate wave solutions of symmetric and antisymmetric modes, and the influence of thickness and radius on wave propagation were examined through the asymptotic approximations of Bessel functions. The analytical solutions to dispersion characteristics were compared with numerical simulations using the Local Interaction Simulation Approach and experimental validation.

KEYWORDS : *cylindrical guided waves, thick walled structures, modelling, LISA*

INTRODUCTION

The train axles are deemed as one of the most critical components of train bogies. Bearing most payloads and dead loads of carriages, they are prone to fatigue cracks, leading to catastrophic failure of the whole bogie structure without timely awareness. Most of the fatigue cracks in axles develop from surface defects, which are often exposed to harsh environmental conditions such as snow, high humidity and temperature changes; although some cases in which the cracks were initiated inside the axle wall have also been reported [1]. Therefore, means for online, nondestructive and in-operational methods to evaluate structural health of such critical components are sought. Currently existing methods for the axles inspection are mainly based on non-destructive techniques (NDT) (e.g. eddy-current), which are costly and can be ineffective. Most of the axle inspections are performed during pre-scheduled checkups. This approach requires however, that time between crack occurrence and failure of axle to be longer than the inspections period. Because the NDT-based maintenance terminate trains from normal operation, the schedule is a trade-off between the cost and probability of terminal crack development.

Recently, of particular interest are continuous monitoring strategies based on the guided wave propagation [2]. Since train axles are typically designed as thick-walled hollow cylinders a detailed study of guided wave propagation characteristics in tubular waveguides is required. An exact solution for a homogeneous, hollow cylinder was first introduced by Gazis [3]. Since then a common classification of the waves propagating in the axial direction of a cylinder as axisymmetric modes (i.e. decoupled longitudinal and torsional modes), and non-axisymmetric modes (i.e. flexural modes) has been established. The axisymmetric modes gained particular attention for damage detection in thin walled pipes, as they allow inspection of the whole circumference of the tube with constant-amplitude wave mode. The applications of techniques based on axisymmetric modes include structure evaluation for corrosion [4] and tapered notches [5]. However, since the symmetry condition of the mode shapes

requires an array of transducers located around the circumference of the pipe to excite the structure, practical applications are limited. To address this issue, the flexural modes in thin walled cylinders were studied by Li and Rose [6]. The authors proposed the concept based on the non-axisymmetric modes at the area of inspection to facilitate crack detection.

Although guided wave propagation in thin tubular structures has been intensively investigated over the last decades, less research focus has been paid to the scenario of cylindrical waveguides with significant thickness-to-wavelength ratio, i.e. thick-walled cylinders. The phenomena associated with wave propagation in such waveguides are substantially different from those in thin walled structures. Thus, inspection approaches applicable to relatively thin wall tubular structures may not be effective and/or applicable to thick-walled cylinders such as train axles.

The paper presents a theoretical investigation of the axisymmetric longitudinal wave modes in thick-walled pipes, followed by a numerical simulation results using the Local Interaction Simulation Approach (LISA) and experimental validation with 1D laser vibrometer measurements. The main goal of the analysis is to facilitate an NDT/SHM monitoring strategy development through a detailed comparison of wave features common and different for guided waves in plates, thin- and thick-walled cylinders.

1. THEORETICAL BACKGROUND

The theoretical framework for wave propagation in an isotropic, homogeneous, hollow cylinder with traction free boundary conditions developed by Gazis [3] was followed by the authors to numerically evaluate the dispersion characteristics. For brevity, only axisymmetric, $L(0,m)$ (involving only axial and radial displacements), and torsional, $T(0,m)$ (with only angular displacements) modes are considered in this work.

First, the general solutions for a hollow cylinder and a plate are considered. Figure 1a shows the dispersion curves for plate and a hollow cylinder. For both structures the thickness of 75 mm and material properties for steel have been selected. Inner radius of 25 mm was taken for the cylinder. The limits of bulk compressional ($c_l = 5.96$ km/s) and shear ($c_t = 3.26$ km/s) waves velocities and the Rayleigh wave velocity ($c_r = 3.02$ km/s) are marked in dashed lines to facilitate further discussion. As can be seen, the results show substantial differences for the plate and the cylinder, which is attributed to specimen's curvature only, as reported in [6].

The results indicate that, in the case of thick wall cylindrical structures, the approximation of the longitudinal dispersion curves by the Lamb modes of a thick plate can be introduced. The overlapping of the high order modes of the plate and the hollow cylinder in Figure 1a, reveals a close correlation between physical phenomena existing in these two types of waveguides. In both structures, the curvature effect on waves dispersion characteristics, is significant only for a few first modes and diminish with increasing thickness to wavelength ratio. A detailed analysis of dispersion characteristics of thick walled structures (Figure 1b) has also shown a repeating interlacing pairs of longitudinal modes in the phase velocity range above the compressional wave velocity value. Below, the longitudinal modes decouple and, except for the first two modes, approach the shear wave velocity. With an increase of frequency, the two lowest modes tend to Rayleigh wave velocity and their displacement profiles are confined to the surfaces of the cylinder.

As indicated, in the range of wavelengths much smaller than the wall thickness, a repetitive interlacing phenomenon of the modes can be observed. A consecutive pairs of longitudinal modes, which are coupled through a radial stress component, follow a similar path interweaving each other. In the high frequency region, the cross-section points of the longitudinal modes overlap with intersections of longitudinal and torsional modes following a regular pattern (Figure 1b). At these points, the wavenumber-frequency pairs satisfy the free boundary conditions for both of the axisymmetric types of modes. A similar behaviour has been observed by Mindlin [7] for high frequency Lamb waves, where the symmetric and antisymmetric modes form so called terrace like structures. Such behaviour,

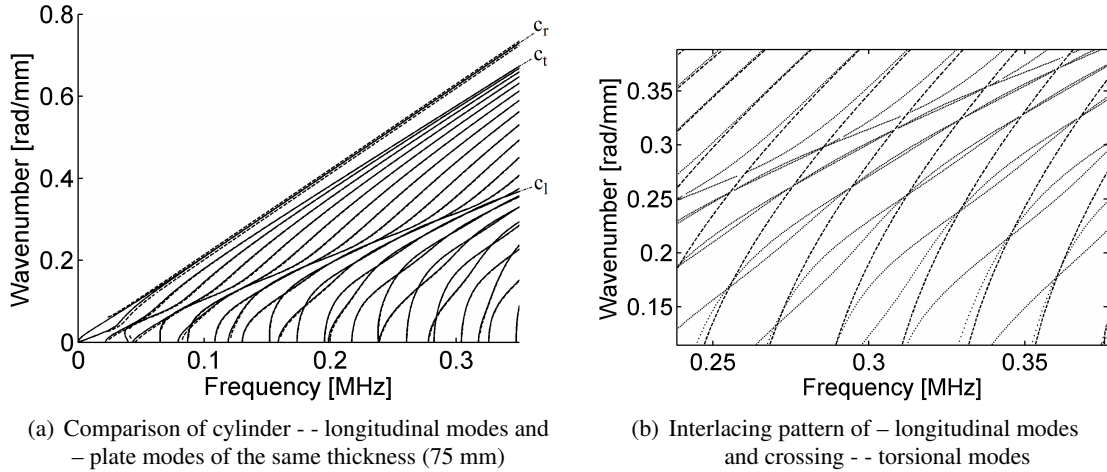


Figure 1: Analytically evaluated dispersion curves of a axisymmetric modes of a hollow cylinder (inner rad. 25 mm, thickness 75 mm, steel);
 --- bulk velocities limits: compressional $c_l = 5.96$ km/s, shear $c_t = 3.26$ km/s,
 Rayleigh wave $c_r = 3.02$ km/s

was recently experimentally confirmed for plates by Veres [8]. The modes interlacing phenomenon discussed here using an example of a thick wall cylindrical structure is also present in thin wall pipes. However, this requires higher order modes excitation at high frequencies, thus the repetitive pattern is hardly discernible.

To better describe the interlacing phenomenon and the correspondence between a hollow cylinder and plate solutions, an asymptotic approximation of Bessel functions was used. It can be shown that for integer order n and argument $x > n$, both Bessel functions qualitatively resemble sine and cosine functions which amplitude decay as $x^{-1/2}$. The asymptotic forms for $x \gg n$ are [9]

$$\begin{aligned}
 J_n(x) &\sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right) \\
 Y_n(x) &\sim \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right)
 \end{aligned}
 \tag{1}$$

where $J_n(x), Y_n(x)$ denote Bessel functions of first and second kind, respectively. For axisymmetric modes, the characteristic equation involves Bessel functions of orders zero and one. The argument is the multiplication of inner or outer radius with α or β calculated from $\alpha^2 = \omega^2/c_p^2 - \xi^2$ and $\beta^2 = \omega^2/c_s^2 - \xi^2$, where ξ, ω, c_p, c_s , are wavenumber, angular frequency, and compressional and shear wave velocities, respectively. Figure 2 shows a comparison of the dispersion characteristics calculated using the exact, analytical formulation, and the Bessel-based approximation. As shown, despite the discrepancies in low frequency range, the asymptotic approximation can be successfully used to evaluate the higher order longitudinal modes.

The plate and hollow cylinder dispersion characteristics can be further investigated through the mode shapes analysis. The substitution of Eq. 1 into wave displacements relations [3], and grouping into pseudo-symmetrical and pseudo-antisymmetrical parts across the wall thickness, yield

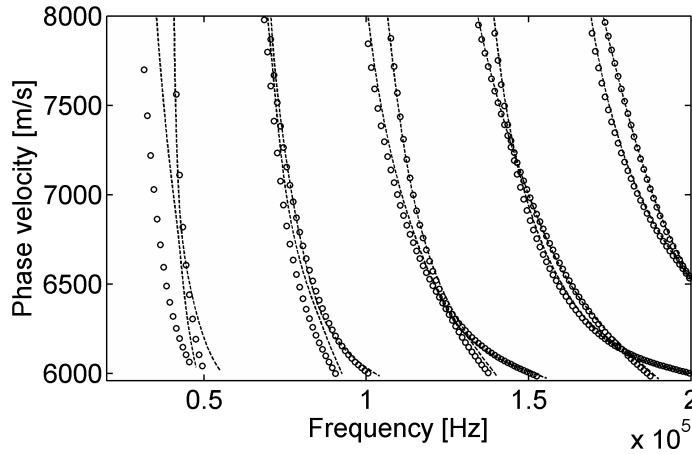


Figure 2: Comparison of - - exact theoretical and \circ approximated dispersion curves of the hollow cylinder (inner rad. 25 mm, thickness 75 mm, steel) longitudinal modes, above compressional velocity limit $c_l = 5.96$ km/s

the following formulas for approximated radial and axial displacements

$$u_r = \left[\frac{\alpha}{\sqrt{\pi\alpha r}} \cos(\alpha r)(A + B) - \frac{\xi}{\sqrt{\pi\beta r}} \cos(\beta r)(A_1 + B_1) \right] + \left[\frac{-\alpha}{\sqrt{\pi\alpha r}} \sin(\alpha r)(A - B) + \frac{\xi}{\sqrt{\pi\beta r}} \sin(\beta r)(A_1 - B_1) \right] \quad (2)$$

$$u_z = \left[\frac{\beta}{\sqrt{\pi\beta r}} \sin(\beta r)(A_1 + B_1) + \frac{\xi}{\sqrt{\pi\alpha r}} \sin(\alpha r)(A + B) \right] + \left[i \frac{-\beta}{\sqrt{\pi\beta r}} \cos(\beta r)(A_1 - B_1) + \frac{\xi}{\sqrt{\pi\alpha r}} \cos(\alpha r)(A - B) \right] \quad (3)$$

where propagation term $e^{i(\xi z - \omega t)}$ was omitted for brevity, r denotes the radius and ξ the wavenumber. α , β are defined as $\alpha^2 = \omega^2/c_l^2 - \xi^2$ and $\beta^2 = \omega^2/c_t^2 - \xi^2$. A , B , A_1 and B_1 are determined from the boundary conditions. The waves with amplitudes $(A - B)$ and $(A_1 - B_1)$ can be considered as the pseudo-symmetric, while the $(A + B)$ and $(A_1 + B_1)$ as pseudo-antisymmetric modes.

To establish the conditions for hollow cylinders solution approximation by the plate, the displacements at internal and external surfaces of the cylinder are calculated. For this purpose a relationship between outer r_o and inner radius r_i is introduced: $r_o = r_i + 2d$, where d is half-thickness. Finally, using the transverse resonance principle [10], i.e. that partial waves must experience a phase shift of an integral multiple of 2π during the round trip from outer-surface back to the same point, the relations of radial and axial displacements at outer $u(z, r_o)$ and inner $u(z, r_i)$ surface of the cylinder for pseudo-antisymmetrical case can be expressed by Eq. 4

$$u_r(z, r_o) = \sqrt{\frac{r_i}{r_i + 2d}} u_r(z, r_i)$$

$$u_z(z, r_o) = -\sqrt{\frac{r_i}{r_i + 2d}} u_z(z, r_i) \quad (4)$$

and for pseudo-symmetrical case by Eq. 5

$$\begin{aligned}
 u_r(z, r_o) &= -\sqrt{\frac{r_i}{r_i + 2d}} u_r(z, r_i) \\
 u_z(z, r_o) &= \sqrt{\frac{r_i}{r_i + 2d}} u_z(z, r_i)
 \end{aligned}
 \tag{5}$$

It can be observed that for inner radius approaching infinity or the thickness being sufficiently small comparing to r_i , Lamb wave modes displacement relations are obtained. However, asymptotic approximations are not valid for r_i equal to zero, i.e. the solid cylinder case. The further limitations include small values of α and β , and the modes of Rayleigh wave velocity.

The categorisation of the mode shapes of a thin walled hollow cylinder into pseudo-symmetrical and pseudo-antisymmetrical using different concept, namely a large radius approximation was discussed in [11]. The proposed approach, based on asymptotic approximation of Bessel functions, provides an efficient tool for analysis and allows for more general discussion on the phenomena characteristic for higher order modes related to Lamb waves without radius and thickness restrictions.

2. NUMERICAL STUDY

Following the theoretical study, numerical simulations with broadband excitation were carried out using the LISA method [12] to determine dispersion characteristics in a virtual experiment. Cubic elements were used with the element size equal to 0.5mm. Time step was 0.05 μ s to ensure the stability of the algorithm. The geometrical model dimensions corresponded to the physical setup with the exception of the length of the model. The model consisted of the total number of 65M elements resulting in 195M degrees of freedom. Material properties equivalent to aluminium were assumed, i.e. $E = 70$ GPa, $\rho = 2700 \frac{kg}{m^3}$ and $\nu = 0.3$. The excitation was introduced as prescribed out-of-plane displacement component for a single node. Displacement responses were acquired at selected nodal points during the simulation.

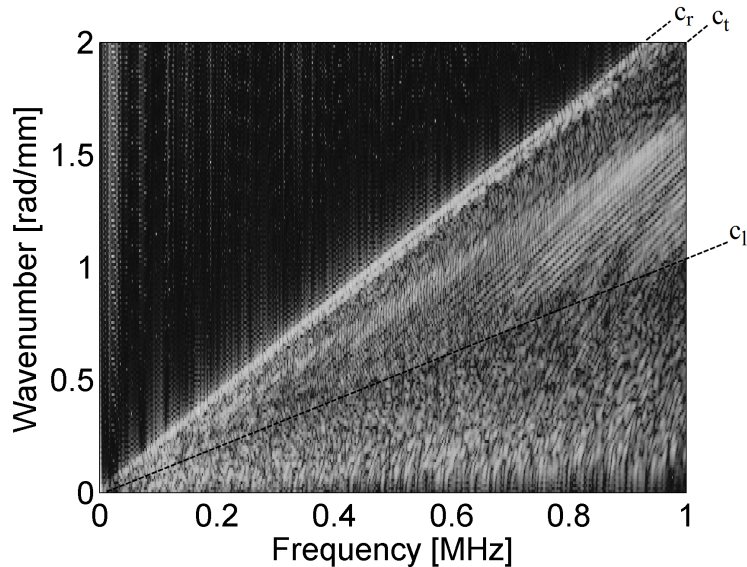


Figure 3: Simulation-based dispersion characteristics of a hollow cylinder (inner rad. 25 mm, thickness 75 mm, aluminium); dashed lines correspond to bulk velocities limits: compressional $c_l = 6.32$ km/s, shear $c_t = 3.13$ km/s and Rayleigh wave $c_r = 2.92$ km/s

To estimate the dispersion characteristics, a broadband excitation was used. The excitation point was selected on the cylindrical surface close to the edge, to effectively capture the longest propagation path and avoid reflected waves superposition on the main excitation pulse. A chirp signal sweeping a frequency range 50-150 kHz was used. The responses were acquired at the outer surface along a line parallel to the cylinder's axis. A total number of 471 responses was recorded from the simulation. Analysis time was set to 0.5ms, to allow double wave reflection from the flat surfaces. Calculation time was 4 minutes using four GPUs. Acquired signals were processed using a two-dimensional spatio-temporal transform resulting in the wavenumber-frequency representation, i.e. the dispersion characteristics. The results are presented in Figure 3.

For considered excitation, complex, multi-modal character of wave propagation was observed. A strong Rayleigh wave at the outer surface, as depicted by a straight diagonal line was noted. As can be seen in Figure 3, except for the Rayleigh mode no clear mode branches can be distinguished. However, a grid formed by a number of closely lying, intersecting modes is observed below the compressional velocity limit. On the other hand, a clear pattern of parallel modes can be seen above this constant-velocity line (for analytical solution refer to Figure 1a). Since all wave modes are present in the experimentally evaluated dispersion curves, i.e. not only longitudinal axisymmetric ones, a very fine resolution in the wavenumber domain should be applied. This is limited by the length of the array of transducers placed at the outer surface and, in consequence, the total length of the cylinder. For this purpose, an investigation of a longer pipe model is required.

3. EXPERIMENTAL VERIFICATION

A hollowed, aluminium cylinder of a total length of 500 mm, with an outer radius 100 mm and inner radius 25 mm was used in the experiments. The structure was excited by a single piezo-stack NOLIAC transducers of 2x2x4 mm. The measurements of the out-of-plane displacements were performed with one-dimensional (PSV-400) Polytec Laser Vibrometer. The same measurement procedures and data processing as for the numerical simulation were followed. The displacements were detected with spacial resolution of 1mm and a sampling frequency of 5.12 MHz. The cylinder was excited with a chirp signal of the frequency range 50-800 kHz and total time 250 μ s. Moreover, to reduce the signal-to-noise ratio, averaging using 100 samples was carried out in all measurements. The dispersion curves were evaluated using two-dimensional Fourier transform. To reduce leakage effect, the Hanning window in space domain was first applied to the signals. The experimentally evaluated wavenumber-frequency dispersion curves are presented in Figure 4.

The experimental results show a good qualitative agreement with the numerical studies. The surface waves dominates the responses (see the diagonal line), however a strong excitation of the higher modes in the region between compressional and shear bulk velocities can be also observed. The interlacing modes pattern is visible around 200 kHz below the c_l line. However, since a single point excitation was used, the results contain both the longitudinal and flexural modes. Therefore the wave pattern is substantially complex. One can also notice the change of the modes character to distinguishable set of parallel lines between the c_l and c_t lines. It should be noted that due to the limitations of the experiential setup, namely the signal amplifier, the results above 500 kHz are disturbed and the mode behaviour is blurred.

CONCLUSIONS

The guided wave behaviour in a thick-walled, hollow cylinder has been studied through analytical, numerical and experimental analyses. A complex, multimodal character of the cylindrical guided waves has been observed.

The theoretical and numerical studies have revealed the phenomena of repetitive, interlacing axisymmetric longitudinal modes in the dispersion characteristics. An asymptotic approximation of

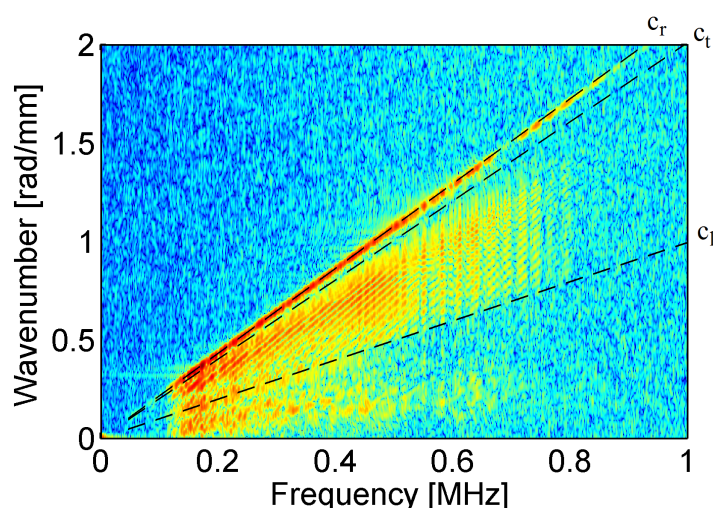


Figure 4: Experimentally evaluated dispersion characteristics of a hollow cylinder (inner rad. 25 mm, outer rad. 100 mm, aluminium); dashed lines correspond to bulk velocities limits: compressional $c_l = 6.32$ km/s, shear $c_t = 3.13$ km/s and Rayleigh wave $c_r = 2.92$ km/s

Bessel functions was used to formulate the relationship between the particle displacement profiles at outer and inner surface of a hollow cylinder.

The numerical and experimental analysis confirmed the presence of higher order modes of the hollow cylinder in the measured signals. Although, the thickness of the analysed structure is considerable and the surface confined wave modes are dominant, the other, faster modes can also be detected. A clear distinction of interlacing modes, indicated by variable thickness of single lobes for low wave numbers in numerical simulation, and hardly excited in experimental verification, can be obtained by increasing the length of measurement path, and thus increasing the wavenumber resolution.

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