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STRUCTURE AND FORCES IN STRESSED 3D PACKINGS

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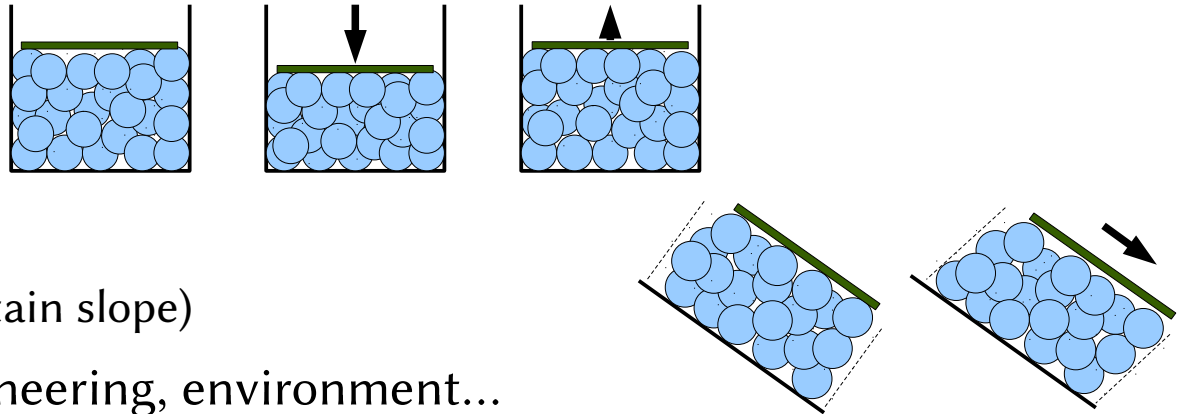
² Geostat team, INRIA Bordeaux

Gdr Phenix, Driven Disordered Systems meeting June 2014

RATIONALE / WHY

Mechanical stresses on granular assemblies are ubiquitous

- Repeated compression
(e.g. trucks passing on a road)
- Shearing
(e.g. gravitational pull on a mountain slope)
- Industrial processes, civil engineering, environment...



Understanding their macroscopic response is necessary

- Requires “seeing” what is the state at the level of grains

Most studies are 2D. Most real cases are 3D.

This work = structure + forces in 3D

ACCESSING THE MICRO-STRUCTURE

X-rays / micro-ct

Fine resolution

Most materials

Costly

Confocal: emulsions

Microscopic

Costly

Difficult to control applied stresses

This work: refractive index matching

Macroscopic grains

Easy to control, tri-axial shearing

Cheap

Submersed

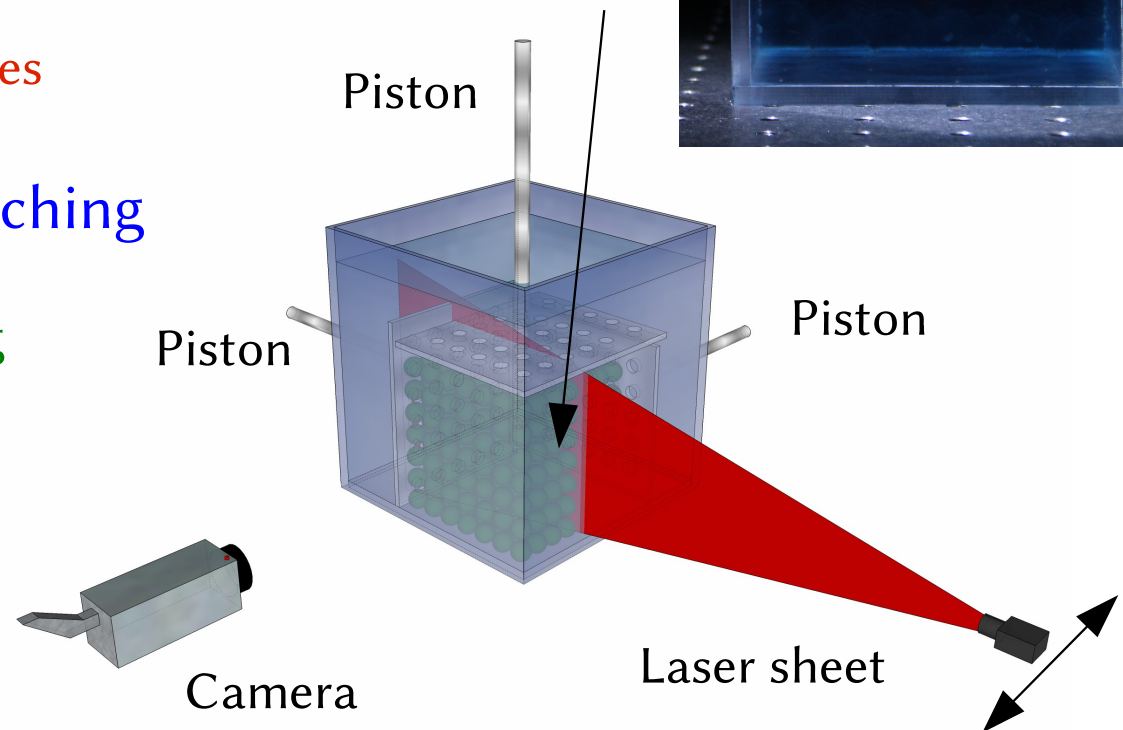
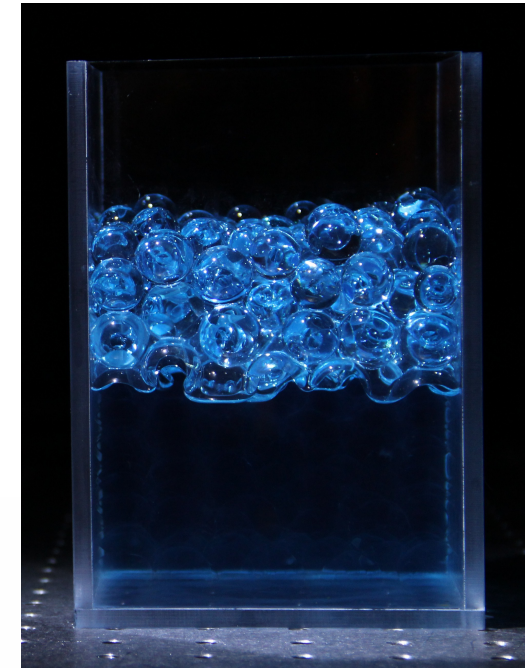
Next slides on:

1. Structure
2. Forces in 3D

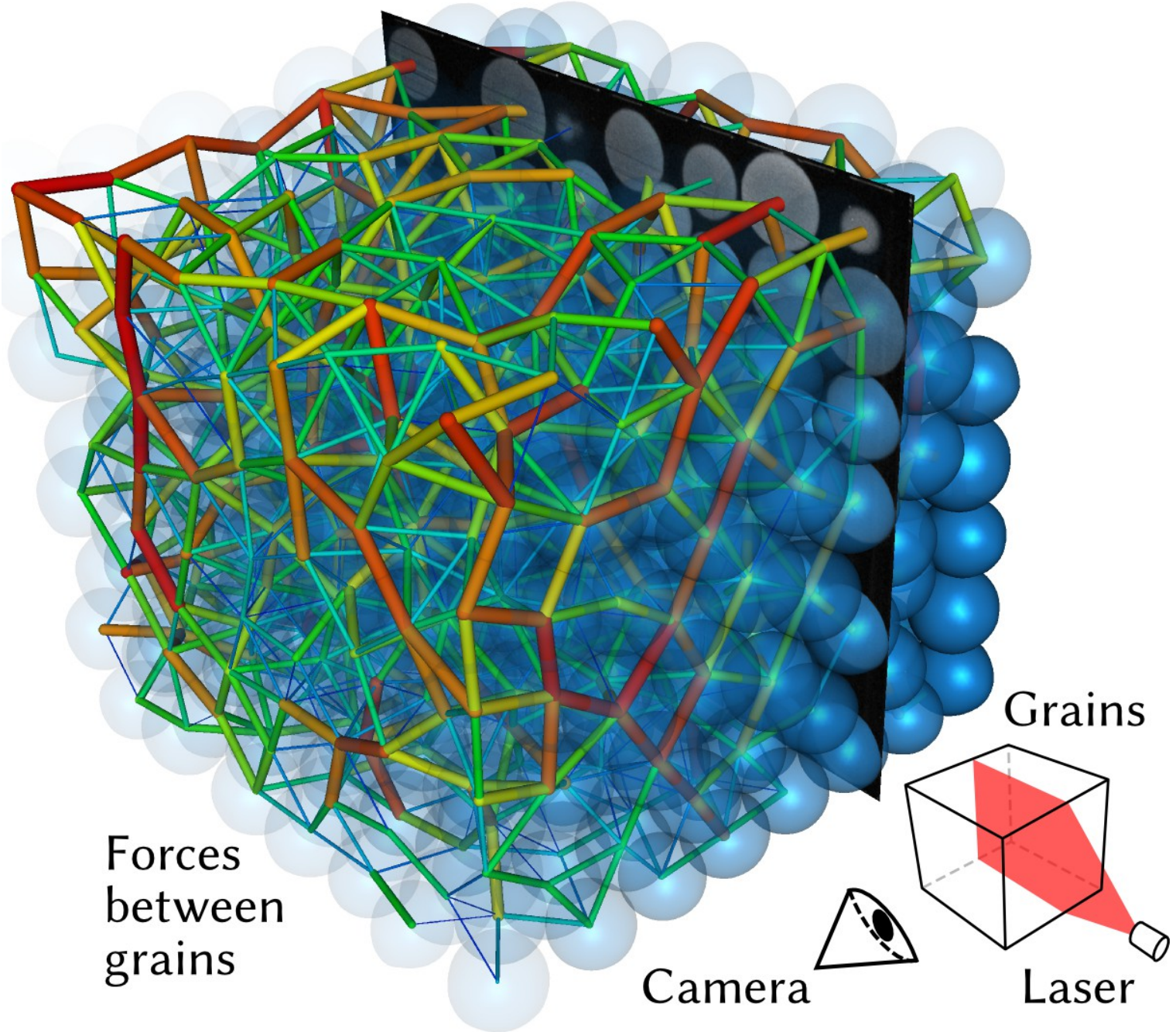
Mukhopadhyay *et al.*
Phys. Rev. E 84, 011302, 2011

Dijksman *et al.*
Rev. Sci. Instrum. 2012

Hydrogel grains
index-matched
+ fluorescent dye



WHAT WE GET



TYPICAL IMAGE

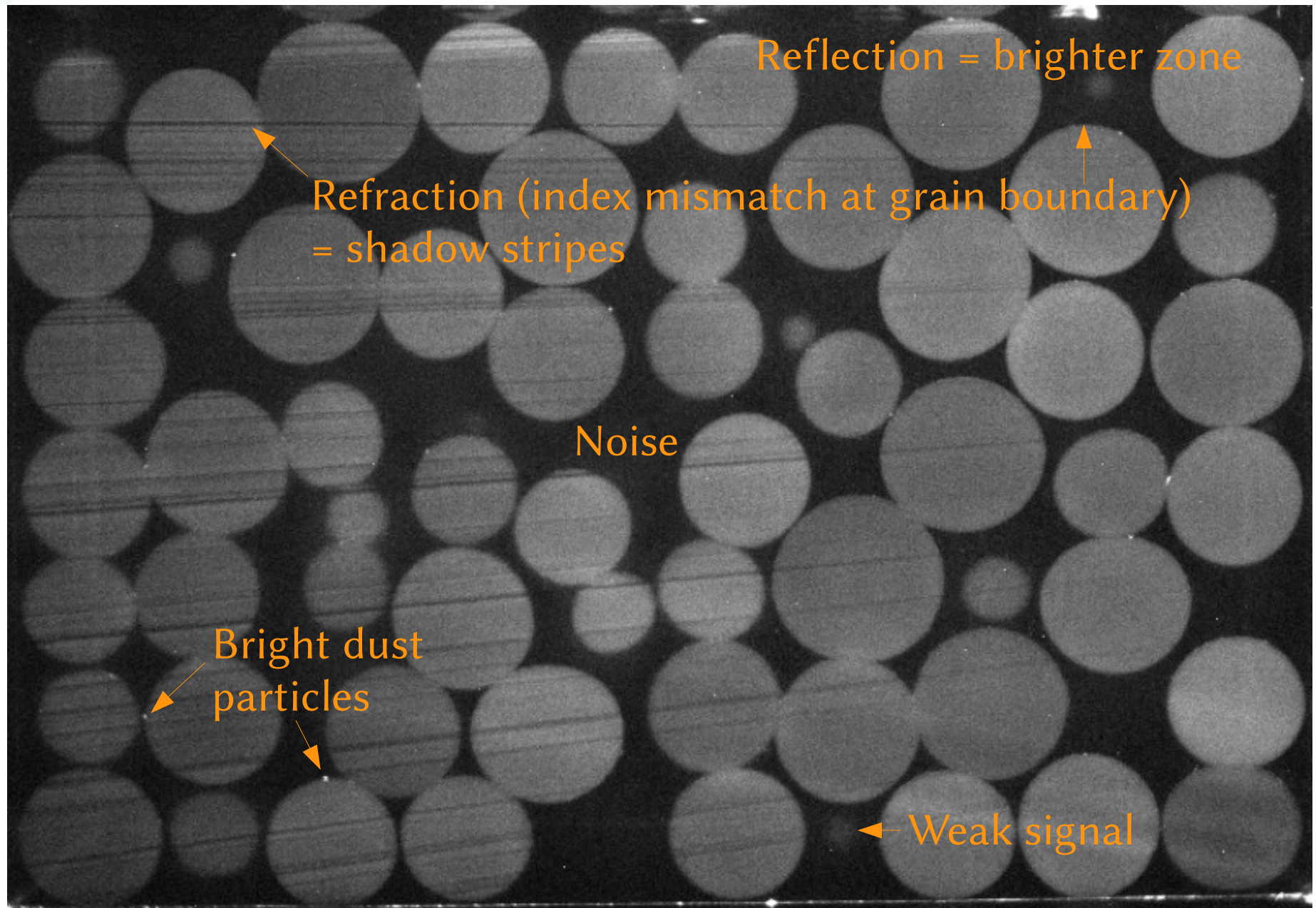
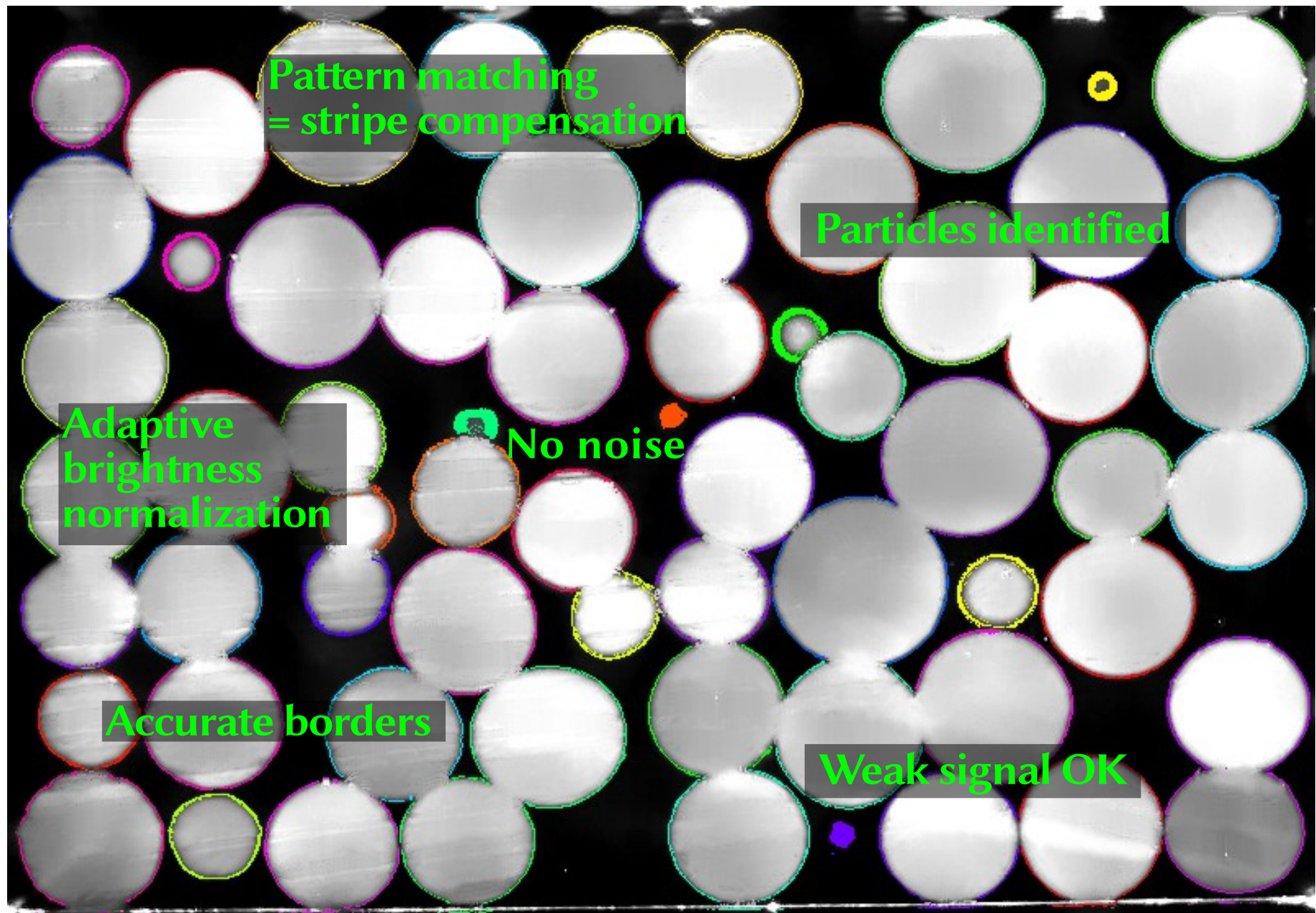


IMAGE PROCESSING



FROM 2D IMAGES TO 3D GRAINS

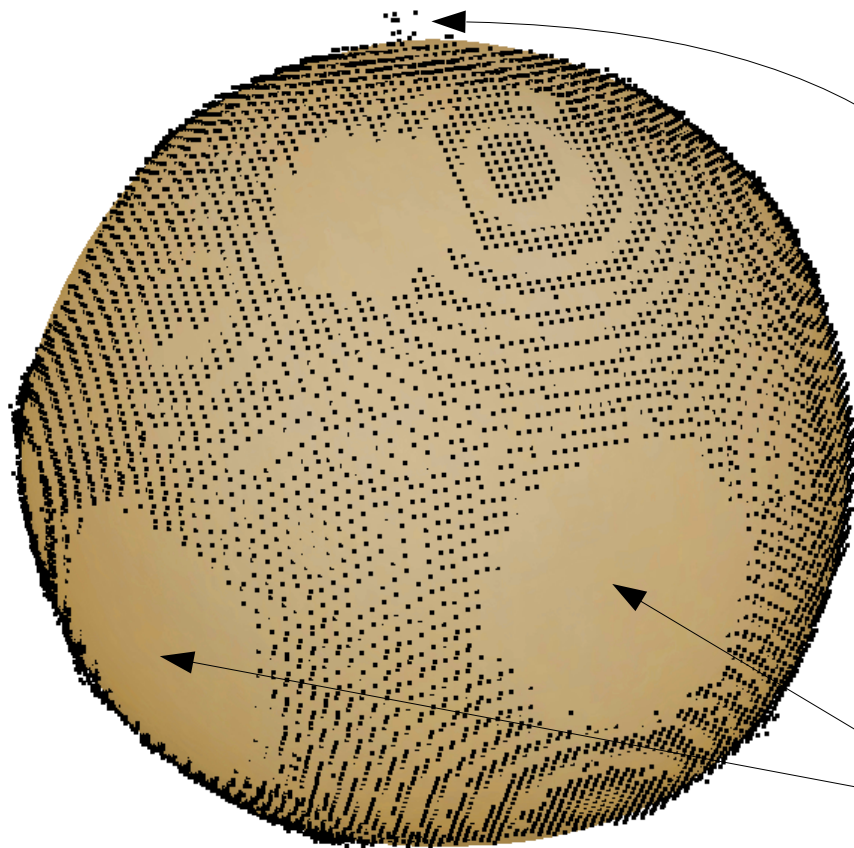
Step 1: Stack the images into 3D voxel

Step 2: Detect border voxels

Step 3: Fit an analytic surface to these borders

Done here using a spline basis of functions on the unit sphere

Step 4: Use these surfaces to get accurate forces

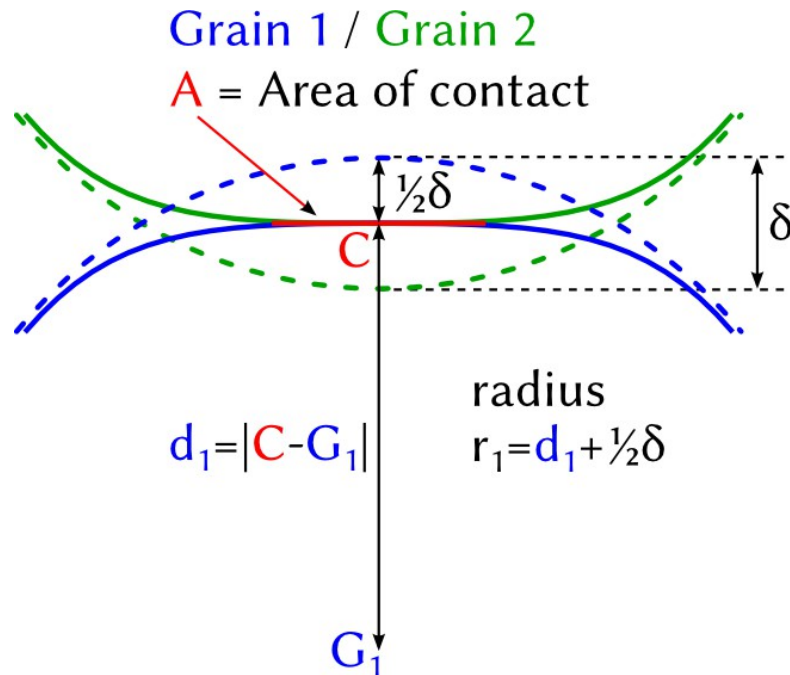


Outliers completely eliminated

Contacts = no border between grains
BUT surface area is well defined

INFERRING FORCES IN FULL 3D

Analytic shape descriptions \Rightarrow contact properties



Measured here: A, C, G_1, G_2, d_1, d_2 .
 Unknown: δ

Contact properties \Rightarrow forces

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{radius of curvature at contact}$$

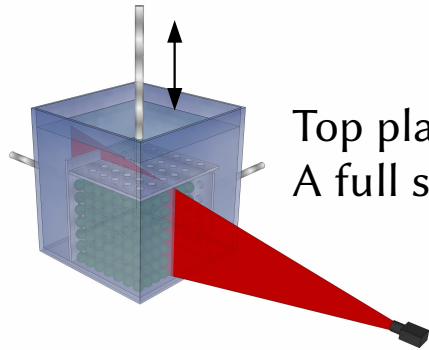
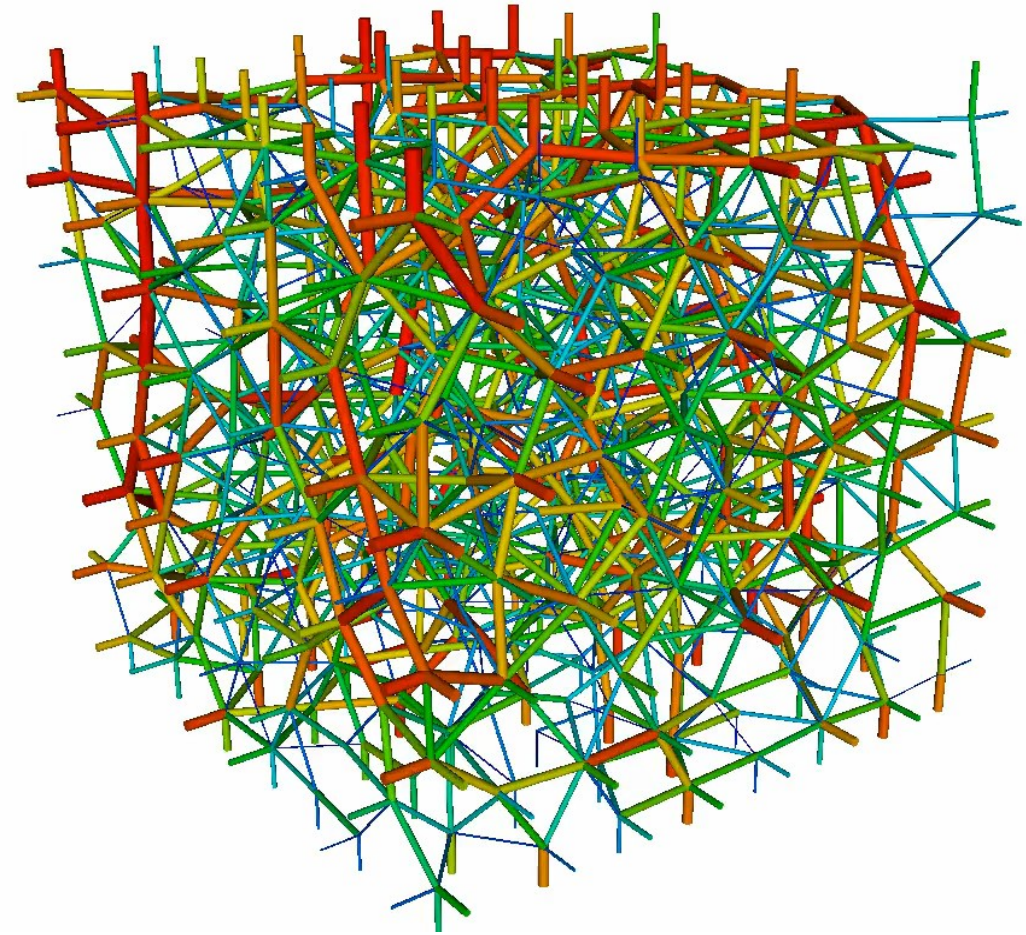
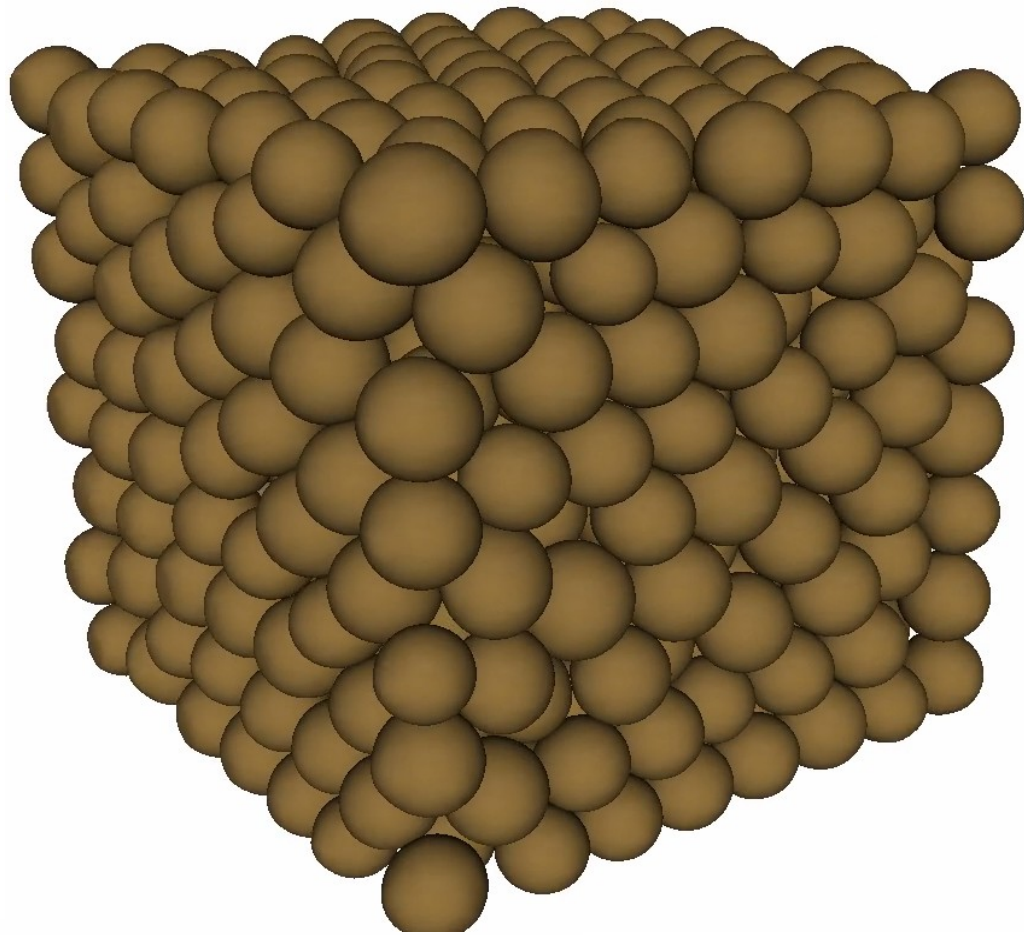
$$F = E r^{1/2} \delta^{3/2} \quad E = \text{effective Young modulus}$$

$$F = E \delta a \quad a = \sqrt{A/\pi} \quad \text{radius of the contact}$$

$$\text{Hence } r \delta = A/\pi \Rightarrow \delta \Rightarrow F \quad (\text{with given } E)$$

\Rightarrow Vector forces in full 3D, with orientation, position, norm
 + grain centers of mass, stress tensor, etc.

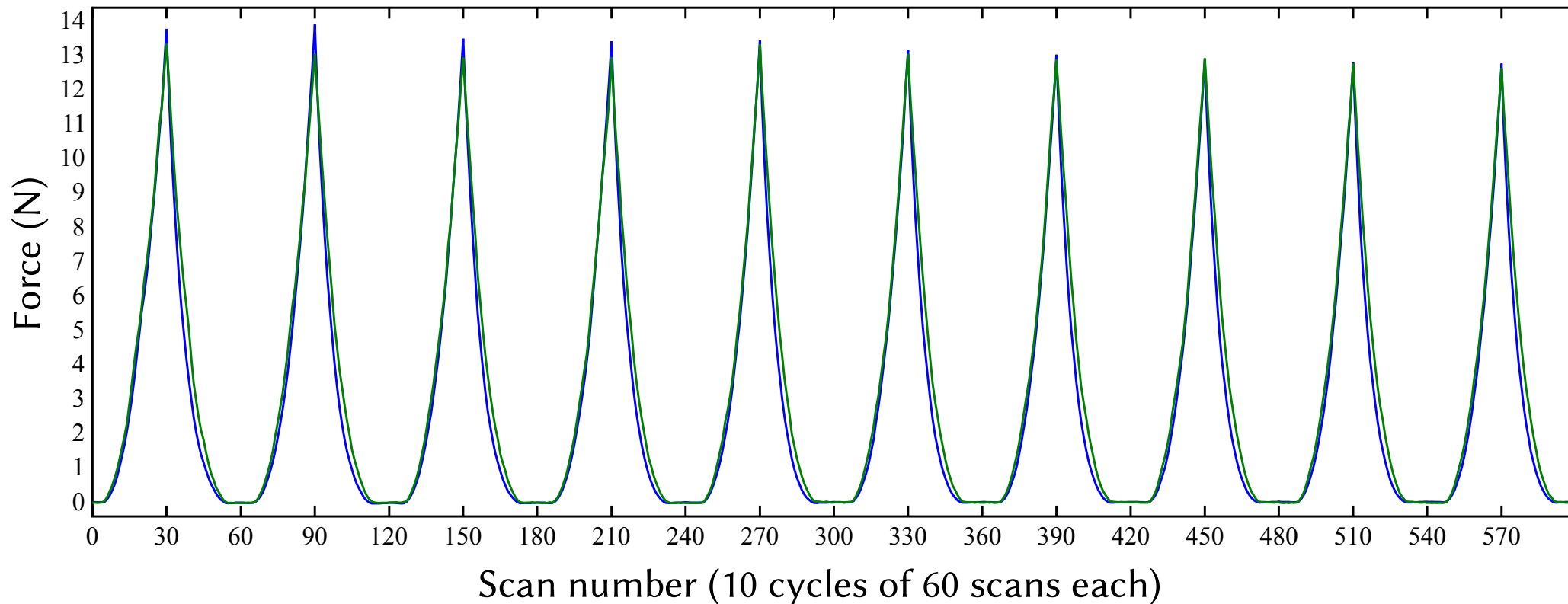
10 UNI-AXIAL COMPRESSION CYCLES



Top plate moves by 1mm increments
A full scan is taken between increments

Forces = struts joining the grain centers
Blue = weakest, Red = strongest

VALIDATION ON COMPRESSION CYCLES



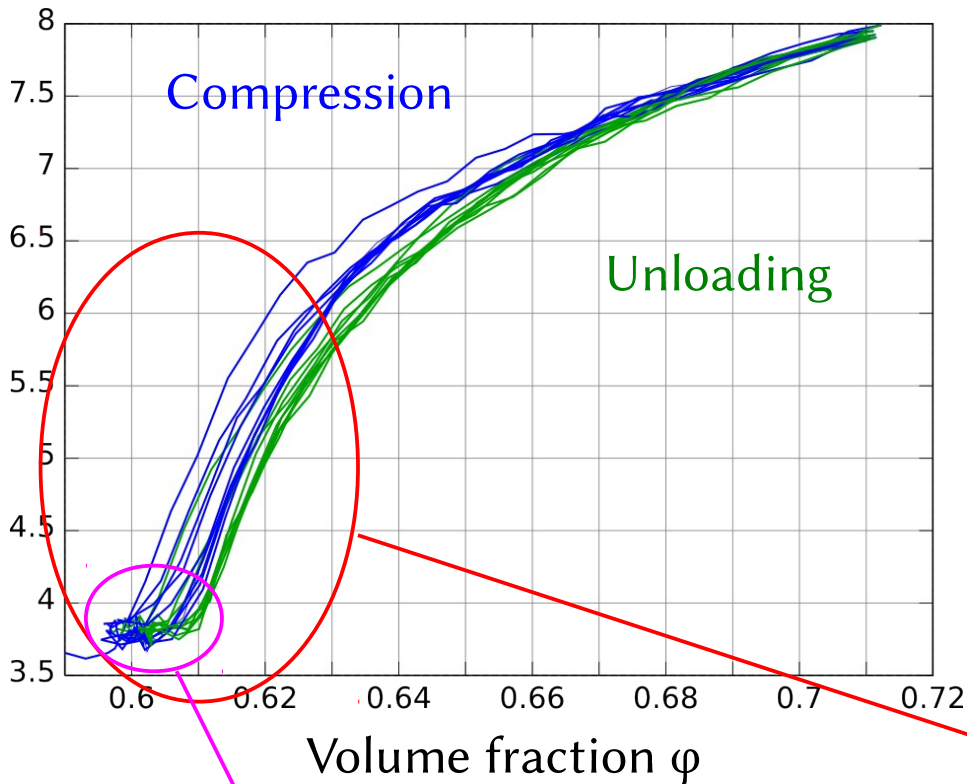
Blue = force measured on the top plate sensor

Green = force inferred from the images + measure of $E=22.4$ kPa

Grain deformation up to $\approx 13\%$, scan processed independently: no global fitting
 $\approx 980 \cdot 10^3$ contacts over 600 scans. Resolution $\approx 10^{-2}$ N.

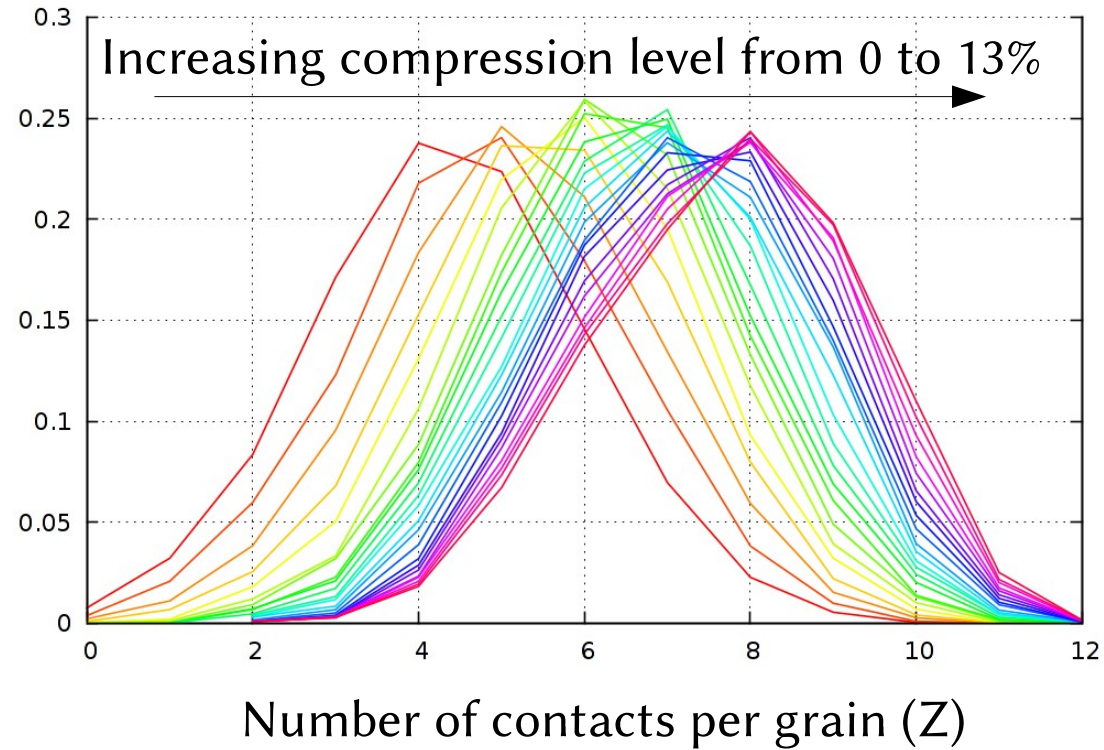
NUMBER OF CONTACTS PER GRAIN Z

Number of contacts per grain (Z)



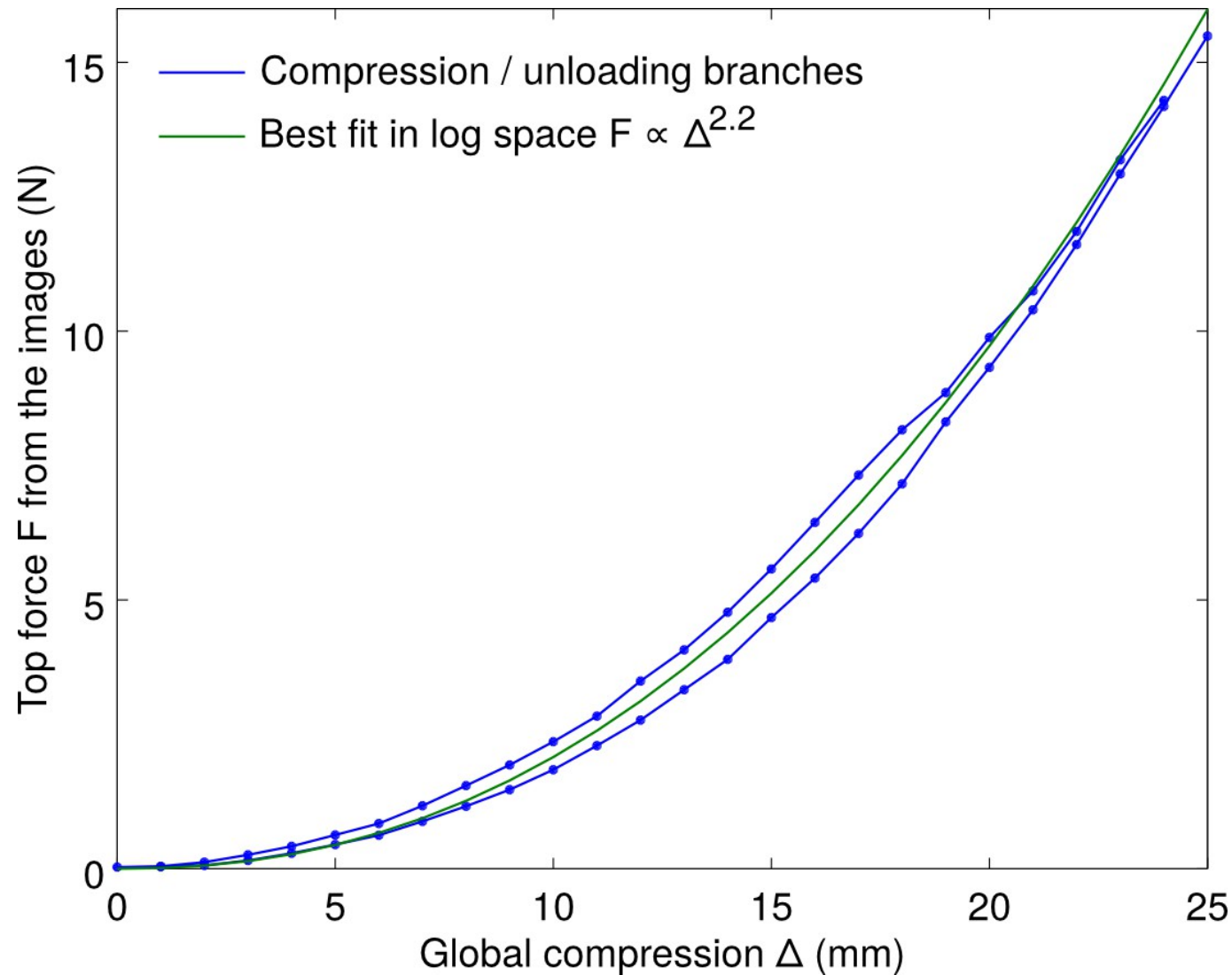
The top plate does not touch the grains
= ignored scans

$p(Z)$

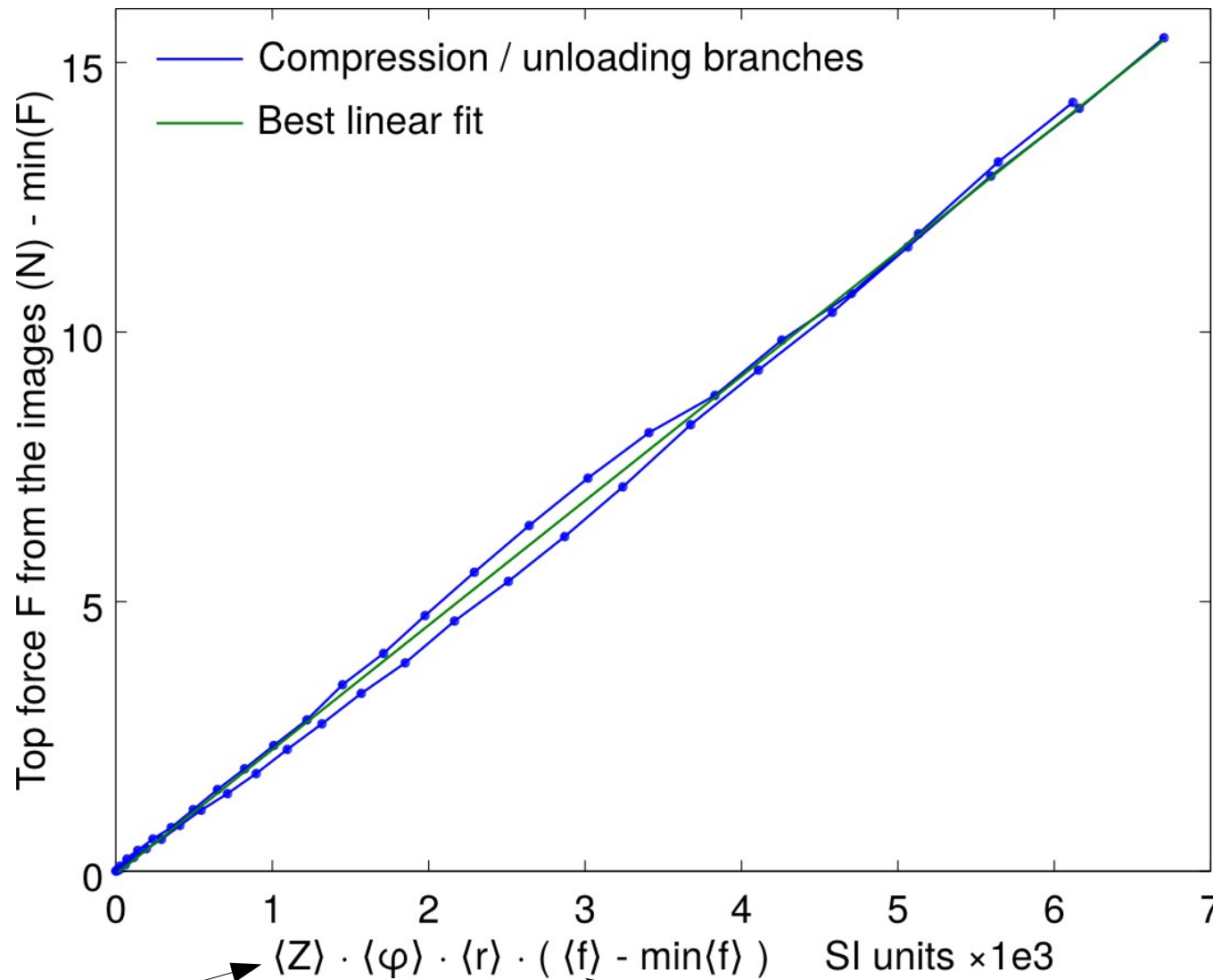


Weak contacts are missing and below the experiment resolution!

NON-HERTZIAN PACKING RESPONSE



A SCALING HOLDS, WHICH IS ...



#contacts / grain

volume fraction

radius

force at contacts

... A MEAN STRESS TENSOR

A mean $\langle Z \rangle$ in the relation \Rightarrow some kind of isotropy between contacts

\Rightarrow Use the isotropic pressure $p = \frac{1}{3} \text{tr} S$ with S the stress tensor.

Without hydrostatic gradient (density match), the force on the top plate $F = p \cdot L \cdot W$

For a given volume element V_e : $S = \frac{1}{V_e} \sum_{c \in V_e} \mathbf{b} \otimes \mathbf{f}_c$

With \mathbf{b} linking the grain centers and \mathbf{f}_c the force vector at contact c .

Sphere approximation: the trace is simply the dot product: $\text{tr} \mathbf{b} \otimes \mathbf{f}_c = \mathbf{b} \cdot \mathbf{f}_c$
and also $\langle \mathbf{b} \rangle = 2 \langle r \rangle$, with r = distance from center to contact for spheres.

The number of terms in the sum depends on the density of contacts v . Incompressible grains \Rightarrow avg. volume $\langle V \rangle$ is constant. With φ the grain volume fraction: $v = \frac{1}{2} Z \varphi / \langle V \rangle$

Replacing these terms by their averages over all contacts $\Rightarrow F \propto \langle Z \rangle \langle \varphi \rangle \langle r \rangle \langle f \rangle$

Note: subtracting min F and min $\langle f \rangle$ on both sides
in noisy experimental data for consistency with $f=0 \Rightarrow F=0$

DEMO + QUESTIONS