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## Stability analysis and exit problem formulation in a 2D model for general anesthesia

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A characteristic pattern with sequences of alternating quiescent ('down') and bursting ('up') states of activity can be observed in neuronal recordings during deep sedation induced by general anesthesia [1,2]. Prevailing models suggest that such bi-stable dynamics might be explained by the existence of notable non-linearities in the neuronal membrane potential as a function of the anesthetic agent (AA) level, with frequent noise-induced transitions between two stable (attracting) branches. Nevertheless, the mathematical tractability of these non-linear scenarios is limited by the 2D phase-space defined by the describing variables: the excitatory ( $V_e$ ) and inhibitory ( $V_i$ ) PSPs at excitatory cells. Exact solutions to these problems are only known for 1D systems.

Here we study the neural mass model presented in [3], formulated in terms of 1st-order (Langevin) SDEs, that we further simplified by replacing the Sigmoid-kind rate functions by others of Heaviside-type. Then, an analytical study is carried out to find the stability ranges and the type of fixed points that appear when the AA level changes, what broadens the analysis presented in [3]. Approximate expressions for the eigenvalues are derived, that consistently permits to classify the kind of fixed points in most of the cases. It is found that stable nodes might bifurcate to unstable fixed points or stable foci only when the effective membrane potential  $V = V_e - V_i$  decreases to values in the vicinity of the spiking excitatory threshold. The appearance of stable foci that could explain some oscillation frequencies found in neuronal recordings, however, can only be considered when the excitatory and inhibitory threshold values are approximately equal.

Finally, equations for the statistics of the residence times in the 'up' and 'down' states are presented. Previous numerical studies have shown that the 2D-trajectory likely escapes from one attractor to the other through a limited section of the saddle-line separating the attractions basins [4]. It is shown how the enunciation of the exit problems, in terms of Fokker-Planck equations, depends and is simplified by this interesting fact.

### References

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- [4] Garcia-Rodriguez P. and Hutt A., CNS meeting, 2014.