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Tag Second-preimage Attack against π -cipher

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Abstract. The π -cipher is one of the candidates of the CAESAR competition. One of the advertised features of the π -cipher is tag second-preimage resistance: it should be hard to generate a message with a given tag, even for the legitimate key holder (insider attack).

In this note, we show that the generalized birthday attack of Wagner gives a practical tag second-preimage attack against the π -cipher.

1 Introduction

The π -cipher [2] is an authenticated encryption algorithm submitted to the CAESAR competition. One of the extra features advertised by the designers is tag second-preimage resistance: it should be hard to produce second-preimages of a given tag, even for an adversary who knows the secret key (most authenticated encryption algorithm do not have this feature, and an insider can easily generate tag second-preimages).

As written in [2, 4.1], the tag generation of an m -block message with the π -cipher can be written as:

$$T = T'' \boxplus_8 e(1, M_1) \boxplus_8 e(2, M_2) \boxplus_8 \cdots \boxplus_8 e(m, M_m)$$

where e denotes a keyed function known to the key holder (the e-triplex), \boxplus_8 is a component-wise addition of vectors of 8 elements in \mathbb{Z}_{2^ω} , and T'' is the associated data tag (known to the insider). The word-size ω is 16, 32, or 64, depending on the security level. In a tag second-preimage attack, an insider wants to build a message M reaching a fixed tag \bar{T} . Without loss of generality, we assume $T'' = 0$ and $\bar{T} = 0$.

2 Generalized birthday attack

In the submission document of π -cipher, the tag second-preimage problem is seen as a knapsack problem, and the main attack considered is a variant of an attack by Camion and Patarin [1]. However, the generalization of this attack due to Wagner [3] can break the problem more efficiently.

The generalized birthday attack of Wagner is an attack against the m -sum problem: given m lists L_1, L_2, \dots, L_m , one find values $l_1 \in L_1, \dots, l_m \in L_m$ such that $\bigoplus_{i=1}^m l_i = 0$. If each list contains at least $2^{r/m}$ elements there is a good probability that a solution exists, but the best known algorithm is a simple birthday attack in time and memory $\tilde{O}(2^{r/2})$. The idea is to build two lists L_A

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and L_B with all the sums of elements in $L_1, \dots, L_{m/2}$ and $L_{m/2+1}, \dots, L_m$ respectively, sort L_A and L_B , and look for a match between the two lists (L_A and L_B contains $2^{r/2}$ elements).

Wagner's algorithm will have a lower complexity, but it requires more elements in the lists. For instance, with $m = 4$, it uses lists of size $2^{r/3}$ and in order to find one solution using $\tilde{O}(2^{r/3})$ time and memory. The basic operation of the algorithm is the general join $\bowtie_j: L \bowtie_j L'$ consists of all elements of $L \times L'$ that agree on their j least significant bits. More precisely, the operation can be defined over list of values with associated data:

$$L \bowtie_j L' = \left\{ (l \oplus l', (a, a')) \mid (l, a) \in L, (l', a') \in L', (l \oplus l')^{[0..j-1]} = 0^j \right\}.$$

The join operation is computed efficiently by sorting the lists L and L' according to the lower j bits, and stepping through the lists simultaneously in order to find values that agree on their low bits. Moreover, the sorting can be done in linear time using a hash table, or a radix sort.

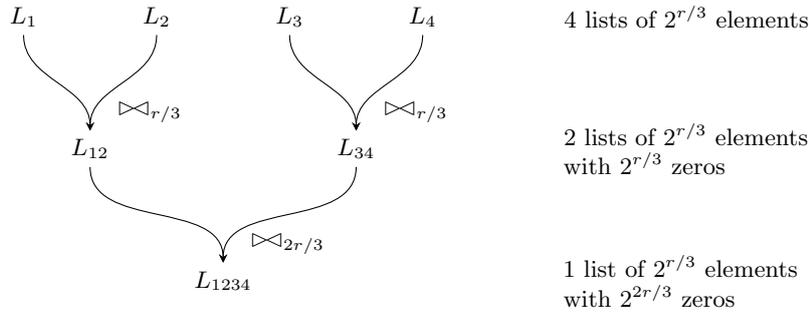


Fig. 1. Wagner's algorithm for $m = 4$

The generalized birthday algorithm for $m = 4$ is described by Figure 1. We first build the lists $L_{12} = L_1 \bowtie_{r/3} L_2$ and $L_{34} = L_3 \bowtie_{r/3} L_4$, containing about $2^{r/3}$ elements. Next, we build $L_{1234} = L_{12} \bowtie_{2r/3} L_{34}$. Since the elements of L_{12} and L_{34} already agree on their $r/3$ lower bits, we are only matching bits $r/3$ to $2r/3$, so we still expect to find $2^{r/3}$ elements. Finally, we expect one of the elements of L_{1234} to be zero. This can be generalized to any m that is a power of two, using a binary tree: if $m = 2^a$, we need m lists of $2^{r/(a+1)}$ elements and the time and memory used by the algorithm is $2^a \cdot r2^{r/(a+1)}$. The algorithm for $m = 8$ is shown by Figure 2.

3 Application to the π -cipher

In order to apply this attack to the π -cipher, we need to solve the m -sum problem for the word-wise modular addition \boxplus_8 . Wagner showed to solve the generalized birthday problem with a modular addition, and his trick also works for the word-wise modular addition. More precisely, we have to modify the join operator:

$$L \bowtie_j L' = \left\{ (l \boxplus_8 l', (a, a')) \mid (l, a) \in L, (l', a') \in L', (l \boxplus_8 l')^{[0..j-1]} = 0^j \right\}.$$

Since the word-wise modular addition \boxplus_8 only has carries from the low order bits to the high order bits, when x and y have their j low-order bits set to zero, $x \boxplus_8 y$ also has j low-order bits set to zero. Moreover, the join \blacktriangleleft can still be computed efficiently. We first negate the list L and define $-L = \{(-l, a) \mid (l, a) \in L\}$, where $-l$ is the additive inverse with regard to the word-wise addition \boxplus_8 , *i.e.* $l \boxplus_8 (-l) = 0$. Then we sort $-L$ and L' according to the lower j bits, and step through the lists in parallel. When an element of $-L$ and an element of L' agree on their low bit, the corresponding sum will have its low bits equal to zero. Therefore, this variant of Wagner's algorithm is suitable for a tag second-preimage attack on the π -cipher.

We give a full description of an attack on the π -cipher with $\omega = 16$ in Algorithm 1; this attack uses 8 lists of size 2^{32} (illustrated by Figure 2), *i.e.* we consider an 8-block message, with 2^{32} possibilities for each block. This gives a complexity of 2^{35} . More generally, we can apply Wagner's attack to different versions of π -cipher (*i.e.* with different values of ω), and several trade-offs between the message length and the attack complexity are possible. We give some parameters in Table 1.

Table 1. Attack parameters

ω	Short messages			Optimal parameters		
	m	$ L $	Complexity	m	$ L $	Complexity
16	2^3	2^{32}	2^{35}	2^{11}	2^{11}	2^{22}
32	2^7	2^{32}	2^{39}	2^{16}	2^{15}	2^{31}
64	2^{15}	2^{32}	2^{47}	2^{22}	2^{23}	2^{45}

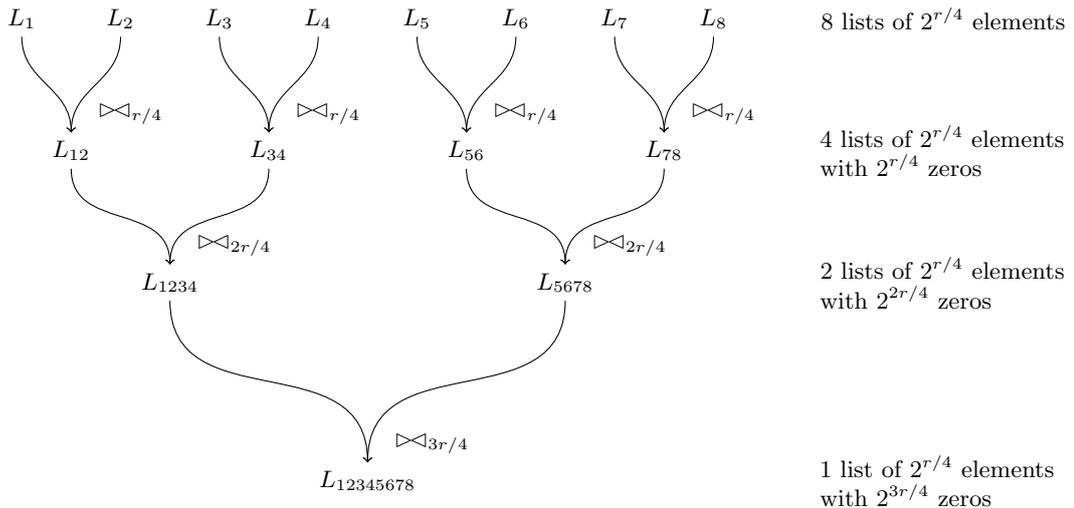


Fig. 2. Wagner's algorithm for $m = 8$

Algorithm 1 Short message attack with $\omega = 16$ and $m = 8$.

```
for  $0 \leq i < 8$  do
  for  $0 \leq j < 2^{32}$  do
     $L[i][j] \leftarrow (e(i, [j]), j)$ 
  end for
end for
 $L[8] \leftarrow \text{MERGE}(L[0], L[1], 32)$ 
 $L[9] \leftarrow \text{MERGE}(L[2], L[3], 32)$ 
 $L[10] \leftarrow \text{MERGE}(L[4], L[5], 32)$ 
 $L[11] \leftarrow \text{MERGE}(L[6], L[7], 32)$ 
 $L[12] \leftarrow \text{MERGE}(L[8], L[9], 64)$ 
 $L[13] \leftarrow \text{MERGE}(L[10], L[11], 64)$ 
 $L[14] \leftarrow \text{MERGE}(L[12], L[13], 96)$ 
for all  $(l, (((a_1, a_2), (a_3, a_4)), ((a_5, a_6), (a_7, a_8)))) \in L[14]$  do
  if  $l = 0$  then
    return  $[a_1] \parallel [a_2] \parallel [a_3] \parallel [a_4] \parallel [a_5] \parallel [a_6] \parallel [a_7] \parallel [a_8]$ 
  end if
end for

function MERGE( $L, L', \tau$ )
  SORT( $L, -\text{low}_\tau$ )
  SORT( $L', \text{low}_\tau$ )
   $i \leftarrow 0$ 
   $j \leftarrow 0$ 
  while  $i < |L|$  and  $j < |L'|$  do
     $(l, a) \leftarrow L[i]$ 
     $(l', a') \leftarrow L'[j]$ 
    if  $\text{low}_\tau(-l) < \text{low}_\tau(l')$  then
       $i \leftarrow i + 1$ 
    else if  $\text{low}_\tau(-l) > \text{low}_\tau(l')$  then
       $j \leftarrow j + 1$ 
    else
       $M \leftarrow M \cup \{(l \boxplus_8 l', (a, a'))\}$ 
    end if
  end while
  return  $M$ 
end function
```

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