

# A Local Frequency Analysis of Light Scattering and Absorption

Laurent Belcour<sup>1</sup> Kavita Bala<sup>2</sup> Cyril Soler<sup>3</sup>

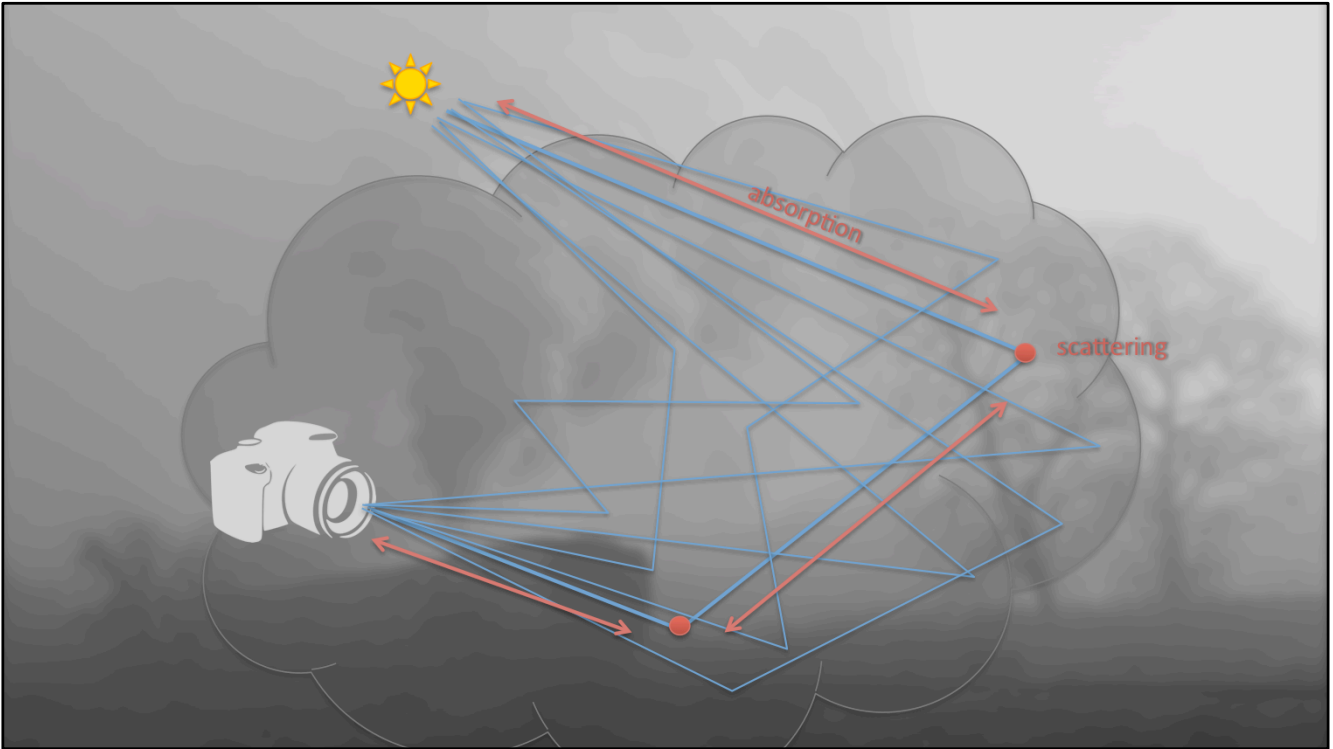
<sup>1</sup>Inria Bordeaux <sup>2</sup>Cornell University <sup>3</sup>Inria Rhône-Alpes



Participating media such as fog or clouds are very complex to render. They require switching from a surface based light transport scheme to a volumetric one.

$$\frac{dL}{dt}(\mathbf{x} + t\boldsymbol{\omega}, \boldsymbol{\omega}) = \overbrace{-\sigma_a L(\mathbf{x}, \boldsymbol{\omega})}^{\text{absorption}} + \overbrace{\sigma_s \int_{\boldsymbol{\omega}'} \rho(\boldsymbol{\omega}, \boldsymbol{\omega}') L(\mathbf{x}, \boldsymbol{\omega}') d\boldsymbol{\omega}'}^{\text{scattering}}$$

Light interaction with a participating medium is described by the radiative transfer equation. This equation tells us how much light passes through an infinitesimal portion of volume in a given direction. \*click\* It has a differential form because it tells us how light changes after traveling through the portion of volume. \*click\* The amount of the light stopped by the medium from travelling in the current direction is described by the absorption term. \*click\* and how much light coming is scattered into the current direction from other directions is described by the scattering term.



Ray tracing is used In modern off-line rendering engines to render such media thanks to its simplicity. \*click\* Here, the radiative transfer equation is stochastically solved by sampling the attenuation first and the scattering second to generate light paths. \*click\* Because of the complexity of the equation, generating a picture requires averaging an enormous amount of those light paths.



And it is so frustrating and wasteful to have to generate this quantity of light paths when the resulting effect is smooth like here in the picture.



But on the other side, density variations in the volume can create very subtle effects that we want to render accurately.

“Do it where it matters, no kidding.”

*C. Shannon*

Having to compute so much for sometimes so little goes against the ideas developed in signal processing. Computation should happen where the signal has complex behavior. And should be saved where the final function is smooth.

# Previous Work

- Adaptive rendering
  - [Engelhardt and Dachsbacher 2010]
  - [Baran *et al.* 2010; Chen *et al.* 2011]
- Frequency analysis
  - [Durand *et al.* 2005]
  - [Belcour *et al.* 2013]

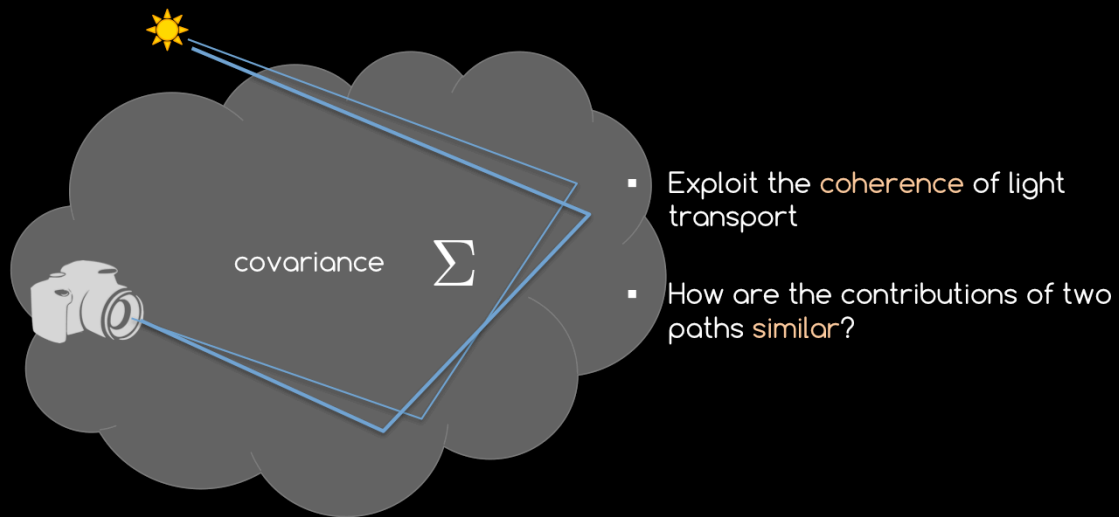
Participating media ?



Some previous work try to adapt the computation to regions where it is needed \*click\* such as the adaptive epipolar works, but they are restricted to single scattering. \*click\* Frequency analysis methods following the one of Durand and colleagues can predict where the computation effort should be, \*click\* last year, we showed how this analysis could be made both generic and practical by studying covariance information. \*click\* But the question of participating media has been left open.



# Frequency Analysis

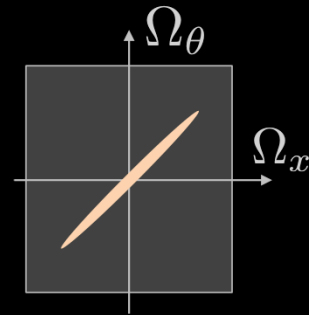


Our main claim is that we should exploit the coherence in light transport simulation. \*click\* To do so, we need to estimate how much elements that carry radiance are related. How much two pixels are related, how much two paths are related. \*click\* To determine this similarity, we build on covariance tracing which uses the covariance of the Fourier transform of the radiance as a measure of the similarity.

# Covariance Tracing

- Tracks **statistics** of radiance's Fourier spectrum
  - Second-order statistics: **covariance**
  - **Simple** to implement: 5x5 matrices

$$\Sigma_{i,j} = \int \Omega_i \Omega_j \mathcal{F}[L](\Omega) d\Omega$$



Covariance tracing is a practical method to track second order statistics of the radiance's Fourier transform during light path construction. Covariance is a measure of the spectrum's spread and orientation with respect to tangential variations in space and angle, here denoted  $\Omega_x$  and  $\Omega_\theta$ . It is lightweight and simple to implement as it only requires tracking a symmetric 5x5 matrix during light-path construction.

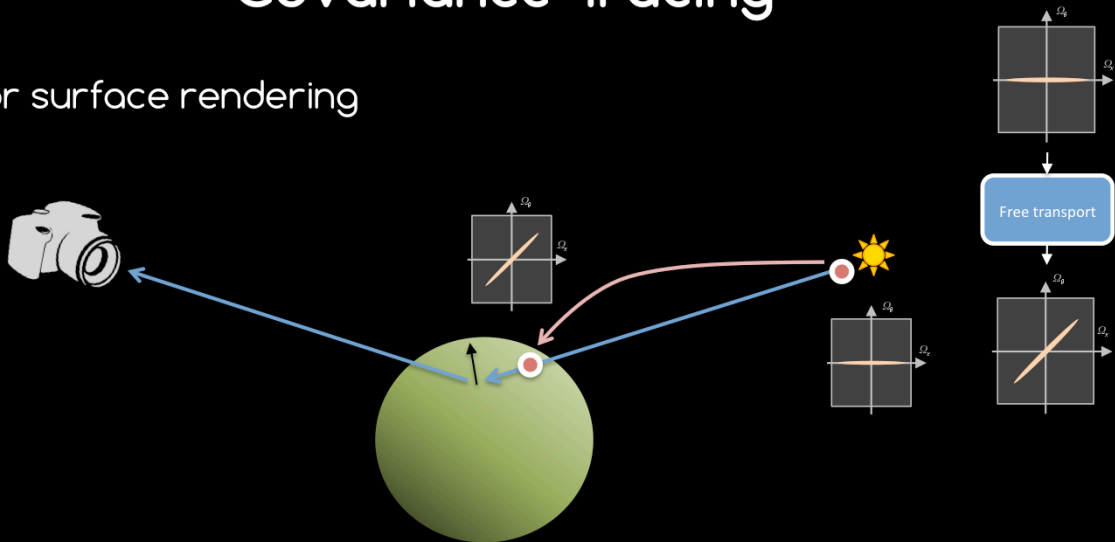
# Covariance Tracing

- Tracks **statistics** of radiance's Fourier spectrum
  - Second-order statistics: **covariance**
  - **Simple** to implement: 5x5 matrices
- Compatible with **Global Illumination**
  - Atomic operators, easily extensible

A good property of covariance tracing is that it is fully compatible with global illumination. Any light path can be analyzed by decomposing it into a set of atomic operators. New effects can be added with the addition of their respective operator.

# Covariance Tracing

- For surface rendering

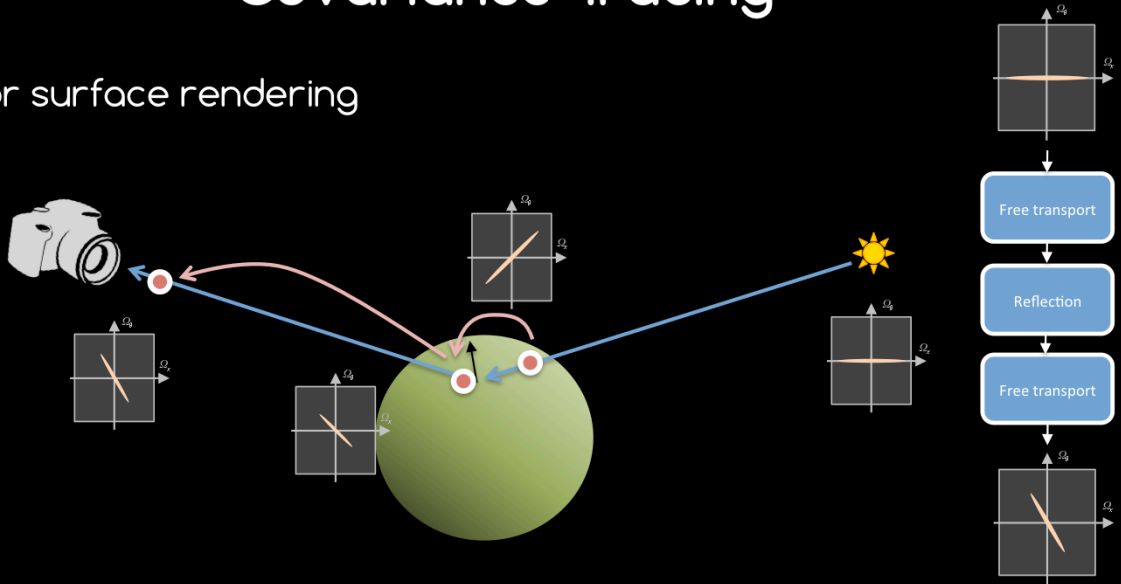


Now I will through a simple example to give intuition.

The propagation of covariance information is done by concatenating operators corresponding to a given light path until the sensor. \*click\* we start from the covariance of the spectrum at the light source. For the case of a point light source, the spectrum has no frequency in the angular domain since the light emits uniformly in any direction, but has an infinite spread in space since the light is a spatial dirac. \*click\* Then we propagate this information along the light path with transport first \*click

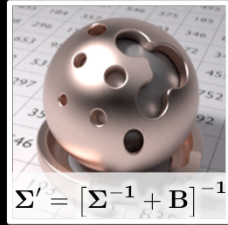
# Covariance Tracing

- For surface rendering



Then any interaction like reflection is treated. \*click\* Then, we continue to build the covariance until the sensor.

# Covariance Tracing



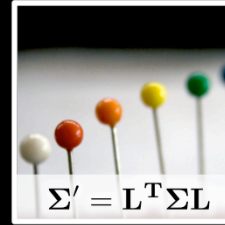
$$\Sigma' = [\Sigma^{-1} + \mathbf{B}]^{-1}$$

reflection



$$\Sigma' = \Sigma + \mathbf{O}$$

shadows



$$\Sigma' = \mathbf{L}^T \Sigma \mathbf{L}$$

lens



$$\Sigma' = \mathbf{T}^T \Sigma \mathbf{T}$$

motion



absorption

?



scattering

Various types of light paths can be analyzed using atomic operators of light transport effects such as \*click\* reflection, \*click\* shadows, \*click\* lens, \*click\* motion. \*click\* Clearly missing are the effects of absorption and scattering.

# Contributions

- Novel frequency analysis
- Derived covariance operators
- Broad application

To overcome this gap, we introduce a novel frequency analysis to meet Shannon's goal in the computation of light interaction with a medium. This analysis is made practical by deriving easy-to-compute matrix representations of the frequency spectrum operators. To show that our method can be applied broadly, we demonstrate three application scenarios.

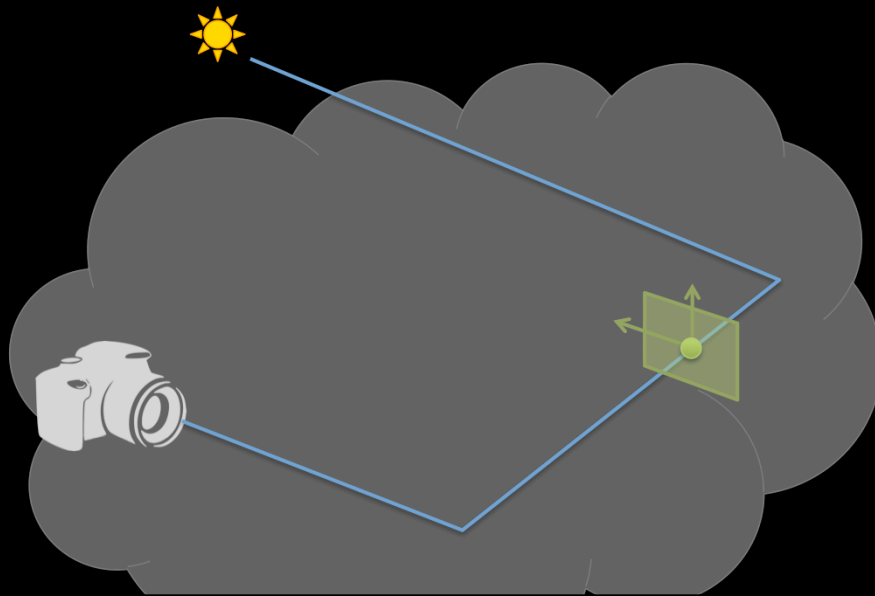
# Contributions

- Novel frequency analysis
- Derived covariance operators
- Broad application

First lets have a look on our theory.

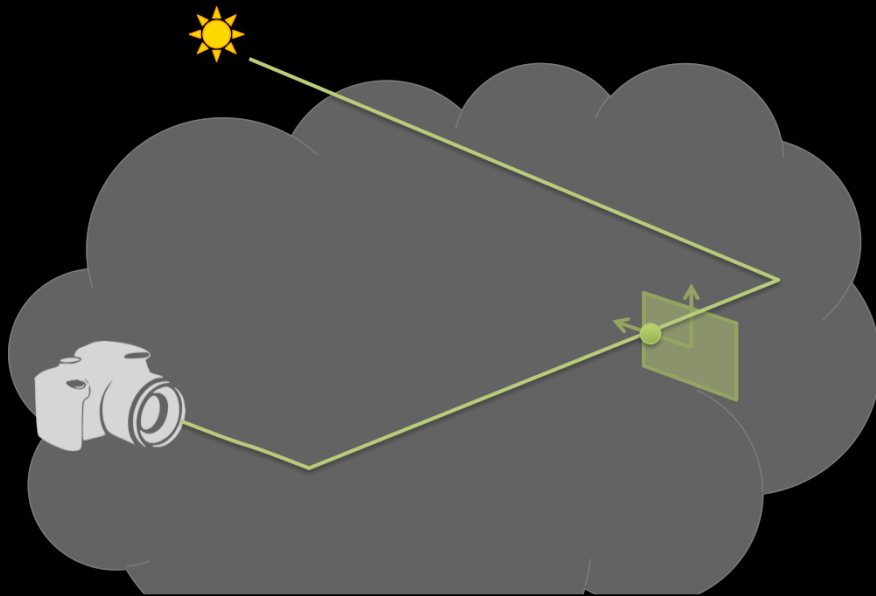


# Frequency Analysis



Our goal is to analyze how much is radiance changing when a light-path is perturbed. \*click\* For any point on the light path, \*click\* and for any perturbation in its tangent plane. \*click\*

# Frequency Analysis



.. we need to predict how much the signal would vary.

# Frequency Analysis

- Formulate local radiance

$$\frac{dL}{dt}(\mathbf{x} + t\boldsymbol{\omega}, \boldsymbol{\omega}) = -\sigma_a L(\mathbf{x}, \boldsymbol{\omega}) + \sigma_s \int_{\boldsymbol{\omega}'} \rho(\boldsymbol{\omega}, \boldsymbol{\omega}') L(\mathbf{x}, \boldsymbol{\omega}') d\boldsymbol{\omega}'$$

↓ ?

$$\mathcal{F}[L]$$

This needs to be inferred from the radiative transfer equation. \*click\* Now the question is to express this equation in the Fourier domain. The main issue \*click\* is that the RTE is a differential equation, and \*click\* we need to have it defined for any perturbation of its input spatial and angular parameters.

# First-order Analysis

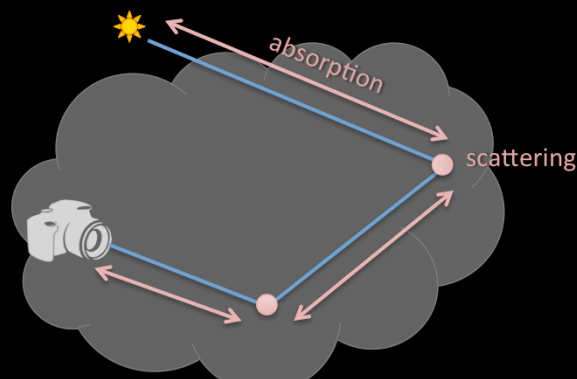
- Formulate local radiance
- Use a first-order expansion

$$L(\mathbf{x} + \Delta t \boldsymbol{\omega}, \boldsymbol{\omega}) = \underbrace{(1 - \Delta t \sigma_a) L(\mathbf{x}, \boldsymbol{\omega})}_{\text{attenuation}} + \underbrace{\Delta t \sigma_s \int_{\boldsymbol{\omega}'} \rho(\boldsymbol{\omega}, \boldsymbol{\omega}') L(\mathbf{x}, \boldsymbol{\omega}') d\boldsymbol{\omega}'}_{\text{scattering}}$$

We perform a first-order expansion of the RTE to formulate the radiance in the Fourier domain. This first-order expression can still be expressed as \*click\* attenuation and \*click\* scattering. To continue our analysis, we need to separate them.

# First-order Analysis

- Formulate local radiance
- Use a first-order expansion
- Absorption and scattering treated separately



Fortunately, in the Monte Carlo formulation used nowadays, absorption and scattering are treated separately on a light path. Absorption affects edges while scattering affects vertices.

# First-order Analysis

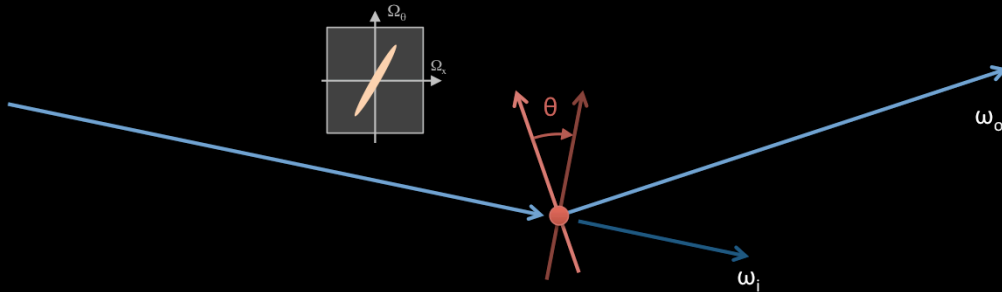
- Formulate local radiance
- Use a first-order expansion
- Absorption and scattering treated separately

$$L(\mathbf{x} + \Delta t \boldsymbol{\omega}, \boldsymbol{\omega}) = \Delta t \sigma_s \int_{\boldsymbol{\omega}'} \rho(\boldsymbol{\omega}, \boldsymbol{\omega}') L(\mathbf{x}, \boldsymbol{\omega}') d\boldsymbol{\omega}'$$

scattering selected by Russian Roulette

Remember that we analyze already sampled light path, so we can treat absorption or scattering as it has been selected by a russian roulette whether we are on an edge or a vertex. I will not go into the details of the absorption equation, please refer to the paper for this part.

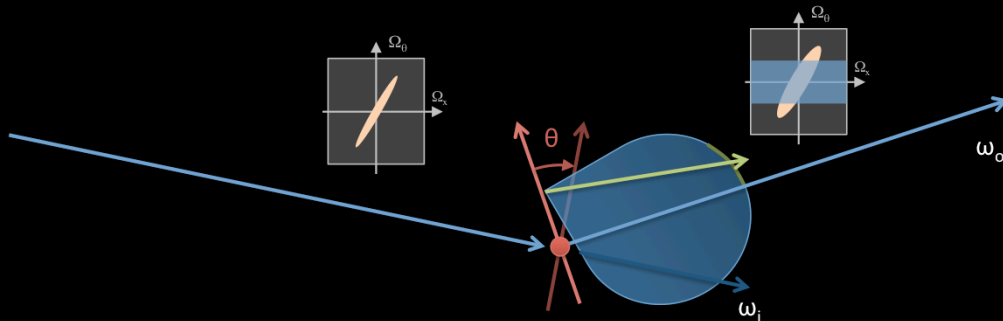
# Scattering



$$\mathcal{F}[L_{\text{in}}](\Omega_x, \Omega_u)$$

We analyze the transformation of the input light field Fourier transform \*click\* before a scattering event. We first \*click\* need to project the input spatial coordinate system to the outgoing one with respect to the scattering angle theta.

# Scattering



small angle assumption → scattering has a convolution form

$$\mathcal{F}[L_{\text{out}}](\Omega_x, \Omega_u) = \mathcal{F}[L_{\text{in}}]\left(\frac{\Omega_x}{\cos(\theta)}, \Omega_u\right) \mathcal{F}[\rho](\Omega_u)$$

This introduces a spatial scaling in the Fourier domain. \*click\* Then, we apply the scattering integral. We show that under a small angle assumption, \*click\* isotropic scattering has a convolution form over the projected local light field.

\*click\* This translates in the Fourier domain to a product of the input light field spectrum with the phase function spectrum. \*click\* That will act as an angular low pass filter.

\*click\* This final equation describes the outgoing radiance Fourier transform.



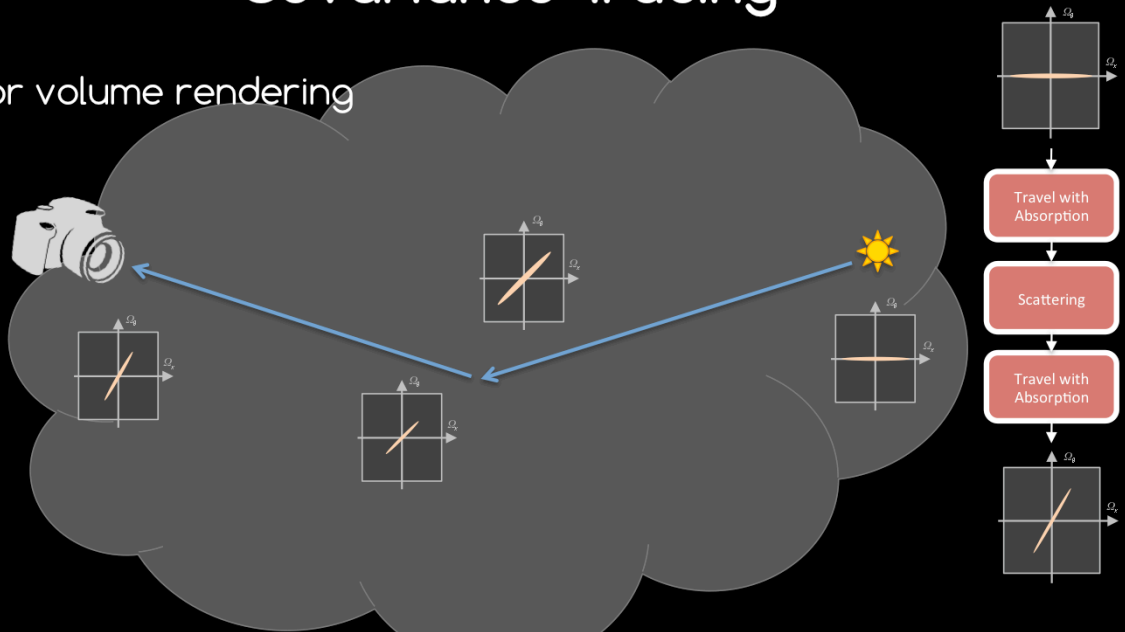
# Contributions

- Novel frequency analysis
- Derived covariance operators
- Broad application

Having this analysis is a good thing, but we need a way to make it practical. For that we rely on covariance tracing and provide the covariance operators that will be used during light-path construction.

# Covariance Tracing

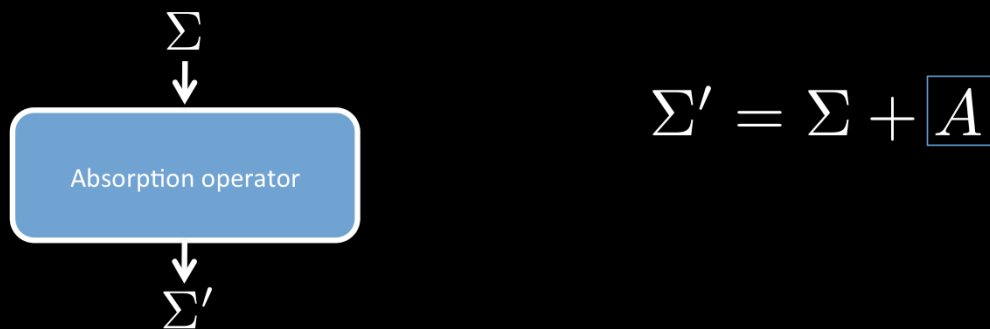
- For volume rendering



Our method provides covariance operators when a participating media is present. \*click\* We start again from the light. But instead of simple free transport, we need to account for the absorption along the edge of the path while we perform the sheared transport, \*click\* and the scattering operator at the vertices. \*click\* Each updates the covariance. By combining the two we can perform frequency analysis on any light path.

# Absorption Operator

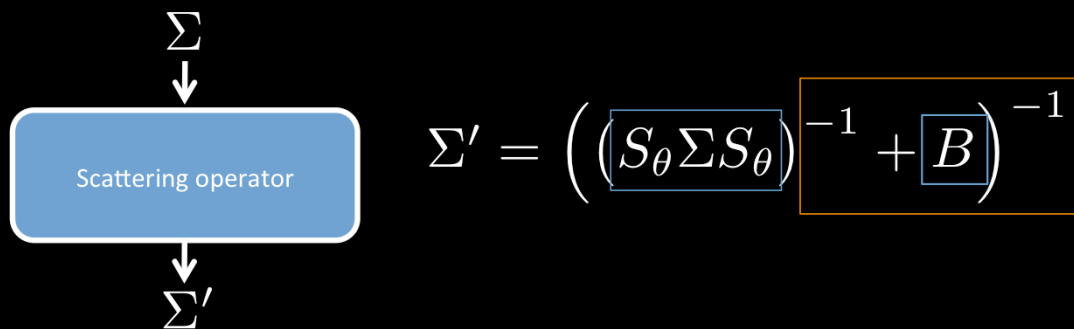
- Density variations **increase** spatial frequencies
  - Same operator as occlusion
  - Requires the density Hessian



In homogeneous media, the absorption behaves like free transport. BUT when the density varies, spatial frequencies increase. We model it using a generalization to the occlusion operator from the surface based covariance tracing. \*click\* Instead of an infinite spectrum, the occlusion spread accounts for the density hessian. The more highly varying in the medium is, the larger the bandwidth will be.

# Scattering Operator

- Scattering **decreases** angular frequencies
  - Same operator as BRDF low-pass filter and scaling
  - Requires the phase function's Hessian matrix
  - Analytical form for Henyey-Greenstein



The other operator, scattering, is applied when there is a change of direction in the light path and accounts for the integration with the phase function.

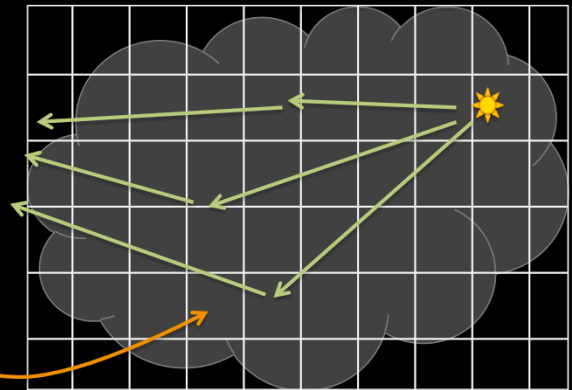
\*click\* It has the same form as the BRDF operator in the surface based covariance tracing. This operator decomposes into \*click\* spatial scaling to account for frame projection and \*click\* low pass filtering with a double inversion and sum. The bandlimiting matrix is the phase function Hessian matrix  $B$ .

\*click\* We provide an analytical form for the commonly used Henyey-Greenstein phase function's Hessian in the paper.

# Covariance Reuse

- Covariance tracing is **costly**
  - Not all paths should carry it
  - Need to **reuse** covariance paths
- **Store** covariance in a grid
  - lightweight representation

$\Sigma$



However, covariance is too costly to compute for all light paths. This is due to the local occlusion computation. To distribute the cost of analysis, *\*click\** we choose to compute covariance information on a subset of all the paths generated. In our example, we use as much as 10 percent. *\*click\** We then reuse this covariance information to improve the integration of all the other light paths. *\*click\**

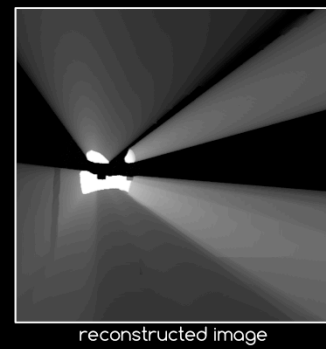
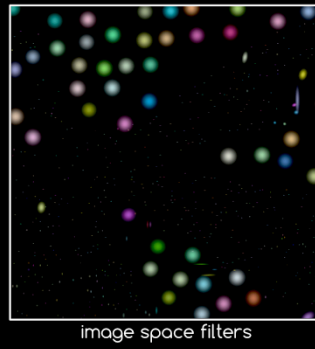
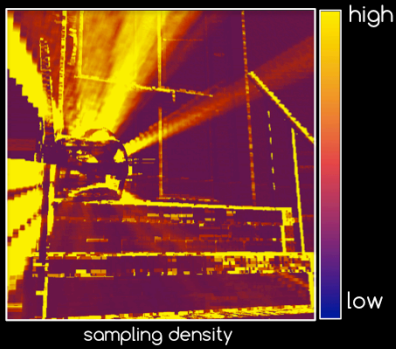
We store in a voxel grid the covariance of the fluence, derived from the covariance carried by paths. This provides a lightweight storage per voxel as only 3X3 matrices are required and provide reasonable frequency information.

# Contributions

- Novel frequency analysis
- Derived covariance operators
- Broad application

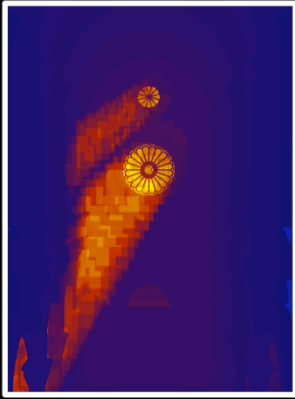
Now we will tell you how our approach can speed up many application in volume rendering.

# 1 – Image Space Sampling & Reconstruction



The first application is to adapt image space sampling. We use the covariance information to sample more sparsely and build image space filters to accelerate image rendering.

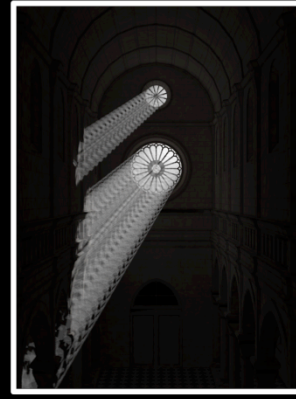
## 2 – Single Scattering Integration



Our method, amount of samples along eye ray



Our method



Uniform sampling, equal time

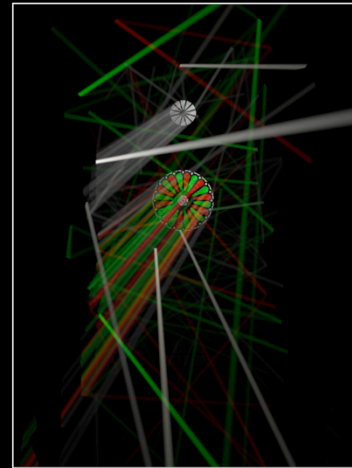
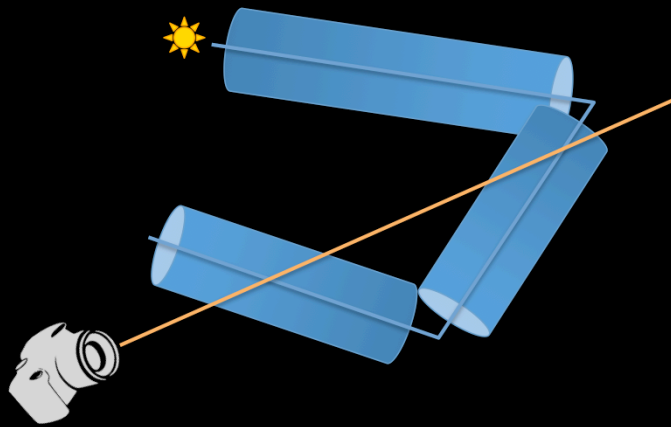
The next application is to accelerate single scattering by adaptively sampling scattering positions using our covariance information. This method proved useful to efficiently sample light shafts.



## 3 – Adaptive Density Estimation

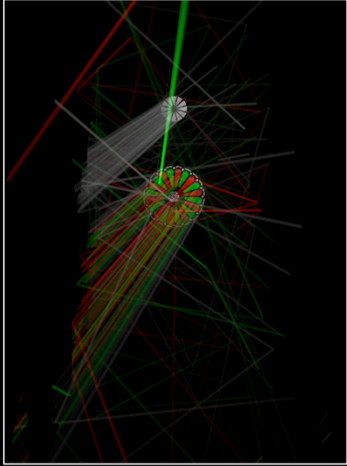
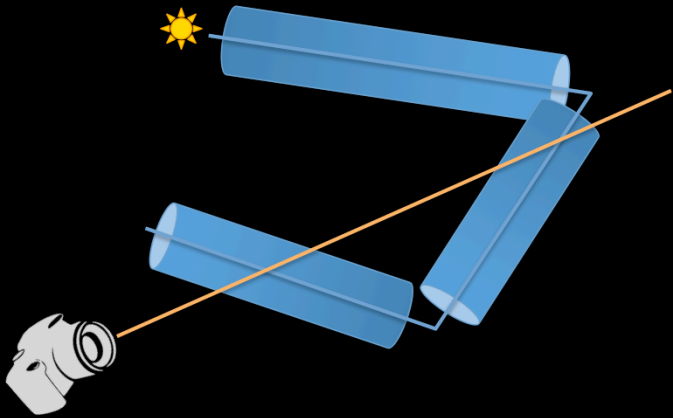
Finally, we provide an improved density estimation method that uses frequency analysis to drive the kernel bandwidth. I will describe this later method in details, please refer to our paper for the complete description of the two other mentioned methods.

## Progressive Photon Beams [Jarosz *et al.* 2011]



To efficiently perform density estimation in participating media, Jarosz and colleagues developed the photon beams estimate. The idea is to perform density estimation on path edges instead of vertices.

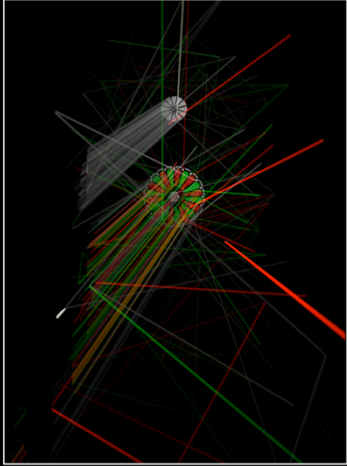
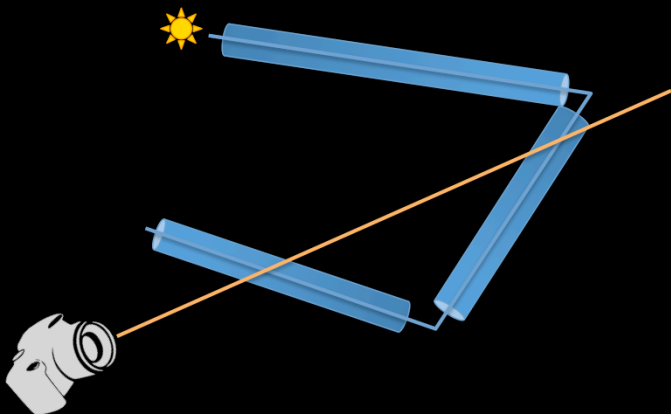
# Progressive Photon Beams [Jarosz et al. 2011]



pass 2

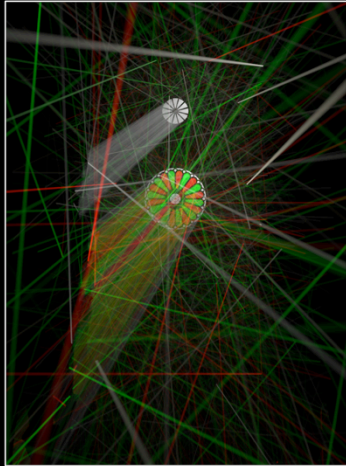
And to reduce the kernel bandwidth over time to ensure convergence.

# Progressive Photon Beams [Jarosz et al. 2011]



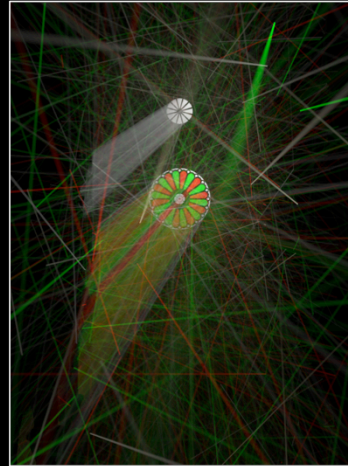
pass 3

## Progressive Photon Beams [Jarosz *et al.* 2011]



Average after 3 passes

...

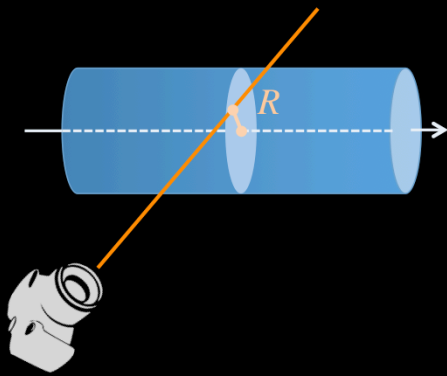


Average after 100 passes

While this method is very efficient to the estimation of specular paths from the light. It is not so efficient to estimate high order scattering events.

# Adaptive Progressive Photon Beams

- Density estimation introduces bias
  - Decrease with radius reduction
  - Proportional to the radiance's Laplacian



$$\text{bias} = \frac{1}{2} R a \Delta L$$

Density estimation introduce bias which decrease with the radius reduction. \*click\* To reduce this bias, the progressive algorithm must control the radius reduction to overcome the local variations of the integrand. \*click\* An approximation of the bias show that it is proportional to the Laplacian of the integrand.

# Adaptive Progressive Photon Beams

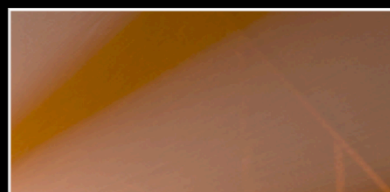
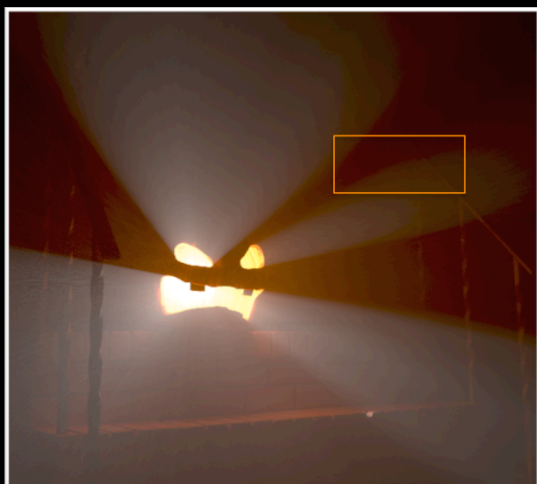
- Density estimation has **variance**
  - Increase with radius reduction
- Need to **balance** radius reduction
  - Reduce variance by increasing the radius
  - Controlling bias thanks to covariance

But but reducing radius too quickly increases the variance of the estimator. \*click\*

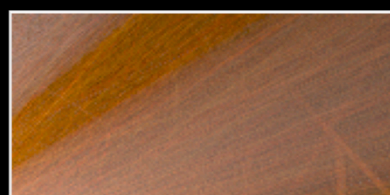
Thus PPB needs to balance the tradeoff between bias and variance and that prevents us from reducing the radius too aggressively. But using our covariance analysis, we can estimate the Laplacian. That means we can adaptively control the radius reduction as needed.

For example, low varying regions can use a larger radius to aggregate more paths and decrease the variance while introducing a controlled bias.

# The Pumpkin Scene



Ours, 25min, 9.7M beams

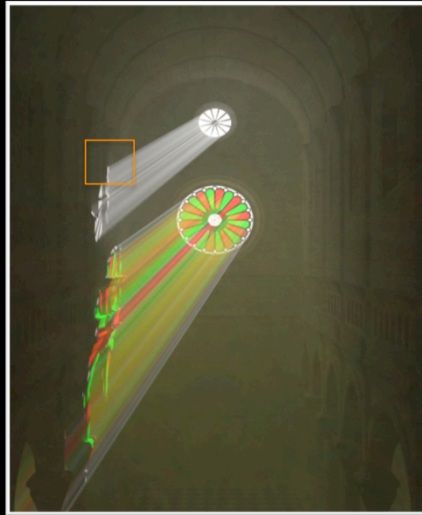


PPB, 25min, 10M beams

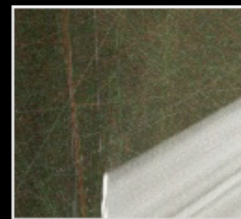
We demonstrate our method in this scene with a carved Pumpkin. \*click\* An equal time comparison between our improved progressive photon beam method and the one of Jarosz and colleagues shows that ours better reconstruct the diffuse component.



# The Sibenik Scene



Ours, 2h 35min,  
20M beams



PPB, 2h 35min,  
20.5M beams

Same for the Sibenik cathedral. Note that ours provide a smoother multiple scattering term, while preserving shafts.

## Limitations / Future Work

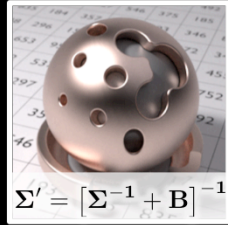
- Our FPPB is **not consistent**
  - Change for APPM style [Kaplanyan & Daschbacher 2012]
- Apply covariance analysis to **modern estimators**
  - VCM++ [Georgiev *et al.* 2014], VBL [Novak *et al.* 2012]



Many improvements are still possible. Our FPPB is not consistent. Consistency can be achieved by extending the radius reduction technique of Kaplanyan and Daschbacher to use our prediction of the Laplacian.

\*click\* We would like to also apply covariance tracing to modern estimators such as the volumetric vertex merging algorithm of Georgiev and colleagues or the virtual beam light of Novak and colleagues.

# Covariance Tracing Framework



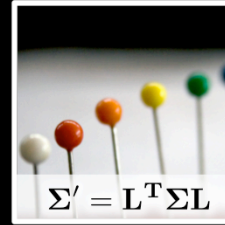
$$\Sigma' = [\Sigma^{-1} + B]^{-1}$$

reflection



$$\Sigma' = \Sigma + O$$

shadows



$$\Sigma' = L^T \Sigma L$$

lens



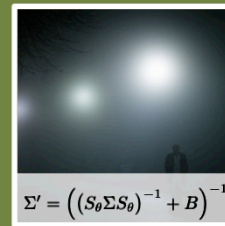
$$\Sigma' = T^T \Sigma T$$

motion



$$\Sigma' = \Sigma + A$$

absorption



$$\Sigma' = (S_\theta \Sigma S_\theta^{-1} + B)^{-1}$$

scattering

In a longer term, our goal is to build a complete framework to predict the frequency content for any of the effects in physically based rendering. In this talk we showed both absorption and scattering operators, \*click\* However, \*click\*

# Covariance Tracing Framework



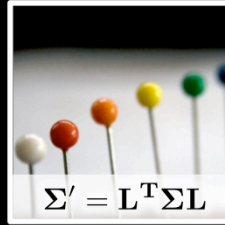
$$\Sigma' = [\Sigma^{-1} + B]^{-1}$$

reflection



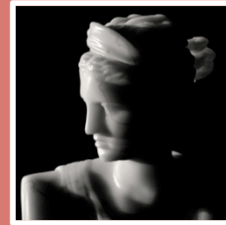
$$\Sigma' = \Sigma + O$$

shadows



$$\Sigma' = L^T \Sigma L$$

lens



subsurface scattering



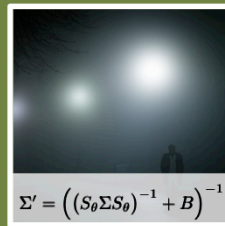
$$\Sigma' = T^T \Sigma T$$

motion



$$\Sigma' = \Sigma + A$$

absorption



$$\Sigma' = (S_\theta \Sigma S_\theta^{-1} + B)^{-1}$$

scattering

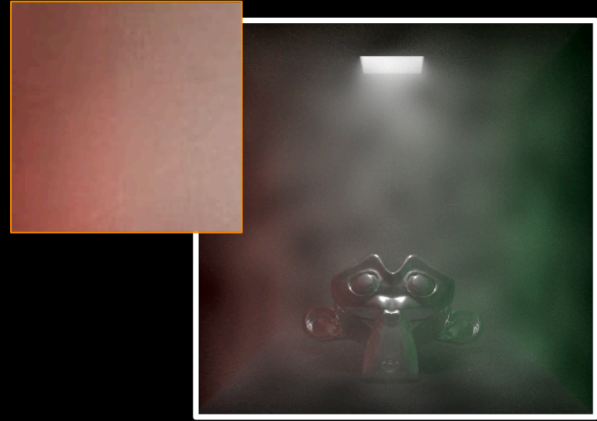


anisotropic media

some effects are missing such as subsurface scattering models or more complex media such as the micro-flakes model of Jakob and colleagues. Those are interesting avenues.

# Take Home Message

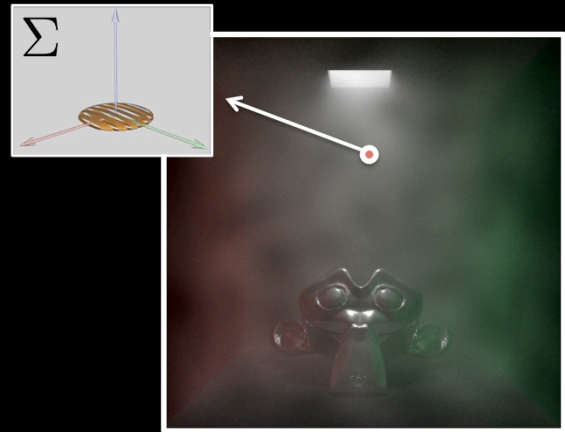
- Efficient noise reduction
  - Know what you are doing
  - Know where to do it



Efficient noise reduction is possible when you \*click\* know what to do and where to do it \*click\*

# Take Home Message

- Efficient noise reduction
  - Know what you are doing
  - Know where to do it
- Our method: **predict correlation**



We presented covariance tracing for participating media with those ideas in mind. Our method predict the local correlation of light's distribution to improve participating media rendering.



Thank you for your attention.

Thank you for your attention.

**Acknowledgments**

ANR 'ALTA'

Explora'Doc funding